Paper

On BELS parameter estimation method of transfer functions

Member Kiyoshi WADA (Kyushu University)
Non-member Lijuan JIA (Kyushu University)
Non-member Takeshi HANADA (Kyushu University)
Member Jun IMAI (Okayama University)

Least squares (LS) method is the most widely used method for the parameter estimation. However when applying directly the LS method to estimate transfer function parameters in the presence of noise, the LS method becomes an asymptotically biased estimator and is unable to be a consistent estimator. There have been many studies to solve the problem of bias and one of them is bias-eliminated least-squares (BELS) method. BELS method is one of consistent estimation methods for unknown parameters of transfer functions. In this paper, we discuss the equivalence of BELS method and IV method under certain condition.

Keywords: parameter estimation, least-squares method, instrumental variables method, transfer function, system identification

1. Introduction

The least-squares (LS) method has been the dominant algorithm for parameter estimation due to its simplicity in concept and convenience in implementation. It is easy to apply but has a substantial drawback: the parameter estimates are consistent only under restrictive conditions (only white noise). When applying the least squares method to estimate transfer function parameters in presence of correlated noise, the LS method becomes an asymptotically biased estimator and is unable to be a consistent estimator. Over the decades, much effort has been devoted to this problem and many kinds of modified least-squares methods in order to overcome this drawback have been developed.

The authors have been proposed the bias compensated LS (BCLS) method which can present consistent estimator on the basis of compensating the noise-induced bias in the LS estimators by applying the estimation of asymptotic bias[1]. On the other hand, another method named BELS method was proposed[2] in which the different estimation method of asymptotic bias is used and further developed to be an efficient method to treat bias problem in system identification[6),8)]. For example, in the BELS method given in the literature[2], a designed filter is inserted into the identified system so that the resulting system has some known zeros which can, based on asymptotic analysis, be used for eliminating the noise-induced bias in the LS estimators.

Recently, literatures[3)∼5),6),7)], and so on, indicate that the BELS method belongs to the class of the instrumental variables (IV) method under certain conditions.

In this paper, we discuss the relationship between the BELS method and the instrumental variables (IV) method in a more general setting. It is illustrated that in the case that BELS method belongs to the class of the instrument variable (IV) method, the estimation accuracy is not affected by the matrices which are introduced in order to estimate the asymptotic bias.

The paper is organized as follows. In the next section, we present the problem statement and the ordinary LS method and then it is shown that the LS estimator is biased, even asymptotically. In section 3, the BELS estimator is derived through introducing the auxiliary vectors and matrices. In section 4, we examine the property of BELS method, and then discuss the relationship between the BELS method and the instrumental variables (IV) method in a more general setting. The simulation results are presented in section 5. Finally section 6 concludes this paper.

2. Problem statement

Consider a linear, single-input single-output, discrete-time system described as follows:

$$A(q^{-1})y(k) = B(q^{-1})u(k) + v(k) \quad \cdots\cdots\cdots\cdots (1)$$

where $u(k)$ is the system input, $y(k)$ is the system output, and q^{-1} is the backward shift operator, i.e. $q^{-i}u(k) = u(k-i)$. $A(q^{-1}), B(q^{-1})$ are polynomials in q^{-1} of the form as follows

$$A(q^{-1}) = 1 + a_1q^{-1} + \cdots + a_nq^{-n},$$
$$B(q^{-1}) = b_1q^{-1} + \cdots + b_nq^{-n}.$$

Assume that $u(k)$, $v(k)$ are the zero mean stationary random processes with finite variance, and are statistically independent of each other.

Let

$$\boldsymbol{\theta} = [\, a_1\, a_2 \ldots a_n\; b_1\, b_2 \ldots b_n \,]^T,$$
$$\boldsymbol{y}(k) = [\, y(k-1)\, y(k-2) \ldots y(k-n) \,]^T,$$
$$\boldsymbol{u}(k) = [\, u(k-1)\, u(k-2) \ldots u(k-n) \,]^T,$$
$$\boldsymbol{p}(k)^T = [\, -\boldsymbol{y}(k)^T\; \boldsymbol{u}(k)^T \,].$$

Then eqn.(1) can be rewritten compactly as

$$y(k) = \boldsymbol{p}(k)^T\boldsymbol{\theta} + v(k). \cdots\cdots\cdots\cdots\cdots\cdots (2)$$

The least-squares estimate $\widehat{\boldsymbol{\theta}}_{LS}$ of the unknown parameter vector $\boldsymbol{\theta}$ is obtained by

$$\widehat{\boldsymbol{\theta}}_{LS} = \left(\sum_{k=1}^{N}\boldsymbol{p}(k)\boldsymbol{p}(k)^T\right)^{-1}\sum_{k=1}^{N}\boldsymbol{p}(k)y(k). \cdots\cdot (3)$$

It is well known that the least-squares estimate $\widehat{\boldsymbol{\theta}}_{LS}$ is not a consistent estimate and thus have an asymptotic bias $\boldsymbol{h}$ unless $v(k)$ is white noise:

$$\boldsymbol{h} = \operatorname*{plim}_{N\to\infty}\widehat{\boldsymbol{\theta}}_{LS} - \boldsymbol{\theta} = -R_{pp}^{-1}\begin{bmatrix} I_n \\ O \end{bmatrix}\boldsymbol{m} \cdots\cdots (4)$$

where

$$R_{pp} = \operatorname*{plim}_{N\to\infty}\frac{1}{N}\sum_{k=1}^{N}\boldsymbol{p}(k)\boldsymbol{p}(k)^T = E[\boldsymbol{p}(k)\boldsymbol{p}(k)^T],$$

$$\boldsymbol{m} = \operatorname*{plim}_{N\to\infty}\frac{1}{N}\sum_{k=1}^{N}\boldsymbol{y}(k)v(k) = E[\boldsymbol{y}(k)v(k)].$$

From eqn.(4) we can deduce that

$$\operatorname*{plim}_{N\to\infty}\widehat{\boldsymbol{m}} = \boldsymbol{m}.$$

Basis of the principle of bias compensation, if we can obtain the estimate $\widehat{\boldsymbol{m}}$ of $\boldsymbol{m}$, then the bias-compensated least-squares estimate[1)] $\widehat{\boldsymbol{\theta}}_{BCLS}$ can be defined as

$$\widehat{\boldsymbol{\theta}}_{BCLS} = \widehat{\boldsymbol{\theta}}_{LS} + \widehat{R}_{pp}^{-1}\begin{bmatrix} I_n \\ O \end{bmatrix}\widehat{\boldsymbol{m}} \cdots\cdots\cdots\cdots (5)$$

would be a consistent estimate of $\boldsymbol{\theta}$,where

$$\widehat{R}_{pp} = \frac{1}{N}\sum_{k=1}^{N}\boldsymbol{p}(k)\boldsymbol{p}(k)^T.$$

In the following Section, we will derive the BELS method which can give consistent parameter estimate according to this principle of bias compensation.

3. BELS method

In order to obtain $\widehat{\boldsymbol{m}}$, we firstly introduce an m dimensional vector $\boldsymbol{\zeta}(k)$ as

$$\operatorname*{plim}_{N\to\infty}\frac{1}{N}\sum_{k=1}^{N}\boldsymbol{\zeta}(k)v(k) = 0.$$

Secondly, let $(2n+m)$ dimensional vectors $\overline{\boldsymbol{p}}(k)$, $\overline{\boldsymbol{\theta}}$ and $\boldsymbol{\psi}(k)$ be, respectively

$$\overline{\boldsymbol{p}}(k) = \begin{bmatrix} \boldsymbol{p}(k) \\ \boldsymbol{\zeta}(k) \end{bmatrix},$$

$$\boldsymbol{\psi}(k) = M\overline{\boldsymbol{p}}(k), \quad \overline{\boldsymbol{\theta}} = M^{-T}\begin{bmatrix} \boldsymbol{\theta} \\ 0 \end{bmatrix}. \cdots\cdots\cdots (6)$$

Then, eqn.(2) can be rewritten as

$$y(k) = \boldsymbol{\psi}(k)^T\overline{\boldsymbol{\theta}} + v(k) \cdots\cdots\cdots\cdots\cdots\cdots (7)$$

where M is a $(2n+m)\times(2n+m)$ non-singular matrix. Then, the least-squares estimate $\widehat{\overline{\boldsymbol{\theta}}}_{LS}$ of $\overline{\boldsymbol{\theta}}$ is obtained as follows:

$$\widehat{\overline{\boldsymbol{\theta}}}_{LS} = \widehat{R}_{\psi\psi}^{-1}\widehat{\boldsymbol{r}}_{\psi y} \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (8)$$

where

$$\widehat{R}_{\psi\psi} = \frac{1}{N}\sum_{k=1}^{N}\boldsymbol{\psi}(k)\boldsymbol{\psi}(k)^T,$$

$$\widehat{\boldsymbol{r}}_{\psi y} = \frac{1}{N}\sum_{k=1}^{N}\boldsymbol{\psi}(k)y(k).$$

Substituting eqn.(7) into eqn.(8) and re-arranging the result, we can get

$$\widehat{\overline{\boldsymbol{\theta}}}_{LS} = \overline{\boldsymbol{\theta}} + \widehat{R}_{\psi\psi}^{-1}\widehat{\boldsymbol{r}}_{\psi v} \cdots\cdots\cdots\cdots\cdots\cdots\cdots (9)$$

where

$$\widehat{\boldsymbol{r}}_{\psi v} = \frac{1}{N}\sum_{k=1}^{N}\boldsymbol{\psi}(k)v(k).$$

Now we begin to estimate the asympototic bias.
Let the matrix M be partitioned in the form

$$M = \begin{bmatrix} M_1 & M_2 & Z \end{bmatrix}, \quad M_1, M_2 : (2n+m)\times n, \quad Z : (2n+m)\times m.$$

and let eqn.(6) be written in the following form

$$M^T\overline{\boldsymbol{\theta}} = \begin{bmatrix} \boldsymbol{\theta} \\ 0 \end{bmatrix},$$

so we can obtain

$$Z^T\overline{\boldsymbol{\theta}} = \boldsymbol{0}.$$

Therefore, according to the eqn.(9), we have

$$Z^T\widehat{\overline{\boldsymbol{\theta}}}_{LS} = Z^T\widehat{R}_{\psi\psi}^{-1}\widehat{\boldsymbol{r}}_{\psi v}.$$

From definition of $\boldsymbol{\psi}$(k), $\overline{\boldsymbol{p}}(k)$ and assumption of $u(k)$ and $\boldsymbol{\zeta}$(k), we have

$$\operatorname*{plim}_{N\to\infty}\widehat{\boldsymbol{r}}_{\psi v} = \operatorname*{plim}_{N\to\infty}\frac{1}{N}\sum_{k=1}^{N}\boldsymbol{\psi}(k)v(k)$$

$$= M\operatorname*{plim}_{N\to\infty}\frac{1}{N}\sum_{k=1}^{N}\begin{bmatrix} \boldsymbol{p}(k) \\ \boldsymbol{\zeta}(k) \end{bmatrix}v(k)$$

$$= M\begin{bmatrix} -\operatorname*{plim}_{N\to\infty}\frac{1}{N}\sum_{k=1}^{N}\boldsymbol{y}(k)v(k) \\ \mathbf{o} \\ \mathbf{o} \end{bmatrix}$$

$$= - \begin{bmatrix} M_1 & M_2 & Z \end{bmatrix} \begin{bmatrix} \boldsymbol{m} \\ \mathbf{o} \\ \mathbf{o} \end{bmatrix}$$

$$= -M_1 \boldsymbol{m},$$

so we can get the result as

$$Z^T \operatorname*{plim}_{N\to\infty} \widehat{\overline{\boldsymbol{\theta}}}_{LS} = -Z^T \left(\operatorname*{plim}_{N\to\infty} \widehat{R}_{\psi\psi}^{-1} \right) M_1 \boldsymbol{m}.$$

Hence a consistent estimate $\widehat{\boldsymbol{m}}$ of $\boldsymbol{m}$ can be determined by, in the case of $m \geq n$,

$$\widehat{\boldsymbol{m}} = -\left(\widehat{D}^T W_{LS} \widehat{D} \right)^{-1} \widehat{D}^T W_{LS} Z^T \widehat{\overline{\boldsymbol{\theta}}}_{LS}, \quad \cdots (10)$$

where W_{LS} is a positive definite weighting matrix, and $m \times n$ matrix $\widehat{D}$ is

$$\widehat{D} = Z^T \widehat{R}_{\psi\psi}^{-1} M_1.$$

Therefore, the BELS estimate $\widehat{\overline{\boldsymbol{\theta}}}_{BELS}$ of $\overline{\boldsymbol{\theta}}$ can be obtained as

$$\widehat{\overline{\boldsymbol{\theta}}}_{BELS} = \widehat{\overline{\boldsymbol{\theta}}}_{LS} + \widehat{R}_{\psi\psi}^{-1} M_1 \widehat{\boldsymbol{m}}. \quad \cdots\cdots\cdots\cdots\cdots (11)$$

From eqn.(6), we have

$$\widehat{\boldsymbol{\theta}}_{BELS} = (H^T H)^{-1} H^T \widehat{\overline{\boldsymbol{\theta}}}_{BELS} \quad \cdots\cdots\cdots\cdots (12)$$

where $(2n+m) \times 2n$ matrix H is

$$H = M^{-T} \begin{bmatrix} I_{2n} \\ O \end{bmatrix}.$$

In the next section, we will examine the BELS estimate $\widehat{\boldsymbol{\theta}}_{BELS}$.

4. Property of BELS estimate

We now analyze the properties of the BELS estimate. Defining an $n+m$ dimensional vector $\boldsymbol{\eta}(k)$ as

$$\boldsymbol{\eta}(k) = \begin{bmatrix} \boldsymbol{u}(k) \\ \boldsymbol{\zeta}(k) \end{bmatrix},$$

it follows from the definition of $\boldsymbol{\psi}(k)$ and $\overline{\boldsymbol{p}}(k)$ that the least-squares estimate $\widehat{\overline{\boldsymbol{\theta}}}_{LS}$ in eqn.(8) can be written as

$$\widehat{\overline{\boldsymbol{\theta}}}_{LS} = -\widehat{R}_{\psi\psi}^{-1} M_1 \widehat{\boldsymbol{r}}_{yy} + \widehat{R}_{\psi\psi}^{-1} \begin{bmatrix} M_2 & Z \end{bmatrix} \widehat{\boldsymbol{r}}_{\eta y}$$

where

$$\widehat{\boldsymbol{r}}_{yy} = \frac{1}{N} \sum_{k=1}^{N} \boldsymbol{y}(k) y(k), \quad \widehat{\boldsymbol{r}}_{\eta y} = \frac{1}{N} \sum_{k=1}^{N} \boldsymbol{\eta}(k) y(k).$$

On the other hand defining a matrix P as

$$P = \widehat{D}^T W_{LS} \widehat{D},$$

the BELS estimate $\widehat{\overline{\boldsymbol{\theta}}}_{BELS}$ can be written from eqn.(10) and eqn.(11)

$$\widehat{\overline{\boldsymbol{\theta}}}_{BELS} = \left[I - \widehat{R}_{\psi\psi}^{-1} M_1 P^{-1} \widehat{D}^T W_{LS} Z^T \right] \widehat{\overline{\boldsymbol{\theta}}}_{LS}.$$

Noting that

$$\left[I - \widehat{R}_{\psi\psi}^{-1} M_1 P^{-1} \widehat{D}^T W_{LS} Z^T \right] \widehat{R}_{\psi\psi}^{-1} M_1 = 0,$$

we have

$$\widehat{\overline{\boldsymbol{\theta}}}_{BELS} = \left[I - \widehat{R}_{\psi\psi}^{-1} M_1 P^{-1} \widehat{D}^T W_{LS} Z^T \right] \widehat{R}_{\psi\psi}^{-1} \cdot \begin{bmatrix} M_2 & Z \end{bmatrix} \widehat{\boldsymbol{r}}_{\eta y}.$$

Then defining a $2n \times n$ matrix $\widehat{C}$ and a $2n \times (n+m)$ matrix $\widehat{K}$ as respectively,

$$\begin{bmatrix} \widehat{C} & \widehat{K} \end{bmatrix} = (H^T H)^{-1} H^T \widehat{R}_{\psi\psi}^{-1} M,$$

so $\widehat{\boldsymbol{\theta}}_{BELS}$ can be described by

$$\widehat{\boldsymbol{\theta}}_{BELS} = \left[\widehat{K} - \widehat{C} P^{-1} \widehat{D}^T W_{LS} \widehat{F} \right] \widehat{\boldsymbol{r}}_{\eta y} \quad \cdots\cdots (13)$$

where $m \times (n+m)$ matrix $\widehat{F}$ is

$$\widehat{F} = Z^T \widehat{R}_{\psi\psi}^{-1} \begin{bmatrix} M_2 & Z \end{bmatrix}.$$

From the definition of $\widehat{C}$, $\widehat{K}$, $\widehat{D}$ and $\widehat{F}$, we have

$$\begin{bmatrix} \widehat{C} & \widehat{K} \\ \widehat{D} & \widehat{F} \end{bmatrix} = \begin{bmatrix} (H^T H)^{-1} H^T \\ Z^T \end{bmatrix} \widehat{R}_{\psi\psi}^{-1} M,$$

$$\begin{bmatrix} \widehat{C} & \widehat{K} \\ \widehat{D} & \widehat{F} \end{bmatrix} M^{-1} \widehat{R}_{\psi\psi} = \begin{bmatrix} (H^T H)^{-1} H^T \\ Z^T \end{bmatrix}.$$

Post-multiplying by H on both sides yields

$$\begin{bmatrix} \widehat{C} & \widehat{K} \\ \widehat{D} & \widehat{F} \end{bmatrix} M^{-1} \widehat{R}_{\psi\psi} H = \begin{bmatrix} I_{2n} \\ O \end{bmatrix}.$$

Considering $\boldsymbol{\psi}(k) = M \overline{\boldsymbol{p}}(k)$ in above equation gives

$$\begin{bmatrix} \widehat{C} & \widehat{K} \\ \widehat{D} & \widehat{F} \end{bmatrix} \begin{bmatrix} -\widehat{R}_{yp} \\ \widehat{R}_{\eta p} \end{bmatrix} = \begin{bmatrix} I_{2n} \\ O \end{bmatrix},$$

and thus,

$$\left[\widehat{K} - \widehat{C} P^{-1} \widehat{D}^T W_{LS} \widehat{F} \right] \widehat{R}_{\eta p} = I_{2n} \quad \cdots\cdots\cdots (14)$$

where

$$\widehat{R}_{yp} = \frac{1}{N} \sum_{k=1}^{N} \boldsymbol{y}(k) \boldsymbol{p}(k)^T,$$

$$\widehat{R}_{\eta p} = \frac{1}{N} \sum_{k=1}^{N} \boldsymbol{\eta}(k) \boldsymbol{p}(k)^T.$$

If the dimension m of the introduced vector $\boldsymbol{\zeta}$(k) is equal to n, it follows from the definition of $\boldsymbol{\eta}$(k) that the dimension of $\boldsymbol{\eta}$(k) will become to $2n$ and be equal to the dimension of the data vector $\boldsymbol{p}$(k). That means the matrix $\widehat{R}_{\eta p}$ is square matrix.

Therefore, if matrix $\widehat{R}_{\eta p}$ is non-singular, we can see from eqn.(13) and (14) that the BELS estimate $\widehat{\boldsymbol{\theta}}_{BELS}$ becomes as follows:

$$\widehat{\boldsymbol{\theta}}_{BELS} = \widehat{R}_{\eta p}^{-1}\widehat{\boldsymbol{r}}_{\eta y}$$

$$= \left(\sum_{k=1}^{N}\boldsymbol{\eta}(k)\boldsymbol{p}(k)^T\right)^{-1}\sum_{k=1}^{N}\boldsymbol{\eta}(k)y(k). \cdots\cdots\cdots (15)$$

From the assumption of $\boldsymbol{u}(k)$ and $\boldsymbol{\zeta}(k)$, we have

$$\operatorname*{plim}_{N\to\infty}\frac{1}{N}\sum_{k=1}^{N}\boldsymbol{\eta}(k)v(k) = 0.$$

If

$$\operatorname*{plim}_{N\to\infty}\frac{1}{N}\sum_{k=1}^{N}\boldsymbol{\eta}(k)\boldsymbol{p}(k)^T$$

is non-singular, the variable $\boldsymbol{\eta}(k)$ can satisfy the condition of the instrumental variable.

Eqn.(15) indicates that the BELS estimate $\widehat{\boldsymbol{\theta}}_{BELS}$ belongs to the IV estimate when the dimension m of the BELS auxiliary vector $\boldsymbol{\zeta}(k)$ is equal to the system order n. Furthermore, in this case the BELS estimate dose not depend on the choices of M and W_{LS}.

In the following, the matrix M in the BELS methods which have been proposed so far are shown(where $m = n$):

(1) OBELS[2)]:

$$M = \begin{bmatrix} I_n & \begin{bmatrix} 0 & 0 \end{bmatrix} \\ 0 & L^T \\ 0 & \begin{bmatrix} 0 & I_n \end{bmatrix} \end{bmatrix}^{-1}$$

where matrix L is made by the coefficients of the polynomial $F(q^{-1}) = 1 + f_1q^{-1} + \cdots + f_nq^{-n}$.

$$L^T = \begin{bmatrix} 1 & f_1 & f_2 & \cdots & f_n & & & \\ & 1 & f_1 & f_2 & \cdots & f_n & & \\ & & \ddots & \ddots & & & \ddots & \\ & & & 1 & f_1 & f_2 & \cdots & f_n \end{bmatrix},$$

(2) W.X. Zheng's BELS[8)]:

$$M = \begin{bmatrix} I_{2n} & -L \\ 0 & I_n \end{bmatrix},$$

(3) Y. Zhangs' BCLS[5)]:

$$M = \begin{bmatrix} I_n & 0 & -L_1 \\ 0 & I_n & 0 \\ 0 & 0 & I_n \end{bmatrix},$$

(4) DBELS[6)]:

$$M = I_{3n},$$

For this choice, the BELS estimate can be expressed as:

$$\widehat{\boldsymbol{\theta}}_{DBELS} = \widehat{\boldsymbol{\theta}}_{LS} + \widehat{R}_{pp}^{-1}\begin{bmatrix} I_n \\ O \end{bmatrix}\widehat{\boldsymbol{m}},$$

$$\widehat{\boldsymbol{m}} = -\left(\widehat{R}_{p\zeta}^T\widehat{R}_{pp}^{-1}\begin{bmatrix} I_n \\ O \end{bmatrix}\right)^{-1}\left(\widehat{\boldsymbol{r}}_{\zeta y} - \widehat{R}_{p\zeta}^T\widehat{\boldsymbol{\theta}}_{LS}\right).$$

5. Simulation

To illustrate the theoretical results we now present simulation studies.

Simulation setting

Consider the following system:

$$y(t) - 1.5y(t-1) + 0.7y(t-2)$$
$$= 1.0u(t-1) + 0.5u(t-2) + v(t)$$

where

$$v(t) = e(t) - 1.5e(t-1) + 0.7e(t-2).$$

input $u(t)$ is normal random variable with variance 1, $e(t)$ is a white noise and the variance of $e(t)$ is determined by SNR (SNR=5).

Let us examine when the polynomials $F(q^{-1})$ are described as the following case respectively :

Case1: $F(q^{-1}) = (1.0 - 0.6q^{-1})(1.0 - 0.8q^{-1})$;

Case2: $F(q^{-1}) = (1.0 - 0.6q^{-1})(1.0 - 0.75q^{-1})(1.0 - 0.9q^{-1})$.

That is, we will examine the case of m=n and $m > n$.

The simulation results are shown in Table.1, Fig.1 (Case 1: $m = n$) and Table.2, Fig.2 (Case 2: $m > n$). The mean values of the parameter estimates obtained by the LS method (eqn.(3)), the BELS method (eqn.(12)) and the IV method (eqn.(15)) for ten runs are listed in Table.1 and Table.2. The estimation errors are shown in Fig.1 and Fig.2.

Simulation results demonstrates that the LS method is unable to give rise to consistent estimates, whereas the BELS method and the IV method are able to estimate parameters consistently.

Table.1 shows that the values obtained by the BELS method coincides completely with those by the IV method for the case of $m = n$. This observation confirms the validity of the theoretical conclusion that the BELS method belongs to the class of the IV method when the dimension m of the BELS auxiliary variable vector is equal to the system order n.

Table 1. Simulation result in Case 1 (SNR=5)

	N	a_1	a_2	b_1	b_2
LS method	50	1.1414	-0.3626	0.9179	0.7827
	500	1.1310	-0.3558	1.0038	0.8795
	2000	1.1435	-0.3674	0.9849	0.8499
IV method	50	1.5498	-0.7427	0.9301	0.4655
	500	1.4978	-0.6987	0.9835	0.5142
	2000	1.5023	-0.7014	0.9910	0.4997
BELS method	50	1.5498	-0.7427	0.9301	0.4655
	500	1.4978	-0.6987	0.9835	0.5142
	2000	1.5023	-0.7014	0.9910	0.4997
True value		1.5	-0.7	1.0	0.5

6. Conclusion

In this paper, we focus on the analysis and discussion of the BELS method. In order to examine some recently proposed bias-correction based method in a unified fashion, we derived the BELS estimate by introducing the

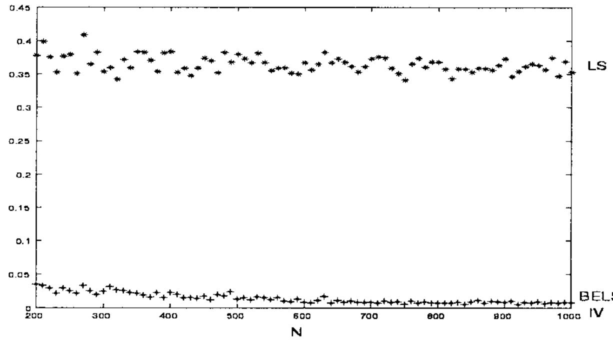


Fig. 1.　$\|\, \hat{\theta} - \theta \,\|_2$ vs N　LS: * , IV: + , BELS: ·

Table 2.　Simulation result in Case 2 (SNR=5)

	N	a_1	a_2	b_1	b_2
LS method	50	1.1414	-0.3626	0.9179	0.7827
	500	1.1310	-0.3558	1.0038	0.8795
	2000	1.1435	-0.3674	0.9849	0.8499
IV method	50	1.4937	-0.6973	0.8989	0.5650
	500	1.5017	-0.7018	0.9849	0.5023
	2000	1.4995	-0.6994	0.9909	0.5096
BELS method	50	1.5334	-0.7394	0.9409	0.4435
	500	1.4980	-0.6976	0.9865	0.5156
	2000	1.5031	-0.7030	0.9913	0.4982
True value		1.5	-0.7	1.0	0.5

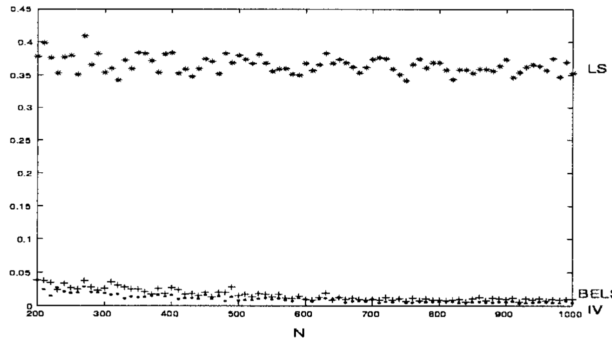


Fig. 2.　$\|\, \hat{\theta} - \theta \,\|_2$ vs N　LS: * , IV: + , BELS: ·

auxiliary vector $\zeta(k)$ and matrix M.

The theoretical analysis indicates that when the dimension m of BELS auxiliary variable vector is equal to the system order n, the BELS method belongs to the class of the IV method and the estimation accuracy is not affected by the matrix M which is introduced in order to estimate the asymptotic bias.

Simulations are performed to validate the theoretical discussions.

(Manuscript received June 2, 2000, revised December 5, 2000)

References

(1) S. Sagara and K. Wada: *On-line modified least-squares parameter estimation on linear discrete dynamic systems.*, Int.J.Control.Vol.25,No.3 329/343(1977)

(2) Chun-Bo Feng and Wei-Xing Zheng: *Robust identification of stochastic linear systems with correlated output noise.*, IEE Proceedings-D,Vol.138,No.5,484-492(1991)

(3) P. Stoica, T. Soderstrom and V. Simonyte: *Study of a bias-free least squares parameter estimator.*, IEE Proc.-Control Theory Appl., Vol.142, No.1, 1-6(1995)

(4) Wada: *On a system identification method proposed by C.B.Feng*, 1440, 898-899(1995)

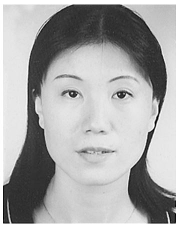
(5) Y. Zhang, T. T. Lie and C. B. Soh: *Consistent parameter estimation of systems disturbed by correlated noise.*, IEE Proc.-Control Theory Appl., Vol.144, No.1, 40-44(1997)

(6) W. X. Zheng: *On a least-squares-based algorithm for identification of stochastc linear systems.* IEEE Trans. Signal Processing, Vol.46, No.6, 1631-1638(1998)

(7) T. Söderstrom, W. X. Zheng and P. Stoica: *Comments on "On a least-squares-based algorithm for identification of stochastc linear systems".* IEEE Trans. Signal Processing, Vol.47, No.5 1395-1396(1999)

(8) W. X. Zheng: *On parameter estimation of transfer function models.* Proc. 38th IEEE CDC, 2412-2413(1998)

Kiyoshi WADA　(Member)　He was born in Shimonoseki, Japan on November 24, 1947. He received the B. E., M. E. and D. E. degrees in electrical engineering from Kyushu University, Fukuoka, Japan in 1970, 1972 and 1978 respectively. He is currently a professor in the department of electrical and electronic systems engineering, graduate school of information science and electrical engineering, Kyushu University. His research interests are in the areas of system identification, digital signal processing and adaptive control, etc. He is a member of the Institue of Systems, Control and Information Engineers; Society of Instrument and Control Engineers; IEEE.



Lijuan JIA　(Non-member)　She received the B.S and M.S degrees in 1990 and 1993 respectively. From 1993 to 1999, she worked in ARIM, Beijing, China. She is currently pursuing the Ph.D. degree at Department of Electrical and Electronic System Engineering of Graduate school of Information Science and Electrical Engineering, Kyushu University, Fukuoka city, Japan.

Takeshi HANADA　(Non-member)　He was born on June 22, 1977. He received the B.S degree from Kyushu University on March,2000. Now he is studying at Department of Electrical and Electronic System Engineering of Graduate school of Information Science and Electrical Engineering, Kyushu University, Fukuoka city, Japan.

Jun IMAI　(Member)　He received B.S., M.S. and Ph.D degrees in Electrical Engineering all from Kyushu University in 1987, 1989, and 1992, respectively. He served as a Research Associate in Kyushu Institute of Technology from 1992-1994, and then in Kyushu University to 2000, respectively. From 2000 he is serving as an assistant professor in the department of Electrical and Electronic Engineering, Okayama University. His main interest is in modeling and control of distributed parameter systems.