Paper

A Method of Model Matching Control for Continuous-Time Systems in the Presence of Arbitrarily Bounded Disturbances

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A method of model matching control for continuous-time systems in the presence of arbitrarily bounded disturbances is proposed. The proposed control system consists of two feedback loops: one performs the model-matching control and the other includes an error feedback controller which could be fully utilized to reduce the effect of external disturbances. In this paper, an explicit design procedure for achieving perfect model matching control with minimal sensitivity will be proposed. Furthermore, we can assure that the system input and output remain bounded at any time for continuous-time system in the presence of arbitrarily bounded disturbances. Finally, the results of computer simulation are presented to illustrate the effectiveness of the proposed method.

Keywords: model matching controller, disturbance, continuous-time system, minimal sensitivity

1. Introduction

The ultimate goal of the model matching is to match the transfer function of the closed loop system to some desired transfer function(1). Due to the fact that the desired system model is usually characterized in terms of a transfer function, it is natural to treat the design of perfect model matching control in the frequency domain directly rather than in the time domain. Thus, recent work concerning perfect model matching control is mostly done in the frequency domain(2)~(4). Furthermore, it is now well known that H_∞-control problem including both sensitivity minimization and robust stabilization is reduced to a mathematical problem called the model matching problem in H_∞(5). Interpolation approach and approximation approach has been proposed to solve the model-matching problem. Both approaches are mathematically equivalent but they are different in computational aspect. The interpolation approach is conceptually appealing(6),(7) but unsuccessful in providing a powerful computational algorithm. The approximation method has made remarkable progress from computational point of view (8)~(10). An extension of the interpolation technique called directional interpolation is a variation of the Nevanlinna-Pick problem(11),(12). This method can be used to design minimal-order controller in the frequency domain(6). State-space version of the classical interpolation theory has been developed for the simplest model-matching problem.

In this paper we introduce a method to solve the perfect model matching problem for continuous-time systems in the presence of disturbances based upon the pole-zero placement method. In this method, we introduce the output loop compensator for the compensation of external disturbances. Furthermore, in this paper, an explicit design procedure for achieving perfect model matching control with minimal sensitivity will be proposed. Here, we adopt the polynomial approach for designing the controller(13)~(16). Section 2 states the problem. In section 3, we precisely state the problem of minimal sensitivity perfect model matching control and give a complete solution and an explicit design procedure. The results of computer simulation are presented in Section 4.

2. Problem Statement

Let the single-input single-output continuous-time plant be given by

$$y(t) = G(s)u(t) + w(t) \quad \cdots\cdots (1)$$

where

$$G(s) = \frac{B(s)}{A(s)} \quad \cdots\cdots (2)$$

$$A(s) = s^n + \sum_{i=1}^{n-1} a_{n-i}s^{n-i}$$

$$B(s) = \sum_{i=0}^{m} b_{m-i}s^{m-i}$$

$A(s)$ and $B(s)$ are stable coprime polynomials in s and $m \leq n$. $u(t)$ and $y(t)$ are the plant input and output, respectively. $w(t)$ is the external arbitrarily bounded disturbances. Moreover, the continuous-time plant is a minimum phase system ($B(s)$ is a stable polynomial).

The reference model is described by

$$y_m(t) = \frac{B_m(s)}{A_m(s)}r(t) \quad \cdots\cdots (3)$$

where

$$A_m(s) = s^\nu + \sum_{i=1}^{\nu-1} a_{m(\nu-i)} s^{\nu-i}$$

$$B_m(s) = \sum_{i=0}^{\mu} b_{m(\mu-i)} s^{\mu-i}$$

$A_m(s)$ is a stable polynomial and $B_m(s)$ is a polynomial, and $\nu \geq \mu$ and $\nu - \mu \geq n - m$ are satisfied. $r(t)$ is the reference input and $y_m(t)$ is the reference model output. Furthermore, the polynomial $A_m(s)$ and $B_m(s)$ are relatively coprime polynomials. The objective of the control is to design a controller such that the transfer function of the closed-loop from the reference input $r(t)$ to the plant output $y(t)$ becomes $B_m(s)/A_m(s)$ and the system should be robustly stable. Thus, in steady state, output $y(t)$ of the plant will converge to the reference model output $y_m(t)$.

3. Controller Design

Now, using the pole-zero placement method, it is possible to design a controller such that the closed-loop transfer function of the system from the reference input $r(t)$ to plant output $y(t)$ matches some desired transfer function. If $T_0(s)$ is an asymptotically stable polynomial, then there exist unique polynomials $R(s)$ and $S(s)$, which satisfy the following Diophantine equation[13],[17]

$$A(s)R(s) + B(s)S(s) = A_m(s)T_0(s) \quad \cdots\cdots\cdots (4)$$

where, $degT_0(s) = l(l > n-1)$, $degR(s) = \nu + l - n$ and $degS(s) = n - 1$. $T_0(s)$ is the part of the desired closed-loop characteristic polynomial which should not influence the reference tracking. It is interpreted as observer polynomial. The polynomial $A_m(s)$ is the desired closed-loop characteristic polynomial. Here, it is the denominator polynomial of the reference model.

We define the control input $u(t)$ by

$$u(t) = T_2(s)[r(t) - e^0(t)] - T_1(s)y(t) \quad \cdots\cdots\cdots (5)$$

where

$$T_1(s) = \frac{S(s)}{R(s)} \quad \cdots\cdots\cdots (6)$$

$$T_2(s) = \frac{T_0(s)B_m(s)}{B(s)R(s)} \quad \cdots\cdots\cdots (7)$$

$$e^0(t) = \frac{N(s)}{M(s)}e(t) \quad \cdots\cdots\cdots (8)$$

$$e(t) = y(t) - y_m(t) \quad \cdots\cdots\cdots (9)$$

and $M(s)$ and $N(s)$ are polynomials. This perfect model matching controller in eq. (5) could be thought of as a combination of feedback having the transfer function $T_1(s)$, a feedforward with the transfer function $T_2(s)$ and the output loop compensator $N(s)/M(s)$.

The block diagram of the proposed system is shown in Fig. 1.

From eqs. (1), (4), the following equation holds.

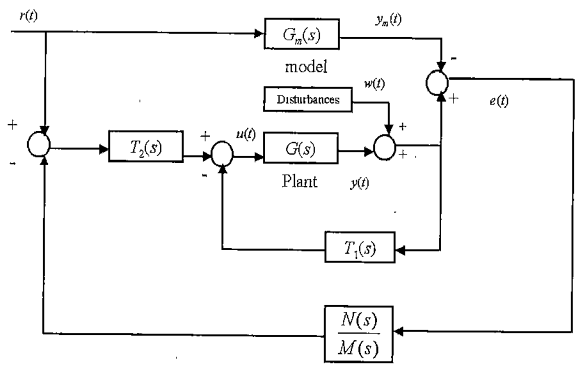


Fig. 1. Block diagram of the proposed system

$$y(t) = \frac{B(s)R(s)}{T_0(s)A_m(s)}u(t) + \frac{B(s)S(s)}{T_0(s)A_m(s)}y(t) + \frac{A(s)R(s)}{T_0(s)A_m(s)}w(t) \quad \cdots\cdots\cdots (10)$$

Using eqs.(5) ~ (9), it is possible to write eq.(10) as

$$y(t) = \frac{1}{T_0(s)A_m(s)}[T_0(s)B_m(s)r(t) - T_0(s)B_m(s)\frac{N(s)}{M(s)}y(t) + T_0(s)B_m(s)\frac{N(s)}{M(s)}y_m(t) + A(s)R(s)w(t)] \quad (11)$$

Moreover, if the polynamial $[A_m(s)M(s) + B_m(s)N(s)]$ and $M(s)$ are stable, by using eq. (11), the following relation can be derived

$$y(t) = \frac{B_m(s)}{A_m(s)}r(t) + \frac{M(s)R(s)A(s)}{T_0(s)(A_m(s)M(s) + B_m(s)N(s))}w(t) \quad \cdots (12)$$

From eq. (12), the sensitivity function relating the disturbance $w(t)$ to output $y(t)$ can be given by

$$SS(s) = \frac{M(s)R(s)A(s)}{T_0(s)(A_m(s)M(s) + B_m(s)N(s))} \cdot (13)$$

which can be interpreted as a sensitivity function relating output to disturbance. To have perfect model matching control with minimal sensitivity to disturbance $w(t)$, it is required to find the design parameters $M(s)$ and $N(s)$ which not only satisfy the stability of $A_m(s)M(s) + B_m(s)N(s)$, but also minimize the following meaningful measure of sensitivity in the low frequency region.

$$J = \left\| \frac{M(s)R(s)A(s)V(s)}{T_0(s)(A_m(s)M(s) + B_m(s)N(s))} \right\|_\infty \quad s = j\omega, \quad \forall\omega \quad \cdots\cdots\cdots (14)$$

where $W(s)$ is the Laplace transform of $w(t)$, $|W(j\omega)| \leq |V(j\omega)|$, and $\|\cdot\|_\infty$ is defined as

$$\|G(s)\|_\infty = \sup_\omega |G(j\omega)| \quad \cdots\cdots\cdots (15)$$

For a given any stable polynomial $T_3(s)$, $\tilde{M}(s)$ and $\tilde{N}(s)$ always exist, which satisfy the following equation[18]

$$T_3(s) = A_m(s)\tilde{M}(s) + B_m(s)\tilde{N}(s) \quad \cdots\cdots\cdots (16)$$

where $deg\ T_3(s) = \xi$. The degrees of the polynomials $\tilde{M}(s)$ and $\tilde{N}(s)$ can be fixed as $deg\ \tilde{M}(s) = \xi - \nu$ and $deg\ \tilde{N}(s) = \nu - 1$, respectively.

Furthermore, we define two polynomials $M(s)$ and $N(s)$ as

$$M(s) = \tilde{M}(s) - L(s)B_m(s) \quad \cdots\cdots\cdots\cdots\cdots (17)$$

$$N(s) = \tilde{N}(s) + L(s)A_m(s) \quad \cdots\cdots\cdots\cdots\cdots (18)$$

In this paper, we introduce a new idea to design the parameters of the controller. The parameters of the controller are designed using polynomials $\tilde{M}(s)$ and $\tilde{N}(s)$, obtained from the Diophantine equation (16). We can obtain these polynomials such that the control input obtained from eq. (5) robustly stabilizes the plant and also the closed-loop transfer function from the reference input $r(t)$ to output $y(t)$ matches some desired transfer function, as in eq. (3).

Using eqs. (13), (16) ~ (18) the sensitivity function relating output $y(t)$ to the disturbance $w(t)$ can be given by

$$SS(s) = \frac{(\tilde{M}(s) - L(s)B_m(s))R(s)A(s)}{T_0(s)T_3(s)} \quad \cdots\cdots (19)$$

The effect of $w(t)$ can be minimized by choosing a suitable sensitivity transfer function $\| S(s)V(s) \|_\infty$. Here, we define the following criterion

$$J = \left\| \frac{(\tilde{M}(s) - L(s)B_m(s))R(s)A(s)V(s)}{T_0(s)T_3(s)} \right\|_\infty$$

$$= \|SS(s)V(s)\|_\infty < \gamma, \quad s = j\omega, \quad \forall\omega \quad \cdots\cdots (20)$$

where γ is a small constant $\gamma > 0$.

Then, the problem becomes equivalent to finding the rational function $L(s)$ such that J given in eq. (20) can be satisfy $J < \gamma$. In this way, the rational function $L(s)$ is independent of the disturbance and the plant, and the model matching will not be destroyed. Furthermore, the rational function $L(s)$ can easily be implemented by using following methods.

3.1 In case of stable polynomial $A(s)$ When $A(s)$ is a stable polynomial, the transfer function $L(s)$ can be obtained from the following relation

$$L(s) = \frac{\tilde{M}(s)R(s)A(s)V_n(s) - \rho T_0(s)T_3(s)V_d(s)}{B_m(s)R(s)A(s)V_n(s)} \quad (21)$$

where $V(s) = V_n(s)/V_d(s)$. ρ is an arbitrarily chosen constant and $\rho < \gamma$. Now, using eqs. (20) and (21), the minimal value of J can be given by

$$J = |\rho| < \gamma \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (22)$$

Here J depends upon ρ and ρ can be selected arbitrarily. Thus J can be minimized upto the desired level.

Furthermore, substituting eq. (21) into eqs. (17), (18), it can be shown that $N(s)/M(s)$ is stable.

3.2 In case of unstable polynomial $A(s)$ In the case when $A(s)$ is an unstable polynomial, $A(s)$ is divided into a stable polynomial component $A^+(s)$ and unstable polynomial component $A^-(s)$, where $A(s) = A^+(s)A^-(s)$. The degree of $A^+(s)$ is n_1 and the degree of $A^-(s)$ is n_2, and $n_1 + n_2 = n$. Since $A^-(-s)$ is a stable polynomial, the transfer function $L(s)$ can be chosen as

$$L(s) = \frac{\tilde{M}(s)R(s)A^+(s)A^-(-s)V_n(s)}{B_m(s)R(s)A^+(s)A^-(-s)V_n(s)} - \frac{\rho T_0(s)T_3(s)V_d(s)}{B_m(s)R(s)A^+(s)A^-(-s)V_n(s)} \quad \cdots (23)$$

Now, using eq. (20) and eq. (23), the minimal value of J can be given by

$$J = \left\| \rho \frac{A^-(s)}{A^-(-s)} \right\|_\infty = |\rho| < \gamma \quad \cdots\cdots\cdots\cdots\cdots (24)$$

and J can be minimized upto the desired level.

Furthermore, substituting eq. (23) into eqs. (17), (18), it can be shown that $N(s)/M(s)$ is stable.

Consequently, a simple and direct design algorithm for achieving perfect model matching control with minimal sensitivity can be outlined as follows.

Design Algorithm:

Step 1: Choose a stable polynomials $T_0(s)$, $T_3(s)$.
Step 2: Solve Diophantine equations (4), (16).
Step 3: Choose $V(s)$ and the small constant ρ.
Step 4: Obtain $L(s)$ from eq. (21) or eq. (23).
Step 5: Obtain $M(s)$ and $N(s)$ from eqs. (17), (18).
Step 6: Then the controller takes the from eq. (5).

4. Simulation Results

In this section, the results of simulations are presented to give an indication of the performance of the model matching scheme.

Example 1: Let us consider the case when the polynomial $A(s)$ has stable poles, and the plant with external disturbance described by the following equation

$$y(t) = \frac{B(s)}{A(s)}r(t) + w(t) = \frac{s^2 + 7s + 6}{s^2 + 5s + 6}r(t) + w(t)$$

where $A(s)$ has two stable poles, at -2 and -3, and $B(s)$ has two stable zeros at -1 and -6. Furthermore, the disturbance is $w(t) = 0.5$, $t > 0$.

The reference model $G_m(s)$ is given by

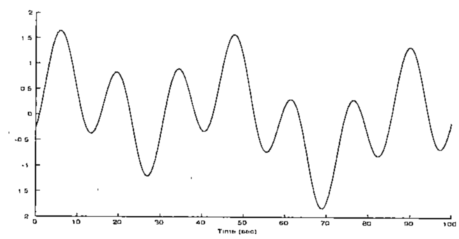
$$G_m(s) = \frac{s^2 + 12s + 35}{s^2 + 12s + 32}$$

Choose stable polynomials $T_0(s) = s^2 + 4s + 3$, $T_3(s) = s^3 + 6s^2 + 11s + 6$.

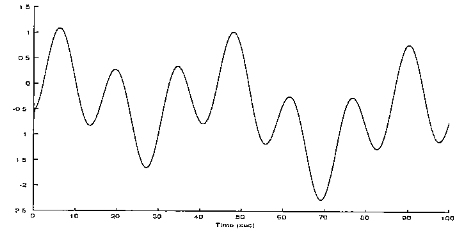
Furthermore, choose $V(s) = 2.011(s+1)/(s+0.1005)$, the small constant $\rho = 0.01$.

Form eq. (21), we obtain

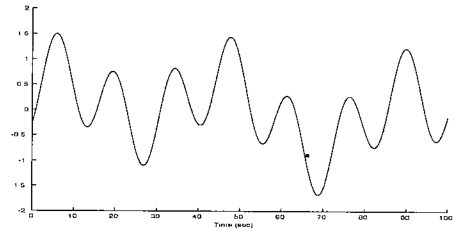
$$L(s) = \frac{-16s^2 - 184s - 504}{s^3 + 19s^2 + 119s + 245}$$



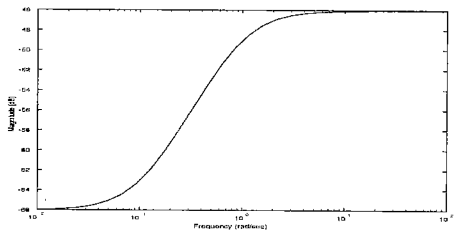
(a) --:$y(t)$(plant output); —:$y_m(t)$(reference output)



(b) $u(t)$(plant input)

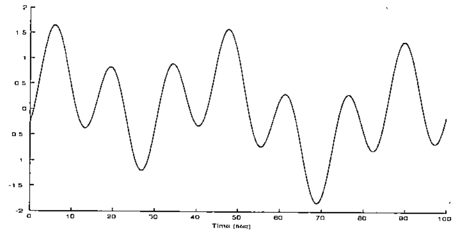


(c) $r(t)$(reference input)

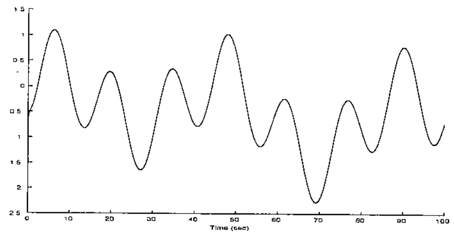


(d) $SS(s)$(sensitivity function)

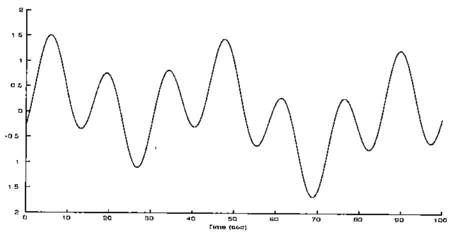
Fig. 2. Result of model matching control with step disturbance ($w(t) = 0.5$, $\rho = 0.01$)



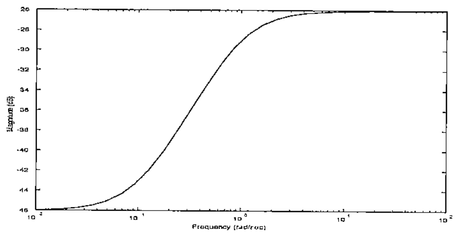
(a) --:$y(t)$(plant output); —:$y_m(t)$(reference output)



(b) $u(t)$(plant input)



(c) $r(t)$(reference input)



(d) $SS(s)$(sensitivity function)

Fig. 3. Result of model matching control with step disturbance ($w(t) = 0.5$, $\rho = 0.1$)

Solve Diophantine equation (16), and using eqs. (17), (18), then $N(s)/M(s)$ is

$$\frac{N(s)}{M(s)} = \frac{200.1s^5 + 4799s^4 + 4.237 \times 10^4 s^3}{s^5 + 26.1s^4 + 254.6s^3} * \frac{+1.650 \times 10^5 s^2 + 2.643 \times 10^5 s + 1.378 \times 10^5}{+1103s^2 + 1823s + 172.4}$$

Furthermore, substituting the result of Diophantine equation (4) into eqs. (6), (7), we arrive at

$$T_1(s) = \frac{3s + 9}{s^2 + 8s + 7}$$

$$T_2(s) = \frac{s^4 + 16s^3 + 86s^2 + 176s + 105}{s^4 + 15s^3 + 69s^2 + 97s + 42}$$

Fig. 2 (a) shows the output response of the proposed model matching control for continuous-time system with step disturbance ($w(t) = 0.5$), when $\rho = 0.01$. Fig. 2 shows that the proposed method is applicable to continuous-time system, and the plant output can converge to the desired output $y_m(t)$ ($y_m(t) = G_m(s)r(t)$) quickly in the presence of disturbance. Furthermore, when $\rho = 0.1$, the model matching control for continuous-time system with step disturbance ($w(t) = 0.5$) is shown in Fig. 3. From Fig. 2 and Fig. 3, it is seen that when we choose small ρ, the effect of the disturbance can be decoupled from the plant output.

Example 2: Let us consider the case when the polynomial $A(s)$ has unstable poles, and the plant with external disturbance described by the following equation

$$y(t) = \frac{B(s)}{A(s)}r(t) + w(t) = \frac{s^2 + 6s + 5}{s^2 + s - 6}r(t) + w(t)$$

where $A(s)$ has a stable pole at -3 and an unstable pole at 2, and $B(s)$ has two stable zeros at -1 and -5.

Furthermore, Fig. 4 shows the external disturbance. The reference model $G_m(s)$ is given by

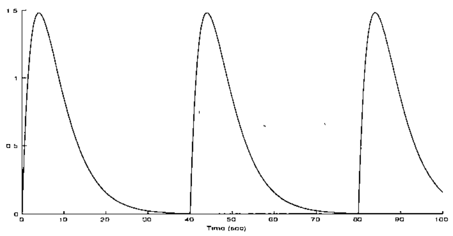
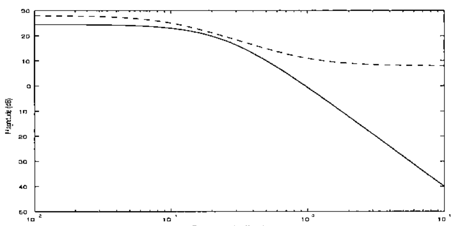


Fig. 4. External disturbance



—: $W(s)$; – –: $V(s)$

Fig. 5. $W(s)$ and $V(s)$

$$G_m(s) = \frac{s^2+7s+12}{s^2+10s+16}$$

Thus, we have $A^+(s) = s+3$, $A^-(s) = s-2$.

Choose the small constant $\rho = 0.01$, and stable polynomials $T_0(s) = s^2+4s+3$, $T_3(s) = s^3+6s^2+11s+6$, $V(s) = 2.513(s+1)/(s+0.1005)$.

The norms of $W(s)$ and $V(s)$ are shown in Fig. 5.

From eq. (23), we obtain

$$L(s) = \frac{-0.746s-4.714}{s^2+10.29s+25.14}$$

Solve Diophantine equation (16), and from eqs. (17), (18), $N(s)/M(s)$ is

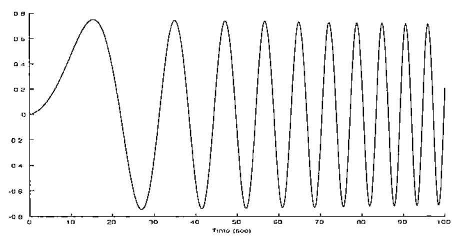
$$\frac{N(s)}{M(s)} = \frac{-252.3s^3-2344s^2-5259s-3161}{s^3+7.101s^2+12.7s+1.206}$$

Furthermore, substituting the result obtained that solve Diophantine equation (4) into eqs (6), (7), we arrive at

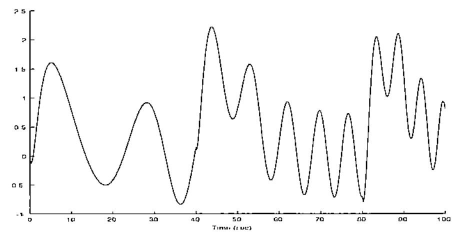
$$T_1(s) = \frac{5.714s+17.14}{s^2+7.286s+6.286}$$

$$T_2(s) = \frac{s^4+11s^3+43s^2+69s+36}{s^4+13.29s^3+55s^2+74.14s+31.43}$$

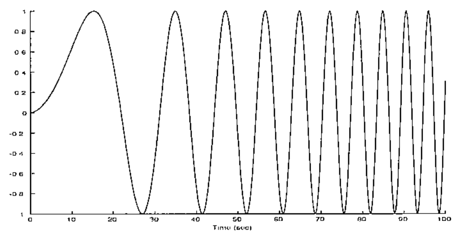
Fig. 6 (a) shows the output response of the proposed model matching control for continuous-time system with periodic disturbance, when $\rho = 0.01$. Fig. 6 shows that the proposed method is applicable to continuous-time system, and the plant output can converge to the desired output $y_m(t)$ quickly in the presence of disturbance. Furthermore, when $\rho = 0.1$, the model matching control for continuous-time system with periodic disturbance is shown in Fig. 7. From Fig. 6 and Fig. 7, it is seen that when we choose small ρ, the effect of the disturbance can be decoupled from the plant output.



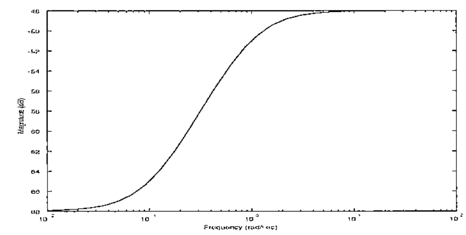
(a) – –: $y(t)$(plant output); —: $y_m(t)$(reference output)



(b) $u(t)$(plant input)



(c) $r(t)$(reference input)



(d) $SS(s)$(sensitivity function)

Fig. 6. Result of model matching control with disturbance ($\rho = 0.01$)

Example 3: Let us consider the case when the plant with external disturbance described by the following equation

$$y(t) = \frac{B(s)}{A(s)}r(t) + w(t) = \frac{s+1}{s^2+s-6}r(t) + w(t)$$

where $A(s)$ has a stable pole at -3 and an unstable pole at 2, and $B(s)$ has a stable zeros at -1.

The reference model $G_m(s)$ is given by

$$G_m(s) = \frac{s+3}{s^2+10s+16}$$

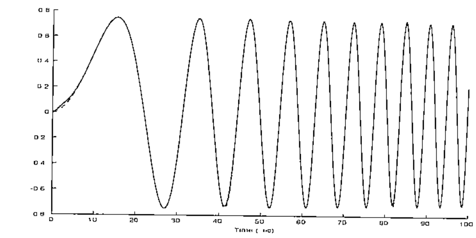
Furthermore, Fig. 8 shows the external disturbance.

Choose the small constant $\rho = 0.01$, and stable polynomials $T_0(s) = s^2+3.6s+3.15$, $T_3(s) = s^3+180s^2+10724s+211200$, $V(s) = 1.508/(s+0.1005)$.

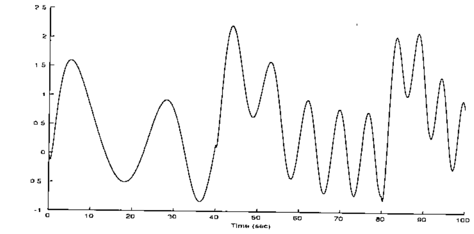
The norms of $W(s)$ and $V(s)$ are shown in Fig. 9.

From eq. (27), we obtain

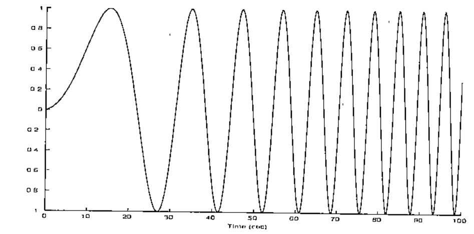
$$L(s) = \frac{1.508\times10^8 s - 5.447\times10^{12}}{s^2+1.508\times10^8 s+4.524\times10^8}$$



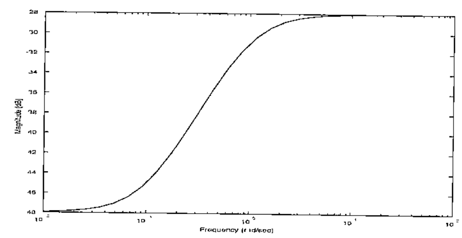
(a) – –:$y(t)$(plant output); —:$y_m(t)$(reference output)



(b) $u(t)$(plant input)



(c) $r(t)$(reference input)



(d) $SS(s)$(sensitivity function)

Fig. 7. Result of model matching control with disturbance ($\rho = 0.1$)

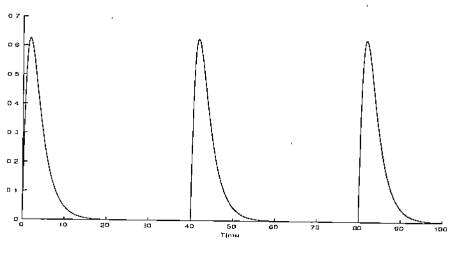
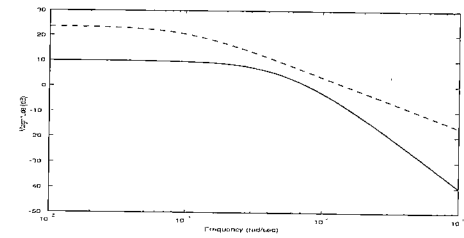


Fig. 8. External disturbance

Solve Diophantine equation (16), and from eqs. (17), (18), $N(s)/M(s)$ is

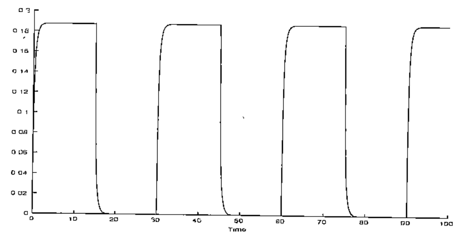
$$\frac{N(s)}{M(s)} = \frac{1.508 \times 10^8 s^3 + 2.714 \times 10^{10} s^2}{s^3 + 3995 s^2} * \frac{+1.617 \times 10^{12} s + 3.185 \times 10^{13}}{+6.353 \times 10^5 s + 3.061 \times 10^8}$$

Furthermore, substituting the result obtained that solve Diophantine equation (4) into eqs (6), (7), we arrive at

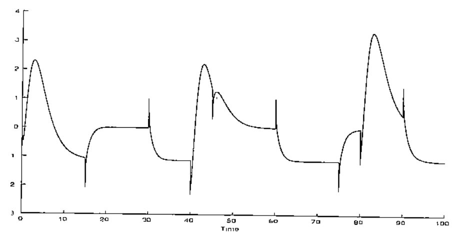


—:$W(s)$; – –:$V(s)$

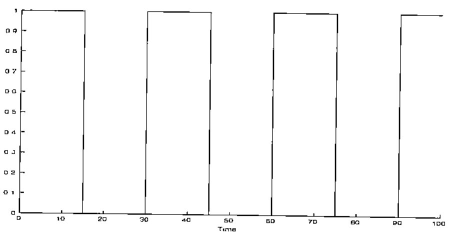
Fig. 9. $W(s)$ and $V(s)$



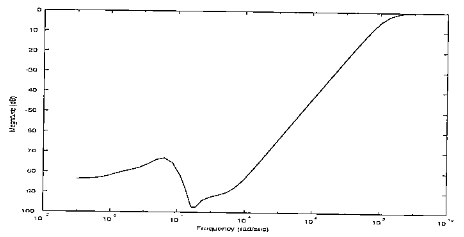
(a) – –:$y(t)$(plant output); —:$y_m(t)$(reference output)



(b) $u(t)$(plant input)



(c) $r(t)$(reference input)



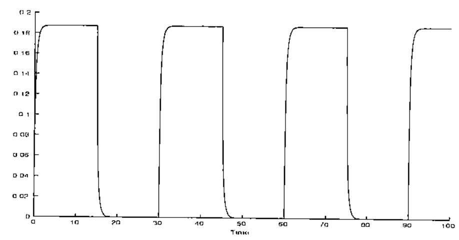
(d) $SS(s)$(sensitivity function)

Fig. 10. Result of model matching control with disturbance ($\gamma = 0.01$)

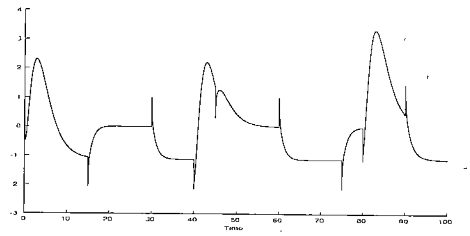
$$T_1(s) = \frac{37.59s + 116.1}{s^2 + 12.6s + 10.96}$$

$$T_2(s) = \frac{s^3 + 6.6s^2 + 13.95s + 9.45}{s^3 + 13.6s^2 + 23.56s + 10.96}$$

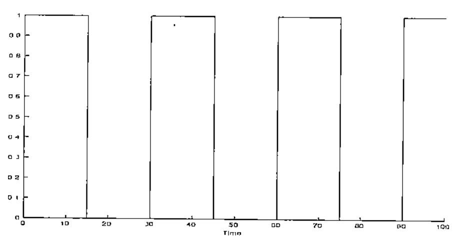
Fig. 10 shows the output response of the proposed model matching control for continuous-time system with the periodic disturbance, when $\rho = 0.01$. Fig. 10 shows that the proposed method is applicable to continuous-time system, and the plant output can



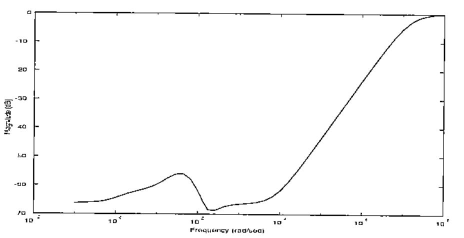
(a) - - -: $y(t)$ (plant output); —: $y_m(t)$ (reference output)



(b) $u(t)$ (plant input)



(c) $r(t)$ (reference input)



(d) $SS(s)$ (sensitivity function)

Fig. 11. Result of model matching control with disturbance ($\gamma = 0.1$)

converge to the desired output $y_m(t)$ in the presence of periodic disturbance. Furthermore, when $\rho = 0.1$, the model matching control for continuous-time system with the periodic disturbance is shown in Fig. 11. From Fig. 10 and Fig. 11, it is seen that when we choose small ρ, the effect of the disturbance can be decoupled from the plant output.

Furthermore, it can also be seen from simulation results that when we use the input $u(t)$ in eq. 5, we can assure that the system input and output remain bounded at any time for continuous-time system with arbitrarily bounded disturbance.

5. Conclusion

We have proposed a new technique to design a controller with two degree of freedom such that the stability of the system can be assured in presence of disturbances. Furthermore, the transfer function of the system from the reference input $r(t)$ to plant output $y(t)$ can be made to match some reference model transfer function. The system input and output remain bounded at any time for continuous-time system with arbitrarily bounded disturbance.

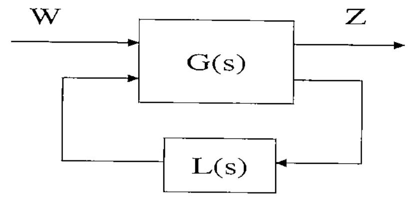


Fig. 12. H_∞ control system

Appendix: The case of $n > m$ and $\nu - \mu \geq n - m$

Let us define the following equation

$$\bar{A}(s) = \frac{\tilde{M}(s)R(s)A(s)V(s)}{\gamma T_0(s)T_3(s)} = \frac{\bar{B}_1(s)}{\bar{A}_1(s)} \quad \cdots\cdots (25)$$

where $V(s) = V_n(s)/V_d(s)$, $deg\ V_n(s) = l_1$, $deg\ V_d(s) = l_2$, $deg\ \bar{A}_1(s) = l + \xi + l_2$, $deg\ \bar{B}_1(s) = l + l_1 + \xi$. Furthermore, $\bar{B}(s)$ is defined as

$$\bar{B}(s) = \frac{B_m(s)R(s)A(s)V(s)}{\gamma T_0(s)T_3(s)} = \frac{\bar{B}_2(s)}{\bar{A}_1(s)} \quad \cdots\cdots (26)$$

where $deg\ \bar{B}_2(s) = \mu + \nu + l + l_1$.

From eqs. (25) and (26), it is possible to write

$$\bar{A}(s) = C_1^T(sI - A_1)^{-1}B_1 \quad \cdots\cdots (27)$$

$$\bar{B}(s) = C_2^T(sI - A_1)^{-1}B_2 \quad \cdots\cdots (28)$$

where, A_1 is a $(l + \xi + l_2) \times (l + \xi + l_2)$ matrix, B_1, C_1, B_2 and C_2 are $(l + \xi + l_2) \times 1$ vectors. Furthermore, using eqs. (20), (25), (26), we obtain

$$|\ \bar{A}(s) - \bar{B}(s)L(s)\ | < 1, \quad s = j\omega, \forall\omega \quad \cdots\cdots (29)$$

Equation (29) can be rewritten as

$$\bar{A}(s) - \bar{B}(s)L(s) = \bar{A}(s) - \bar{B}(s)L(s)(I - 0 \cdot L(s))^{-1}I \quad \cdots (30)$$

Let the state space controller be $L(s)$, then the block diagram of the system can be given as in Fig. 12.

$$G(s) = \begin{bmatrix} \bar{A}(s) & -\bar{B}(s) \\ I & 0 \end{bmatrix} \quad \cdots\cdots (31)$$

Using eqs. (27), (28), and (31), we obtain

$$G(s) = \left[\begin{array}{cc|cc} A_1 & 0 & B_1 & 0 \\ 0 & A_1 & 0 & B_2 \\ \hline C_1 & -C_2 & 0 & 0 \\ 0 & 0 & I & 0 \end{array}\right] \quad \cdots\cdots (32)$$

The state feedback H_∞ controller $L(s)$ can be obtained by solving the H_∞ problem given in Fig. 12[(5),(19),(20)].

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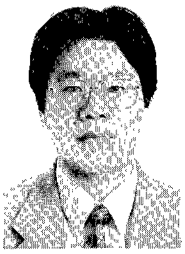

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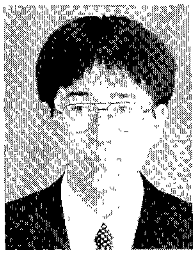

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