

Decentralized PVFC for Cooperative Mobile Robots

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Traditionally, tasks for a robotic system are specified by a desired timed trajectory in the manipulator's workspace, which the manipulator is required to track at every instant of time. However, there are many tasks in which the desired motions are specified by the state of system rather than time, such as contour following tasks. For such tasks, Passive Velocity Field Control(PVFC) has been proposed and the geometry of the controlled systems was analyzed. In this paper, a method to apply a decentralized PVFC to a cooperative multiple mobile robotic system whose sub-system is under non-holonomic constraints and which conveys a common rigid object in a horizontal plain is proposed, and the effectiveness of the control method is demonstrated by experiments.

Keywords: Cooperative control, PVFC(Passive Velocity Field Control), Multiple mobile robotic system, Velocity Field

1. Introduction

Traditionally, tasks for a robotic system are specified by a desired timed trajectory in the manipulator's workspace, which the manipulator is required to track at every instant of time. However, there are many tasks in which the desired motions are specified by the state of system rather than time, such as contour following tasks, painting so on in which the target point of the system should keep a contact with an environment. If the desired trajectory were specified in terms of time and rigid servo controller was implemented, big forces would give damage to the system due to the tracking error.

Based on the considerations, Passive Velocity Field Control(PVFC) has been proposed and the geometry of the controlled systems was analyzed.^{(1) (2)} In the control, the desired motion is specified by a desired velocity field depending on the state of the system, e.g., joint angles or task space coordinates. The methodology encodes tasks using time invariant desired velocity fields instead of the more traditional method of timed trajectories and guarantees that the closed loop system behave passively with environment power as the supply rate. By maintaining the passivity property of the closed loop system, stability and robustness will be enhanced, especially when interaction with uncertain environments. The most important feature of the control algorithm is that the whole system becomes a virtual passive system even though the velocity of the system converges to the desired one. Since the system is passive, contact tasks with human being are safely realized and stability of the system against an environment is also easily analyzed. To these ends, the formulation of PVFC has two distinct features as follows :

(1) The task is encoded using a velocity field on

the configuration space of the system.

(2) Controllers are constrained so that the closed loop system appears to the physical environment to be a passive system.

Please refer the details to^{(1) (2)}.

In our previous works^{(4) (5)}, decentralized implementation of PVFC including internal force control was proposed in a case where an object is grasped rigidly by multiple manipulators. In this paper, we propose a method to apply the PVFC with some modification to a cooperative multiple mobile robot system which consists of two planar mobile robots which convey a rigid object attached to the robots with passive rotational joints. Each mobile robot is a three-wheeled mobile robot and is under non-holonomic constraints. (See Fig. 3) The specifications of the proposed controller are as follows:

- (1) The center of the object follows a desired velocity field without external disturbances.
- (2) The orientation of the object tracks a desired value specified in terms of the position of the center of the object.
- (3) Linear motion of the object has properties of a system controlled by the original PVFC.
- (4) Internal force is controlled in a certain direction.

In this paper, we will focus on how to realize the specification above by a decentralized PVFC controller though a centralized PVFC was applied for the same system in⁽⁷⁾. In chapter 2, the original PVFC and an extended PVFC are summarized since the proposed control method is based on the PVFC. In chapter 3, dynamics of the cooperative mobile robot is analyzed and in chapter 4, a proposed control method is described. In chapter 5, experimental results are shown to demonstrate the validity of the proposed control algorithm. In

chapter 6, concluding remarks and future problems are discussed.

2. Passive Velocity Field Control(PVFC)

In this section the original PVFC and an extension are briefly summarized to capture the function of PVFC. The passivity can be defined in mathematical term, however, it can be simply understood that the energy of the system is preserved if no external force is exerted.

2.1 Original PVFC In PVFC, instead of requiring the motion of the system to track a desired timed trajectory, as is traditionally the case, the task is encoded using a velocity field on the configuration space of the system, i.e., specifying a desired velocity $V(q)$ at each configuration q . By requiring that the velocity of the system converges to a scalar multiple of the encoding field, the system will be guided to satisfy the task in an expedient and coordinated manner. In conventional controlled mechanical systems, the energy will change in accordance with the convergence to the desired velocity signals which are not normally constant. In PVFC, the mechanical system is augmented with a fly-wheel system virtually so that the virtual system satisfies the passivity as follows.

Let assume that dynamics of an n degree of freedom (D.O.F) mechanical system is represented in a coordinate system as

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} = T + F_e \dots\dots\dots (1)$$

where $q \in \mathbf{R}^n$ stands for coordinates of the system, T and F_e are control input and external force, respectively. M, C are an inertia matrix and Coriolis, centrifugal force terms, respectively. In that equation we assume that gravity term is completely compensated for by a local feedback.

Then, the system is augmented with a fly-wheel system as

$$\underbrace{\begin{pmatrix} M & 0 \\ 0 & M_{fw} \end{pmatrix}}_{M^a} \ddot{x}_a + \underbrace{\begin{pmatrix} C & 0 \\ 0 & 0 \end{pmatrix}}_D \dot{x}_a = T^a + F_e^a \quad (2)$$

where

$$x_a \triangleq \begin{pmatrix} q \\ x_{fw} \end{pmatrix}, F_e^a \triangleq \begin{pmatrix} F_e \\ 0 \end{pmatrix}, T^a \triangleq \begin{pmatrix} T \\ T_{fw} \end{pmatrix}$$

In the following this system is referred to an augmented system, and the terms are suffixed with a .

It should be noted that in eq.(2), D.O.F. of the system is increased from n to $n + 1$ and its additional freedom can be considered to be a flywheel which stores and discharge the energy. If we consider the flywheel as a part of the system, we can control the energy of the flywheel so that the whole energy does not change or the whole system becomes passive even if the original mechanical system alters the velocity due to a desired velocity. Actually, a desired velocity field for the flywheel is specified based on the desired velocity field for the original system so that the energy of a whole system becomes constant. The original PVFC can be represented in a coordinate free language by

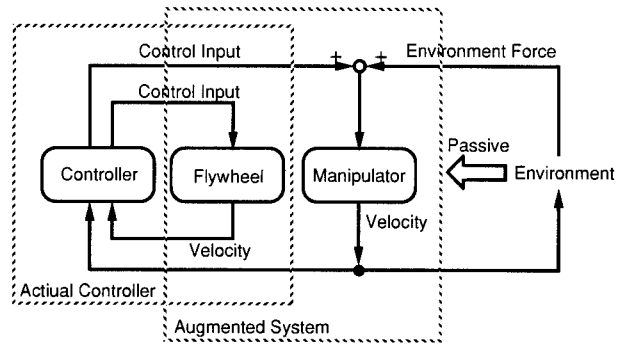


Fig. 1. Passive Velocity Field Control

$$T^a = \dot{x}_a \rfloor \left\{ Q \wedge \left(\frac{1}{2\bar{E}} \nabla_{\dot{x}_a} Q - \gamma p \right) \right\} \dots\dots\dots (3)$$

where p, Q and \bar{E} are momentum, desired momentum of the augmented system and desired energy level, respectively, and \wedge is the wedge product for differential forms, and \rfloor is the contraction operator, and γ is a control constant coefficient. For the details see ^{(1) (2) (3)}. A block diagram of the PVFC is shown in Fig.1.

Since the augmented system is passive with respect to a energy function H_a , it satisfies the following equation:

$$\frac{d}{dt} H_a = \langle F_e^a, \dot{x}_a \rangle = \langle F_e, \dot{x} \rangle \dots\dots\dots (4)$$

where $H_a = \frac{1}{2} v_a^T M^a v_a$ ($v_a = \dot{x}_a$, i.e., v is used for \dot{x} alternatively.) is a kinematic energy of the augmented system, and $\langle \cdot, \cdot \rangle$ is an ordinary inner product. Note that the resultant velocity of the system is a multiple of the desired velocity vector V_a depending on the total energy, that is

$$v_a \rightarrow \beta V_a \dots\dots\dots (5)$$

where β is given by

$$\beta = \sqrt{\frac{\langle \langle v_a, v_a \rangle \rangle_a}{\langle \langle V_a, V_a \rangle \rangle_a}} =: \sqrt{\frac{v_a^T M_a v_a}{V_a^T M_a V_a}} \dots\dots\dots (6)$$

In the basic PVFC the desired velocity field is time invariant, however, complex trajectories which has intersections can not be realized. 'Self-pacing' which is a energy depending pseudo time was introduced. ⁽¹⁾ Since extensions of the PVFC to multiple coordinated system is not considered in the original work by Li, we discussed an extension including internal force control and coordinations of self-pacings for a system in ^{(4) (5) (7)}.

3. Modeling of the System

3.1 Modeling of a three-wheeled mobile robot

In order to derive a dynamic equation of the considered cooperative mobile robotic system, a basic modeling of a single three wheeled mobile robot moving in a horizontal plain is summarized. Please, see for the details in ^{(8) (9)}. Let consider a three wheeled mobile robot shown in Fig.2. In the figure $O - I_1 I_2$ is a base coordinate system and $Q - X_1 X_2$ is a moving coordinate frame with the mobile robot. R is a radius of the wheel

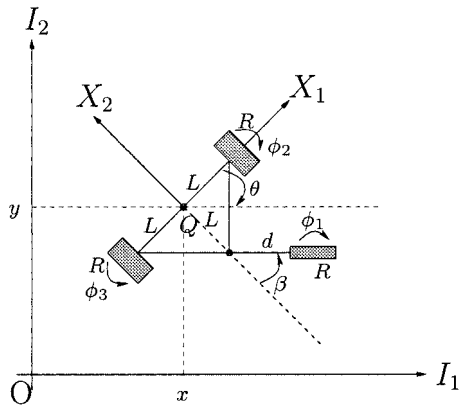


Fig. 2. Single three wheeled mobile robot

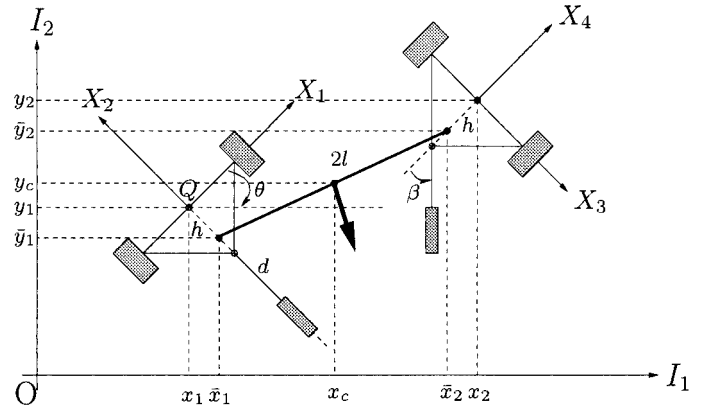


Fig. 3. Coordinated Mobile Robots

and ϕ_1, ϕ_2, ϕ_3 are rotational angles of the wheels. L is a length between a reference point Q and wheels and a steering point, and d is a length between the steering point and the wheel attached to the steering rod. θ is an angle between X_1 and I_1 and β is the steering angle. In the system we assume that ϕ_2 and ϕ_3 are driven by actuators and steering angle ϕ_1 are free.

The motion of the body of the mobile robot is completely specified in terms of position of the reference point $Q, (x, y)$, and the rotation of the body, θ , so we introduce a vector ξ :

$$\xi = [x \ y \ \theta]^T \dots\dots\dots (7)$$

to indicate the motion of the body.

Furthermore, we introduce generalized coordinate vector to describe a whole motion of the system as

$$q = [x \ y \ \theta \ \beta \ \phi_1 \ \phi_2 \ \phi_3]^T \dots\dots\dots (8)$$

Using this generalized coordinates and the following constraints:

- (1) Pure rolling condition : the fact that the component of the velocity of the contact point of the wheel with the ground in the plane of the wheel is zero(i.e., there is no slip between ϕ and floor).
- (2) Non slipping condition : the fact that the component of the velocity of the contact point, orthogonal to the plane of the wheel is zero(i.e., there is no slip of the steering wheel).

we can obtain the dynamic equation as

$$P^T M^*(\beta) P \dot{\zeta} + P^T f^*(\theta, \beta, \zeta) = P^T G(\beta) u, \dots (9)$$

and it is easily shown that these constraints are non-holonomic constraints for the system since two vector fields which satisfy the conditions are not involutive.^{(8) (9)}

If we assume the conditions and use the following coordinate change:

$$\zeta_1 = -\dot{x} \sin \theta + \dot{y} \cos \theta \dots\dots\dots (10)$$

$$\zeta_2 = \dot{\theta} \dots\dots\dots (11)$$

and input transformation, we have the following simplified dynamic equation:

$$\frac{d}{dt} \begin{bmatrix} \psi \\ \theta \\ x \\ y \\ \beta \\ \zeta_1 \\ \zeta_2 \end{bmatrix} = \begin{bmatrix} \zeta_1 \\ \zeta_2 \\ -\zeta_1 \sin \theta \\ \zeta_1 \cos \theta \\ -\frac{1}{d} \zeta_1 \sin \beta - \frac{1}{d} \zeta_2 (d + L \cos \beta) \\ \nu_1 \\ \nu_2 \end{bmatrix} \dots\dots\dots (12)$$

where $\nu = [\nu_1, \nu_2]^T$ is a new input, and the constraints are represented by

$$\dot{x} \cos \theta + \dot{y} \sin \theta = 0 \dots\dots\dots (13)$$

$$-\dot{x} \sin \theta + \dot{y} \cos \theta = \zeta_1 \dots\dots\dots (14)$$

(See^{(8) (9)} for the details.)

3.2 Modeling of the considered system

In this section we consider a dynamic equation of the system shown in Fig. 3. In the system an object denoted by a rod whose length is $2l$ is connected to each mobile robot by a free joint without friction. In this paper we consider a case where two mobile robots convey a rod for simplicity, however, the similar discussion can be applied for cases where more mobile robots carrying a general planer rigid object.

If we assume that mass and inertia of the object are m and I_o , and a position of the mass center and rotational angle of the object from $O - I_1$ in clockwise direction are (x_c, y_c) and φ , we have free dynamic equations of the object as :

$$M_o \ddot{x}_o = 0 \dots\dots\dots (15)$$

$$I_o \ddot{\varphi} = 0 \dots\dots\dots (16)$$

where

$$M_o := \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} \dots\dots\dots (17)$$

$$x_o := \begin{bmatrix} x_c \\ y_c \end{bmatrix} \dots\dots\dots (18)$$

On the other hand, dynamic equations of two mobile robot can be expressed in an augmented dynamic equation as

$$P^T M^* P \dot{\eta}(t) + P^T f^*(\theta, \beta, \eta) = P^T G(\beta) u \dots (19)$$

where

$$P := \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix}, M^* := \begin{bmatrix} M_1^*(\beta_1) & 0 \\ 0 & M_2^*(\beta_2) \end{bmatrix}$$

$$\eta := \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix}, u := \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}, f^* := \begin{bmatrix} f_1^* \\ f_2^* \end{bmatrix}$$

$$G(\beta) := \begin{bmatrix} G_1(\beta_1) & 0 \\ 0 & G_2(\beta_2) \end{bmatrix}$$

and $\eta_i(t) := [\zeta_1, \zeta_2]^T$ for each mobile robot where $\zeta_i \in R$ is defined in the previous subsection (That is, each subscript number indicates a number of mobile robot except ζ_i . As ζ_i is not appeared alone in the following, there should be no confusion.). Therefore, the dynamic equation of the whole system without constraints introduced by the passive joints is given by

$$\begin{bmatrix} P^T M^* P & & \\ & M_o & \\ & & I_o \end{bmatrix} \ddot{x}_s + \begin{bmatrix} P^T f^* \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} P^T G(\beta) \\ 0 \\ 0 \end{bmatrix} u \quad (20)$$

This equation is simply re-expressed as

$$M_s \ddot{x}_s + f_s^* = G_s u \quad (21)$$

where

$$M_s := \begin{bmatrix} P^T M^* P & & \\ & M_o & \\ & & I_o \end{bmatrix}, \dot{x}_s := \begin{bmatrix} \eta_1 \\ \eta_2 \\ \dot{x}_o \\ \dot{\varphi} \end{bmatrix} \quad (22)$$

$$G_s := \begin{bmatrix} P^T G(\beta) \\ 0 \\ 0 \end{bmatrix}, f_s^* := \begin{bmatrix} P^T f^* \\ 0 \\ 0 \end{bmatrix}. \quad (23)$$

From the kinematic constraints by the passive joints, holonomic constraints between the generalized coordinates are given by

$$x_1 + h \sin \theta_1 = x_c - l \cos \varphi \quad (24)$$

$$y_1 - h \cos \theta_1 = y_c - l \sin \varphi \quad (25)$$

$$x_2 + h \sin \theta_2 = x_c + l \cos \varphi \quad (26)$$

$$y_2 - h \cos \theta_2 = y_c + l \sin \varphi. \quad (27)$$

Using this equations and the definition of η_i we can derive velocity constraint of x_s as

$$J_s \dot{x}_s = 0 \quad (28)$$

where

$$J_s := \begin{bmatrix} -\sin \theta_1 & h \cos \theta_1 & 0 & 0 \\ \cos \theta_1 & h \sin \theta_1 & 0 & 0 \\ 0 & 0 & -\sin \theta_2 & h \cos \theta_2 \\ 0 & 0 & \cos \theta_2 & h \sin \theta_2 \\ & & -1 & 0 & -l \sin \varphi \\ & & 0 & -1 & l \cos \varphi \\ & & -1 & 0 & l \sin \varphi \\ & & 0 & -1 & -l \cos \varphi \end{bmatrix}. \quad (29)$$

Therefore an actual dynamic equation of the whole system is given by

$$M_s \ddot{x}_s + f_s^* = G_s u - J_s^T \lambda \quad (30)$$

where $\lambda \in R^4$ is a constraint force vector. If we define λ, λ_i as

$$\lambda^T := [\lambda_1 \quad \lambda_2 \quad \lambda_3 \quad \lambda_4] \quad (31)$$

$$\lambda_1^T := [\lambda_1 \quad \lambda_2] \quad (32)$$

$$\lambda_2^T := [\lambda_3 \quad \lambda_4], \quad (33)$$

the dynamic equation can be decomposed into

$$P_i^T M_i^* P_i \dot{\eta}_i(t) + P_i^T f_i^* = P_i^T G_i(\beta) u_i - J_{xi}^T \lambda_i (i = 1, 2) \quad (34)$$

$$M_o \ddot{x} = -J_{xo}^T \lambda \quad (35)$$

$$I_o \ddot{\varphi} = -J_{\varphi}^T \lambda. \quad (36)$$

where

$$J_{xi}^T := \begin{bmatrix} -\sin \theta_i & \cos \theta_i \\ h \cos \theta_i & h \sin \theta_i \end{bmatrix} \quad (i = 1, 2) \quad (37)$$

$$J_{\varphi}^T := [-l \sin \varphi \quad -l \cos \varphi \quad l \sin \varphi \quad l \cos \varphi]. \quad (38)$$

4. Proposed Control Algorithm

In this section we propose a control method for the system, which satisfies the specification given in the introduction. In order to make a PVFC computationally simple, we consider a minor loop compensation first.

4.1 Minor loop compensation Let consider about eq.(34). Since we assume that the constraint forces λ_i are observed by each force sensor in our control method, then we can define a local control input given by

$$u_i = (P_i^T G_i)^{-1} ((P_i^T M_i^* P_i) v_i + P_i^T f_i^* + J_{xi}^T \lambda_i) - (P_i^T G_i)^{-1} (P_i^T M_i^* P_i) J_{xi}^T \lambda_i. \quad (i = 1, 2) \quad (39)$$

where v_i is a new input. If we inject the control input to the system, the closed loop system becomes

$$\dot{\eta}_i = v_i - J_{xi}^T \lambda_i \quad (i = 1, 2). \quad (40)$$

Therefore a whole dynamic equation becomes

$$\dot{\eta}_i = v_i - J_{xi}^T \lambda_i \quad (i = 1, 2) \quad (41)$$

$$M_o \ddot{x}_o = -J_{xo}^T \lambda \quad (42)$$

$$I_o \ddot{\varphi} = -J_{\varphi}^T \lambda, \quad (43)$$

and the matrix form of the equations is given by

$$\bar{M}_s \ddot{x}_s = \bar{G}_s v - J_s^T \lambda \quad (44)$$

where

$$\bar{M}_s := \begin{bmatrix} I_{4 \times 4} & & \\ & M_o & \\ & & I_o \end{bmatrix} \quad (45)$$

$$\bar{G}_s := \begin{bmatrix} I_{4 \times 4} \\ 0_{3 \times 4} \end{bmatrix} \quad (46)$$

$$v := [v_1^T, v_2^T]^T. \quad (47)$$

(Please note here that the minor loop compensation is not necessary for the design of PVFC, however, the computation for the PVFC would become very complex.)

Since the motions of x_o and φ should be controlled in our problem and direct control input for φ does not exist, the distributed control method proposed in ^{(4) (5)} can not be applied for directly. The dynamic equation is first transformed by a coordinate transformation and input change so that the dynamics of φ disappears in the equation. Control input for φ is realized as an internal force for the motion of x_o .

Let define $\dot{\hat{x}}_i$ as

$$\dot{\hat{x}}_i := J_{xi}\eta_i(t), \quad (i = 1, 2) \dots\dots\dots (48)$$

and using the new coordinate \bar{x}_i the dynamic equation can be rewritten as

$$\begin{bmatrix} J_{x1}^{-1} & & & \\ & J_{x2}^{-1} & & \\ & & M_o & \\ & & & I_o \end{bmatrix} \ddot{\bar{x}}_s + \begin{bmatrix} -J_{x1}^{-1}\dot{J}_{x1}J_{x1}^{-1}\dot{\hat{x}}_1 \\ -J_{x2}^{-1}\dot{J}_{x2}J_{x2}^{-1}\dot{\hat{x}}_2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ 0 \\ 0 \end{bmatrix} - J_s^T \lambda, \dots\dots\dots (49)$$

$$J_s \begin{bmatrix} J_{x1}^{-1} & & & \\ & J_{x2}^{-1} & & \\ & & I_{2 \times 2} & \\ & & & 1 \end{bmatrix} \dot{\bar{x}}_s = 0, \dots\dots\dots (50)$$

where

$$\dot{\bar{x}}_s := \begin{bmatrix} \dot{\hat{x}}_1 \\ \dot{\hat{x}}_2 \\ \dot{x}_o \\ \dot{\varphi} \end{bmatrix} \dots\dots\dots (51)$$

Pre-multiplying a matrix

$$\begin{bmatrix} J_{x1}^{-T} & & & \\ & J_{x2}^{-T} & & \\ & & I_{2 \times 2} & \\ & & & 1 \end{bmatrix} \dots\dots\dots (52)$$

to both hand-sides of eq.(49) and defining a matrix

$$J_c := \begin{bmatrix} I_{2 \times 2} & & -I_{2 \times 2} & \\ & & I_{2 \times 2} & -I_{2 \times 2} \\ & & & J_\varphi \end{bmatrix}, \dots\dots\dots (53)$$

we have a new dynamic equation given by

$$\begin{bmatrix} J_{x1}^{-T} J_{x1}^{-1} & & & \\ & J_{x2}^{-T} J_{x2}^{-1} & & \\ & & M_o & \\ & & & I_o \end{bmatrix} \ddot{\bar{x}}_s + \begin{bmatrix} -J_{x1}^{-T} J_{x1}^{-1} \dot{J}_{x1} J_{x1}^{-1} \dot{\hat{x}}_1 \\ -J_{x2}^{-T} J_{x2}^{-1} \dot{J}_{x2} J_{x2}^{-1} \dot{\hat{x}}_2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} J_{x1}^{-T} v_1 \\ J_{x2}^{-T} v_2 \\ 0 \\ 0 \end{bmatrix} - J_c^T \lambda, \quad (54)$$

$$J_c \dot{\bar{x}}_s = 0. \dots\dots\dots (55)$$

Furthermore, if we define v_i as

$$v_i = J_{xi}^{-1}(\nu_i - \dot{J}_{xi}J_{xi}^{-1}\dot{\hat{x}}_i) + (J_{xi}^T - J_{xi}^{-1}) \lambda_i, \dots (56)$$

finally the dynamic equation becomes

$$\bar{M}_s \ddot{\bar{x}}_s = [\nu_1^T \ \nu_2^T \ 0 \ 0]^T - J_c^T \lambda \dots\dots\dots (57)$$

$$J_c \dot{\bar{x}}_s = 0, \dots\dots\dots (58)$$

which implies

$$\ddot{\bar{x}}_i = \nu_i - \lambda_i \dots\dots\dots (59)$$

$$M_o \ddot{x}_o = \lambda_1 + \lambda_2. \dots\dots\dots (60)$$

From the relationship between x_o to eq.(24) - eq.(27) and \bar{x}_i of eq.(48), we have

$$\dot{x}_o = \dot{\bar{x}}_1 + [I_{2 \times 2} \ 0_{2 \times 2}] J_\varphi \dot{\varphi} \dots\dots\dots (61)$$

$$\dot{x}_o = \dot{\bar{x}}_2 + [0_{2 \times 2} \ I_{2 \times 2}] J_\varphi \dot{\varphi} \dots\dots\dots (62)$$

and

$$\ddot{x}_o - [I_{2 \times 2} \ 0_{2 \times 2}] (J_\varphi \ddot{\varphi} + \dot{J}_\varphi \dot{\varphi}) = \nu_1 - \lambda_1 \dots (63)$$

$$\ddot{x}_o - [0_{2 \times 2} \ I_{2 \times 2}] (J_\varphi \ddot{\varphi} + \dot{J}_\varphi \dot{\varphi}) = \nu_2 - \lambda_2. \dots (64)$$

In order to design a decentralized PVFC, we assume that $\ddot{\varphi}$ is measurable for each subsystem, which is a crucial step to derive a decentralized PVFC since $\ddot{\varphi}$ is determined based on both λ_1 and λ_2 , and both signals can not be used for each subsystem in the decentralized formulation, i.e., λ_1 can be used only in the subsystem 1 so on. Of course, it is difficult to measure the signal in practice, we will estimate the signal from the angle and angular velocity using an observer. The validity of the assumption will be demonstrated in the experiments. If ν_i is defined as

$$\nu_1 = \nu'_1 - [I_{2 \times 2} \ 0_{2 \times 2}] (J_\varphi \ddot{\varphi} + \dot{J}_\varphi \dot{\varphi}) \dots\dots\dots (65)$$

$$\nu_2 = \nu'_1 - [0_{2 \times 2} \ I_{2 \times 2}] (J_\varphi \ddot{\varphi} + \dot{J}_\varphi \dot{\varphi}), \dots\dots\dots (66)$$

then we have

$$\ddot{x}_o = \nu'_i - \lambda_i. \dots\dots\dots (67)$$

From the dynamic equation of x_o and the above equations, the motion of x_o is governed by

$$(I_{2 \times 2} + M_o + I_{2 \times 2}) \ddot{x}_o = \nu'_1 + \nu'_2. \dots\dots\dots (68)$$

From this equation it can be seen that the dynamics of x_o is the same as that of connected three mass, $I_{2 \times 2}, M_o, I_{2 \times 2}$, and the side masses are controlled by ν'_1 and ν'_2 respectively. So we can apply a decentralized PVFC proposed in ^{(4) (5)} to design ν'_i . Basically, the procedure is that an individual PVFC is designed using the following virtual dynamic equation:

$$(I_{2 \times 2} + \rho_1 M_o) \ddot{x}_o = \nu'_1 \dots\dots\dots (69)$$

$$(I_{2 \times 2} + \rho_2 M_o) \ddot{x}_o = \nu'_2, \quad \rho_1 + \rho_2 = 1 \dots\dots\dots (70)$$

where ρ_i is a load sharing coefficient. It has been shown that if ν'_i is defined as

$$\nu'_i = \nu'_{pvfci} + \nu'_{Ii} \dots\dots\dots (71)$$

where ν'_{pvfci} is an original PVFC for eq.(69) or (70), and ν'_{Ii} is desired internal forces satisfying

$$\sum \nu'_{Ii} = 0, \dots\dots\dots (72)$$

then the constraint force λ_i converges to

$$\beta \rho_i M_o \frac{\partial V}{\partial x_o} V + \nu'_{Ii} \dots\dots\dots (73)$$

where β is a scalar number and V is a desired velocity field. So if we set

$$\begin{bmatrix} \nu'_{I1} \\ \nu'_{I2} \end{bmatrix} =: \nu'_I = J_\varphi / \|J_\varphi\|^2 v_\varphi + J_\varphi^\perp v_I \dots\dots\dots (74)$$

where v_φ is a control input for φ and v_I is control for an internal force which does not affect the linear nor angular motion of the object, and

$$J_\varphi^\perp := \begin{bmatrix} \cos \varphi \\ \sin \varphi \\ -\cos \varphi \\ -\sin \varphi \end{bmatrix}, \dots\dots\dots (75)$$

and if $\frac{\partial V}{\partial x_o} V$ is designed to be perpendicular to J_φ , it is easily shown that we have

$$I_o \ddot{\varphi} = -v_\varphi, \dots\dots\dots (76)$$

$$\lambda_I := J_\varphi^{\perp T} \lambda / 2 = v_I. \dots\dots\dots (77)$$

So we can control the linear motion of the object by PVFC, and φ and λ_I can be controlled to desired values.

5. Experimental Results

In this section some experimental results are shown to demonstrate the validity of the proposed control method.

5.1 Experimental conditions As in Fig. 4 the experimental system consists of two Nomad Scouts and an object which is a wooden plate. In the experiment the control period was set to 2 [msec] and the constraint force λ_i was measured through a filter whose transfer function is $10/(s + 10)$ for each subsystem. The position and the orientation of the robot was estimated by dead reckoning. The dynamic parameters of the robot were identified off-line.

The desired velocity field is defined such that if a point moves along the desired velocity, the point converges to a circle whose center and radius are the origin and 1 [m] at a constant speed in a anti-clockwise direction. For the control of φ , desired angle and desired angular velocity, φ_d and $\dot{\varphi}_d$, are defined by

$$\varphi_d := -\tan^{-1}(y_c/x_c) \dots\dots\dots (78)$$

$$\dot{\varphi}_d := \frac{d}{dt}(\varphi_d) \begin{cases} \dot{x}_c = \dot{x}_{cd} \\ \dot{y}_c = \dot{y}_{cd} \end{cases} \dots\dots\dots (79)$$

where $\dot{x}_{cd}, \dot{y}_{cd}$ are the desired velocity specified by a desired velocity field in PVFC. Control input v_φ is determined by

$$v_\varphi = -K_v(\varphi_d - \varphi) - K_p(\dot{\varphi}_d - \dot{\varphi}), \dots\dots\dots (80)$$

and v_I was set to 0. Those control were used for both robot, and the load sharing parameter ρ_i was set to 0.5 since we assumed that each mobile robot has the same capability. For the observation of $\dot{\varphi}$, we used a minimal order observer was designed based on a triple integrator model where the pole of the observer was chosen to -50 .

In Fig. 5 desired trajectory and the actual trajectories of the robots and the object are shown. From the figure it can be seen that the center of the object follows the desired trajectory though some tracking error exists due to uncertainties of the parameters and the effects of the dead reckoning. In Fig 7 tracking performance of the angles for the desired signals is shown. It shows also good tracking performance. Finally, in fig. 6 changes of the virtual energy are plotted. It is seen from the figure that each virtual energy decrease slightly due to energy loss caused by incomplete cancellation of the friction effects.

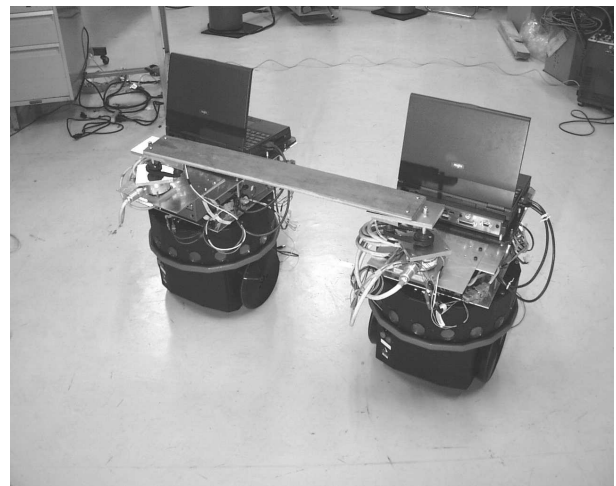


Fig. 4. Constructed experimental system.

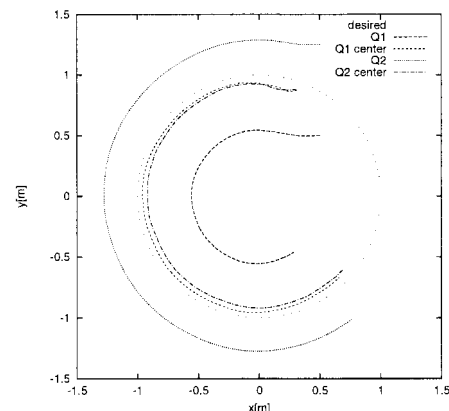


Fig. 5. Trajectories of mobile robots and object.

6. Concluding Remarks

In this paper we have proposed to apply PVFC for cooperative mobile robots conveying an rigid object, and a decentralized control algorithm was given. Though the validity of the proposed method was demonstrated

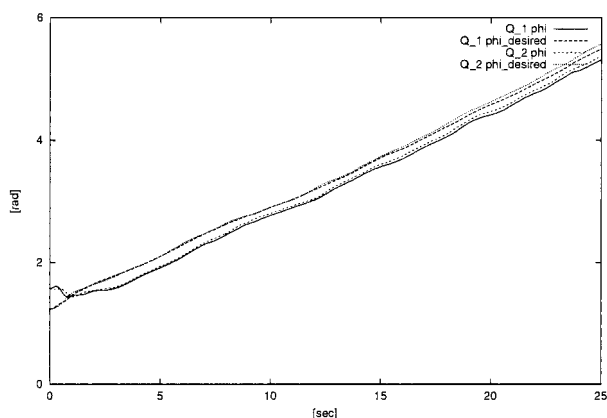


Fig. 6. Tracking of the angle of the object φ .

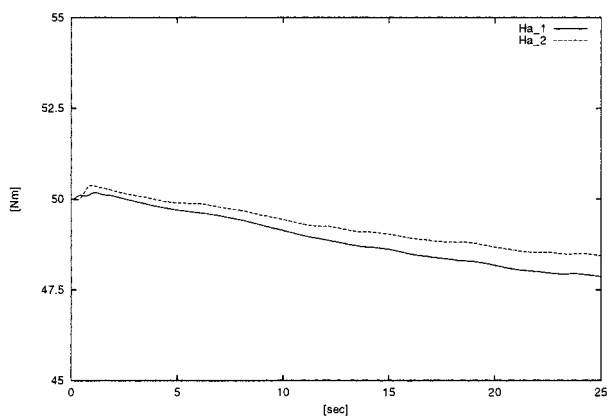


Fig. 7. A virtual energy of each subsystem.

by experiments, the validity should be analyzed theoretically. In this paper we considered time invariant velocity field, the proposed method can be applied for pseudo-time variant cases as in ⁽⁴⁾ ⁽⁵⁾.

Since we assumed that there exists no uncertainties about all parameters in our experiments, the robustness of the proposed method should be checked by further experiments.

This work was partially supported by the Scientific and Research Foundation of the Ministry of Education under Grant COE #104723.

(Manuscript received December 22, 1999)

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