

# Probabilistic assessment of the sag in an overhead transmission line

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**Abstract** -- The sag in an overhead transmission line is very much influenced by the current in the conductor. Until recently, the current carrying capacity of a conductor has been determined by the deterministic approach. However, the probabilistic approach has become popular in recent times, and it is being applied in designing insulation, mechanical components, general electrical apparatus, etc. A new method, based on the probabilistic technique, is used to assess the sag in an overhead line in this paper. This is to avail another method, which looks more promising than the deterministic approach, and can provide utility companies as well as analysts a basis for comparison.

Recorded data on some climatic factors and transmission line current are used in our analyses.

**Keywords:** Sag, conductor clearance, probabilistic approach, conductor temperature and climatic factors.

## 1. Introduction

Power utility companies are required to transmit more power to meet the growing demand. The capacities of existing lines need to be upgraded in order to carry out this task. Upgrading existing line's voltage results in some technical problems. And constructing new transmission lines seems difficult from cost and environmental points of view. It is thus, very necessary to investigate the possibility of increasing the current carrying capacities of the existing transmission lines.

Stability, voltage and other factors need to be considered in the determination of the current carrying capacity of a conductor. However, the current carrying capacity of a conductor is principally militated by its thermal limits. For instance, sag, loss in tensile strength, the degradation at the conductor joints, compression clamps, etc. are all thermal limits.

In this paper, the authors focus on the sag in the line since a similar probabilistic assessment has been carried out on the reduction of tensile strength in Ref. 1.

Sag depends on the conductor temperature, which also depends on climatic factors as well as the line current. All these factors are stochastic variables and thus the probabilistic approach is used for the analyses. The thermal analysis is carried out on the conductor to obtain data on its temperature. This information is then used to assess the sag in the line.

## 2. The probabilistic approach

Climatic factors are all probabilistic variables and, in our view, should be treated as such in analyzing the thermal behavior of a conductor. With the deterministic approach, some specific values of ambient conditions are assumed, with very little or no reference to statistical data, in setting capacity limits for a conductor<sup>(2,3)</sup>. The method is quick and simple<sup>(3)</sup>, but is typically conservative, re-

sulting in underutilization of conductors<sup>(4,5)</sup>.

The probabilistic approach comprises *static* and *dynamic* methods<sup>(6)</sup>. This approach makes due reference to statistical data, by either using the raw data or further treating it to suit its application. For this reason, it has a very high tendency of reflecting the real situation since the prevailing conditions are well incorporated into the analysis. The probabilistic approach as used in this study employs recorded climatic and current data or data based on these to determine the conductor temperature.

In the static method, the probability distributions derived from the available climatic and current data are first obtained. These probability distributions are assumed to be independent of each other; and for each of the possible combinations of these elements in these probability distributions, the conductor temperature and its probability of occurring can be estimated. The time sustained by a given conductor temperature is estimated from the ratio of the occurrence probability of that temperature to the sum of the occurrence probabilities of all temperatures. With the temperature and its sustained time, the probability distribution of the sag can be obtained. The method gives the probability distribution of sag for a long term depending on the duration of the data, say one year, if an annual data is used.

On the contrary, the dynamic method uses the data as recorded for its assessment. A conductor temperature at a given time is calculated by using climatic and current data recorded at the same time and is thus expected to be the more accurate than the static method. This is quite similar to real time assessment of the index. However, acquiring instantaneous data for such assessment can be very difficult at times.

## 3. Climatic/current data

Ambient temperature, wind velocity, wind direction and global solar radiation have significant influence on con-

ductor thermal behavior. Consequently, data on these factors are used in the study. Data on ambient temperature and the wind velocity as used for the study are presented in Figs.1 and 2 respectively.

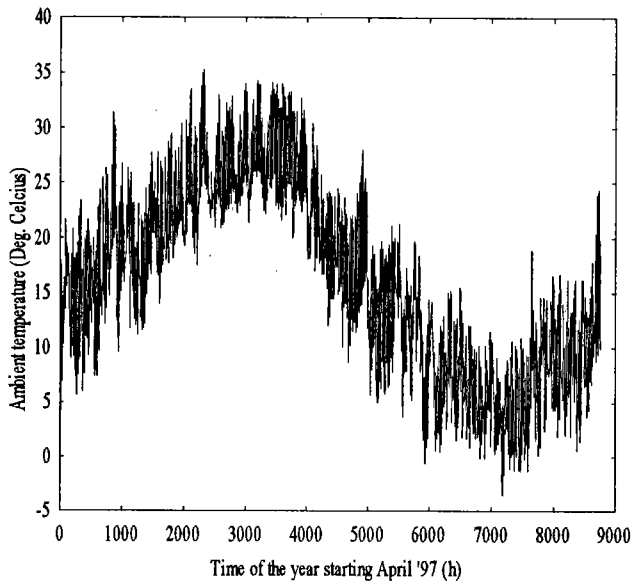


Fig.1: Ambient temperature - time variation

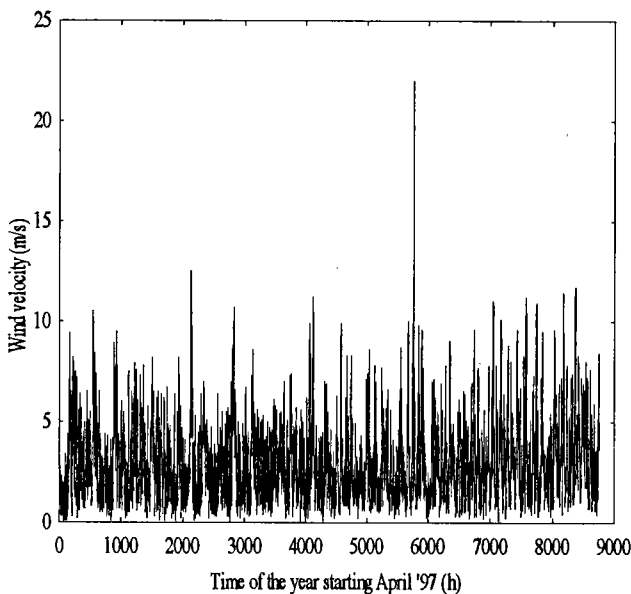


Fig.2: Wind velocity - time variation

These are hourly-recorded data, from April 1997 to March 1998, and were obtained from Nagoya Weather Observatory Station, Japan.

The data obtained on wind direction required some modification before it could be applied in the study. This is because it is the angle of attack, not the wind direction, which is used in the analyses. This angle of attack is the angle subtended between the wind direction and the conductor axis. And this can be estimated from the data on wind direction if the plane of the transmission line is known.

Chubu Electric Power Company in Nagoya also provided us with the hourly-recorded current data for a specified transmission line for the mentioned period, April 1997 to March 1998. The current data as used in this study is shown in Fig.3. To broaden the scope of the investigation, factors of 1.0 (actual data) and 2.0 are applied on the recorded current data and used for the study.

A 410mm<sup>2</sup> ACSR conductor whose characteristics lie between "Rail" and "Condor" is used on this line.

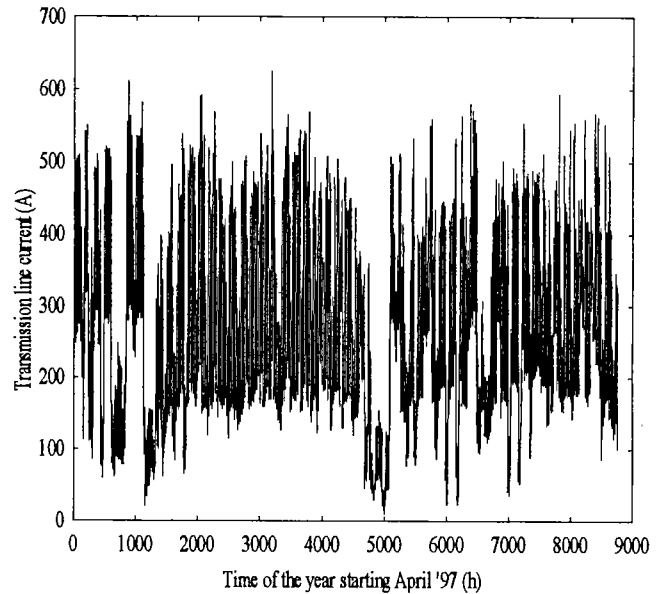


Fig.3: Transmission line current - time variation

#### 4. Thermal analyses of the conductor

Our thermal analyses are based on the CIGRE method<sup>(7)</sup>. The conductor temperature is obtained by solving Eq.1 with the acquired data substituted in the components of Eq.1.

$$P_j + P_s = P_c + P_r \quad (1)$$

$$P_j = I_{dc}^2 R_{dc} [1 + \alpha(T_c - 20)] \quad (2)$$

$$P_s = \alpha_s S D \quad (3)$$

$$P_c = \pi \lambda_f (T_c - T) \cdot Nu \quad (4)$$

$$P_r = \pi D \epsilon \sigma_B [(T_c + 273)^4 - (T + 273)^4] \quad (5)$$

$P_j$  and  $P_s$  are the Joule heating and solar heating respectively with  $P_c$  and  $P_r$  as the convective cooling and radiative cooling respectively. These components have thoroughly been discussed in Ref. 7.

Fig.4 shows the hourly conductor temperature variation for a period of one year, and was obtained from the dynamic analysis with the current and climatic data as recorded. Fig.5 also shows the conductor temperature distributions for the dynamic method for the same duration. The nomenclature "cur1.0" and "cur2.0" used in Fig.5 refer to conductor temperature distributions obtained with a factor of 1.0 and 2.0 respectively applied on the obtained transmission line current. Similar temperature distributions are obtained for the static method.

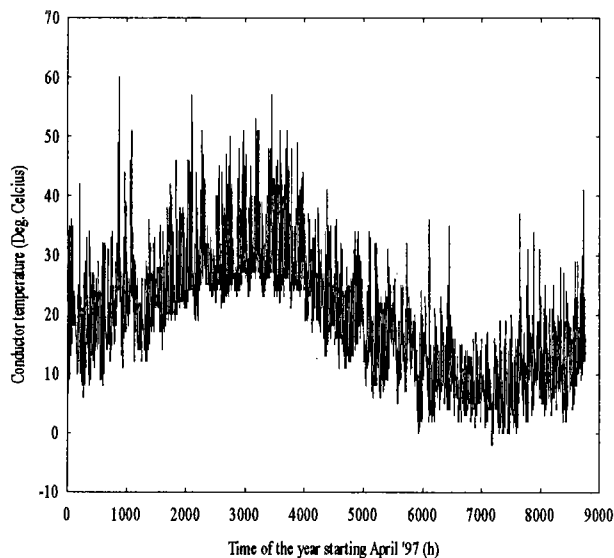


Fig. 4: Conductor temperature - time variation - Dynamic

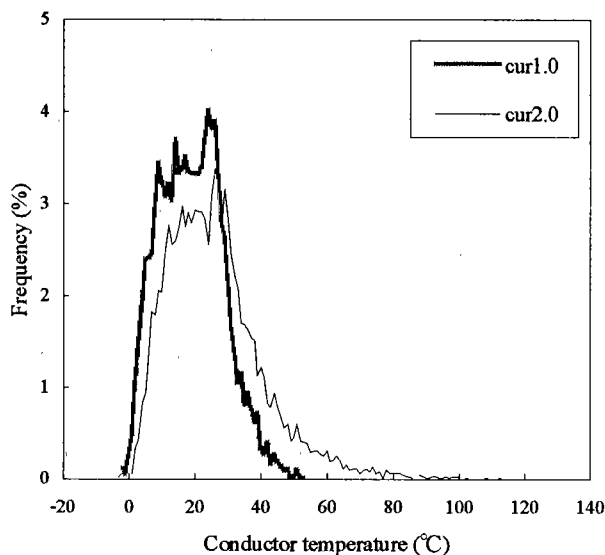


Fig. 5: Conductor temperature distribution - Dynamic

### 5. Definition of used symbols

The following symbols as used in the equations for the assessment of the line's sag are defined below with their units in the square brackets.

- $T_c$  [°C]: conductor temperature
- $\alpha_e$  [1/°C]: temperature coefficient of elongation
- $A$  [m<sup>2</sup>]: total conductor area
- $d$  [m]: conductor sag
- $E$  [kg/m<sup>2</sup>]: conductor's modulus of elasticity
- $H_o$  [kg]: initial horizontal tension in the conductor
- $H$  [kg]: horizontal tension in the conductor
- $T_o$  [°C]: initial conductor temperature
- $l_o$  [m]: initial conductor length in the span
- $l$  [m]: conductor length in the span
- $w$  [kg/m]: total weight of conductor per unit length
- $\xi$  [m]: change in conductor length due to variations in horizontal tension and conductor temperature
- $\Delta l$  [m]: change in conductor length

$x$  [m]: horizontal distance from tower to the mid-span

### 6. Sag and its calculation

Sag is an important index that should be taken into consideration in designing overhead transmission lines. Because of this, there is legislation on conductor clearance in almost every country. For instance, some minimum clearances used in Japan and the United States of America are listed in references 8 and 9 respectively.

The sag in an overhead line determines the conductor to ground clearance as shown in Fig.6. The conductor is supported by the structures A and B in Fig.6 at  $h$  meters above the ground. The clearance,  $c$ , is the difference between the height of the point of conductor support and the sag,  $d$ . The point of conductor support,  $h$ , for this transmission line is 27m above the ground.

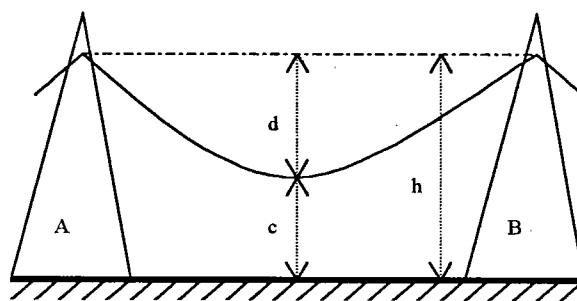


Fig. 6: A typical span of a transmission line

The assumptions used in the *ruling span method* <sup>(10)-(13)</sup> are employed. In addition, a span of 270m with an initial horizontal tension and temperature of 2837kg and 15°C respectively are used for these calculations. Wind and ice loading effects are not factored into the study. The catenary method is used because a conductor suspended between two supports conforms more to a catenary than a parabola <sup>(13)</sup>. The length of the conductor in a span and the sag are given in Eqs.6 and 7 respectively.

$$l = 2 \frac{H}{w} \cdot \sinh(wx/H) \quad (6)$$

$$d = \frac{H}{w} (\cosh(wx/H) - 1) \quad (7)$$

The change in the conductor length is expressed in Eq.8. This is caused by the variations in the horizontal tension in the line as well as the changes in the conductor temperature, and it is expressed in Eq.9. The two changes are the same <sup>(14)</sup> and this is expressed in Eq.10. The tension in the span is solved from Eq.10 using numerical computation method. The determined tension is then used to assess the conductor's sag at that temperature using Eq.7.

$$\Delta l = 2 \frac{H}{w} \cdot \sinh(wx/H) - l_o \quad (8)$$

$$\xi = l_o \left\{ \frac{H - H_o}{AE} + \alpha_e (T_c - T_o) \right\} \quad (9)$$

$$\frac{H - H_o}{AE} + \alpha_e(T_c - T_o) = \frac{2 \frac{H}{w} \cdot \sinh(wx/H) - l_o}{l_o} \quad (10)$$

By repeatedly solving (10), and using the determined tension to estimate the sag in the line, the sag and its distribution for the period of the study can be determined. These are shown in Figs.7 and 8 for the dynamic method.

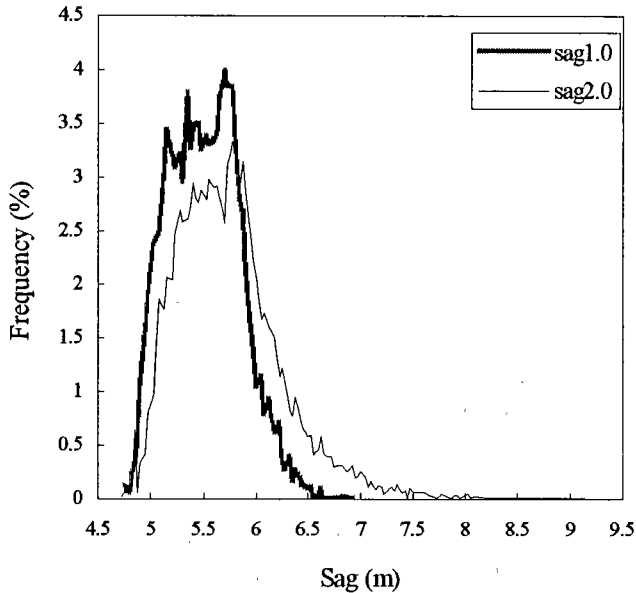


Fig.7: Sag distribution --Dynamic

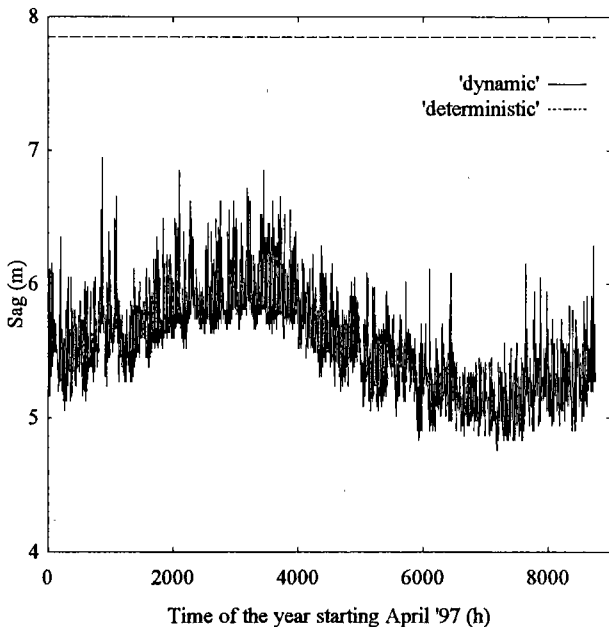


Fig.8: Sag - time variation -- Dynamic

Fig.7 is the sag distributions for the dynamic method for a period of one year. The results of the static analyses are very similar to the dynamic method. The nomenclatures "sag1.0" and "sag2.0" refer to sag distributions obtained with factors 1.0 and 2.0 applied on the obtained current data.

The maximum design temperature for an ACSR conductor is 90°C in Japan under normal conditions. The corresponding sag set by this temperature, using the initial conditions stated above, is 7.85m. This is the maximum design sag calculated by the deterministic approach. Fig.8 shows a comparison of this sag, labeled as 'deterministic', and the hourly-assessed sag on the line, labeled as 'dynamic', for a period of one year. This assessed-sag is obtained by the dynamic method with the transmission current data as recorded.

By referring to Fig.6, it is quite clear that the conductor to ground clearance distribution can be obtained from the sag distribution. A conceptual situation of the line to ground clearance determined by the deterministic approach and the dynamic method, using the actual recorded climatic and current data, is illustrated in Fig.9.

The minimum clearance for the power utility company, which has jurisdiction over this transmission line, is shown in dotted line in Fig. 9.

There is a reasonable margin between the assessed conductor-to-ground clearance and the minimum clearance set by this utility company. A much wider margin is available if the minimum clearance of 6.0 m for railway crossing in Japan is applied.

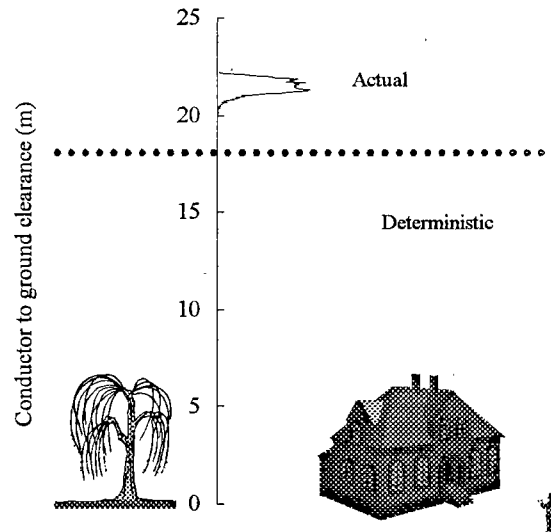


Fig.9: Conductor to ground clearance

With the sag distribution at a specified current levels estimated, it is necessary to make some comparisons with the obtained results. Such comparisons are done with the aid of Figs.10, 11 and 12.

The results of the study indicate that it is possible to increase the current carrying capacity using the set deterministic limit as a reference. As shown in Fig.10, this deterministic limit is exceeded 0.33% of the time when a factor of 2.0 is applied on the actual current data with the dynamic method. This limit is not exceeded when the actual recorded current data is used in the analyses. Similarly, the deterministic limit is exceeded 0.285% of the time when a factor of 2.0 is applied on the current data using the static

method. With the actual current data, this limit is not exceeded using the static method, and it is shown in Fig.11.

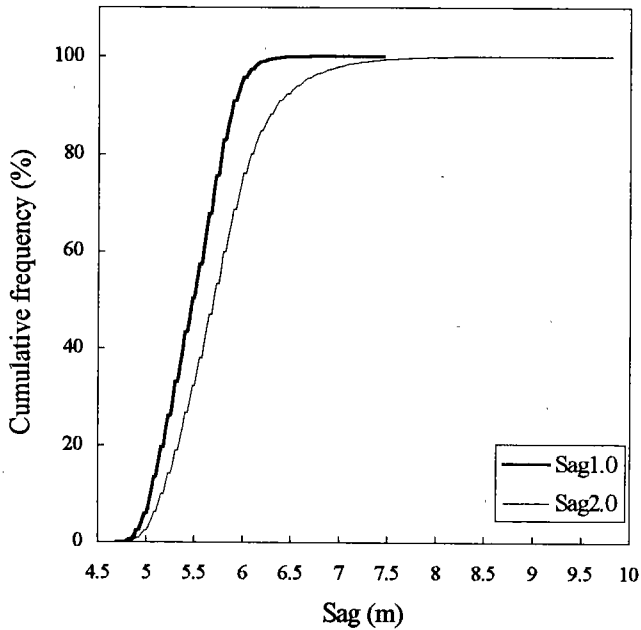


Fig.10: Sag distribution -Dynamic

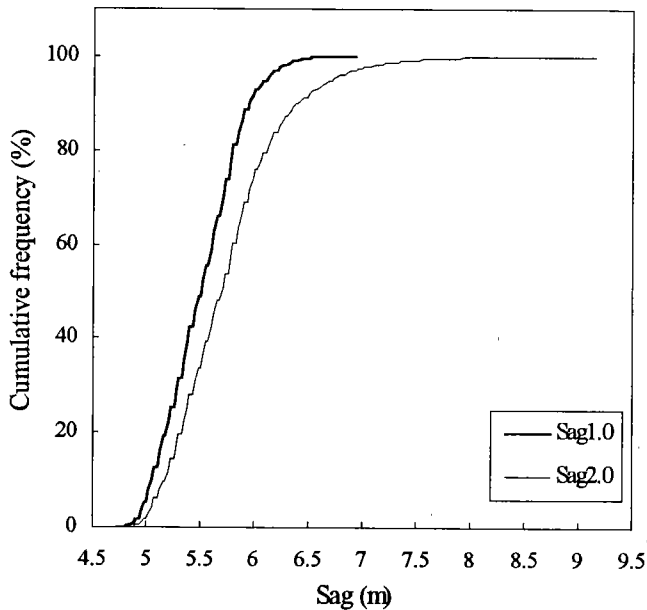


Fig.11: Sag distribution - Static

The difference in the results of the dynamic and static assessments is very insignificant in this study, and this is shown in Fig.12. This comparison is done using the actual transmission current. The small difference in the results of the dynamic and static analyses is due to the fact that the static method treats the used factors as independent variables. The static method assumes no correlation between the parameters used for the study. For instance, there is a correlation between the transmission current and the ambient temperature [1]. This trend is portrayed in 3D in

Fig.13. The probability distribution shown in Fig.13 is the probability of the simultaneous occurrence of a given ambient temperature and transmission line current. It is expected that more energy is required for heating and cooling during winter and summer respectively. A similar correlation is expected between wind velocity and ambient temperature as well as the global solar radiation and the ambient temperature. This makes the dynamic assessment theoretically more accurate of the two.

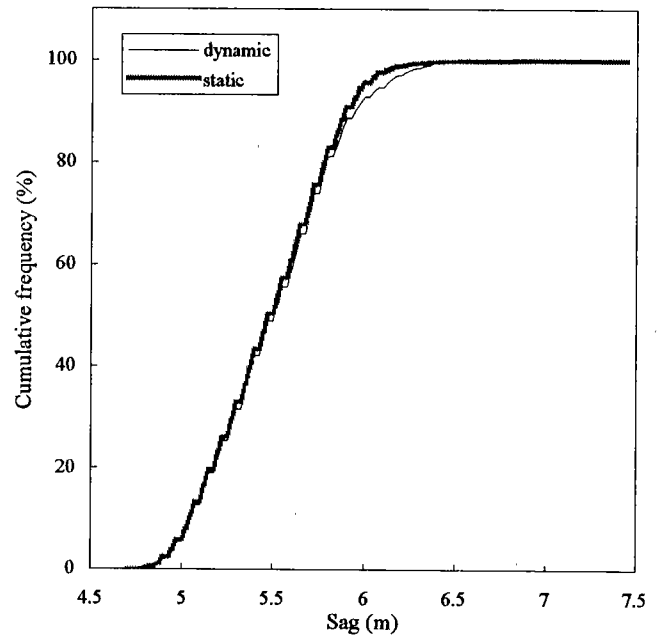


Fig.12: Comparison of sag distribution -Dynamic/Static methods

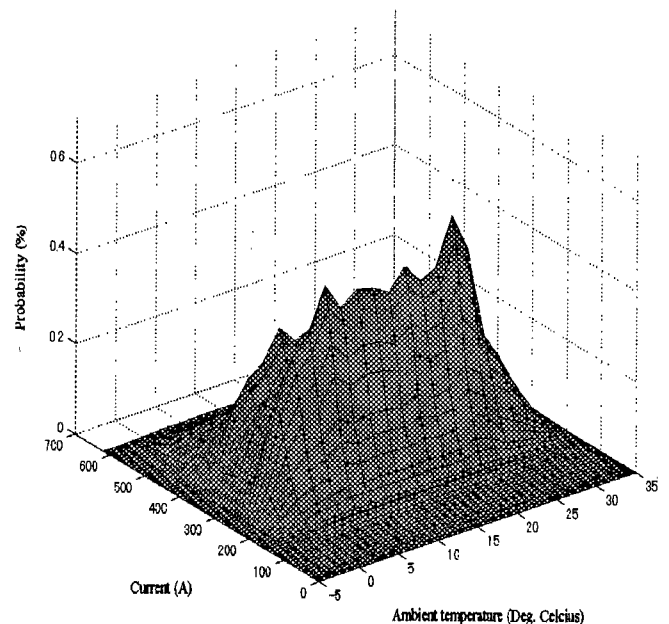


Fig.13: Simultaneous occurrence probability distribution

In comparing the static and the deterministic approaches, it is portrayed that static method makes statistical reference to the data on these ambient conditions, unlike the deterministic approach where specific values are

assumed for these conditions. The static method is therefore more accurate than the deterministic approach, thus making the dynamic method the most accurate of the three.

### 7. Conclusions

A basic study on how the probabilistic approach can be used to assess the sag in overhead line has been discussed. Our main conclusions drawn from the study are that:

1. There is a significant difference between the probabilistic and the deterministic approaches. The former is more realistic and is highly recommended for assessing conductor capacity limits for overhead
2. Using the same current, calculation by the probabilistic approach gives much lower sag compared with the deterministic approach.
3. The difference between the dynamic and static methods is not significant, and the two methods can be said to be in agreement, at least, in this study.
4. Climatic factors along the overhead line may vary. However, the impact of this on the study can be minimized. The stretch can be divided into many climatic zones and the study conducted for each zone if such data are available. Another alternative is to use the data for the area with statistically more extreme conditions.

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