Paper

Robust Nonlinear Control of a Feedback Linearizable Voltage-Controlled Magnetic Levitation System

Member Zi-Jiang Yang (Kyushu University)
Non-member Masayuki Minashima (Kyushu Institute of Technology)

This paper considers the position tracking problem of a voltage-controlled magnetic levitation system. It is well known that the control problem is quite complicated and challenging due to inherent nonlinearities associated with the electromechanical dynamics. And feedback linearization approach via coordinate transformation is considered to be a useful candidate to tackle the control problem. Usually however, feedback linearization control does not guarantee exact linearization and robustness in the presence of model uncertainties. In this paper, we propose a robust nonlinear controller in the presence of parametric uncertainties. The design procedure is carried out in a backstepping design manner, by employing nonlinear damping terms to suppress the effects of parametric uncertainties which may cause instability. Input-to-state stable property of the control system is analyzed, and experimental results are included to show the excellent position tracking performance of the designed control system.

Keywords: Magnetic levitation system, feedback linearization, nonlinear damping term, backstepping design, input-to-state stability.

1. Introduction

Magnetic levitation systems are widely used in various fields, such as frictionless bearings, high-speed maglev passenger trains, levitation of wind tunnel models etc., and it is an important task to construct a high performance feedback controller to control the position of the levitated object since a magnetic levitation system is usually open-loop unstable. Due to inherent nonlinearities associated with the electromechanical dynamics, the control problem is usually quite challenging to the control engineers, since a linear controller is valid only about a small region around a nominal operating point. In recent years, a lot of works have been reported in the literature, for controlling a magnetic levitation system by actively taking nonlinearities of the system model into account [(1)(3)~(5)(7)(11)(12)(14)~(17)(19)~(22)]. In the references raised here, high performance control of a magnetic levitation system in the presence of parametric uncertainties is of particular interest.

In our previous work [(22)], a robust nonlinear controller is proposed for a current-controlled magnetic levitation system governed by an SISO second-order nonlinear differential equation, where a current feedback power amplifier is employed. Control of a voltage-controlled magnetic levitation system is much more difficult and challenging, since in this case the system model is governed by a third-order nonlinear model which is usually not in canonical form. In reference (12), an adaptive partial state feedback controller via backstepping design for a levitated ball is presented, with neither simulation nor experimental results. And in reference (17), a backstepping based controller is presented for an active magnetic bearings and numerical simulation results are provided to verify the control performance. However, further study is required to counteract parametric uncertainties. Applications of the feedback linearization techniques via coordinate transformation [(10)] have also been reported in the literature [(3)(4)(11)(19)(20)]. In reference (19) an H_∞ controller, and in reference (4) a sliding mode controller are introduced to enhance the robustness of the feedback linearization controller. However, stability and control performance in the presence of parametric uncertainties are not analyzed explicitly. In reference (11), a robust feedback linearization controller is presented, and the stability is analyzed strictly based on the Kharitonov's theorem [(2)]. The basic idea of this approach is to transform the nonlinear system with uncertain mass and bounded external disturbance into a linear interval matrix robustness problem. However, uncertainties of the other parameters are not considered, and although the stability of the control system is guaranteed theoretically, it seems that the control results exhibit relatively large overshoots.

In this paper, motivated by the pioneering works mentioned above, we propose a robust nonlinear controller for a volatge-controlled magnetic levitation system in the presence of parametric uncertainties. At first, the third-order nonlinear system model is transformed via coordinate transformation into a more transparent model which is a composite of a canonical nominal model and perturbations due to modelling errors caused by parametric uncertainties. Then a backstepping design procedure is performed by employing nonlinear damping terms [(12)(13)] at each step to suppress the effects of the parametric uncertainties which may cause instability. Input-to-state stability (ISS)† of the control system is analyzed and experimental results are included to show the excellent position tracking performance of the designed control system.

2. Model of the magnetic levitation system

Consider a magnetic levitation system shown in Fig. 1. This is a popular gravity-biased one degree-of-freedom magnetic levitation system, in which an electromagnet exerts attractive force to levitate a steel ball (in some references a steel plate is levitated). The system dynamics can be described in the following equations [3] [11] [18]~[20].

$$\begin{aligned} M\ddot{x} &= Mg + \frac{1}{2}i^2\frac{\partial L}{\partial x} \\ u &= Ri + \frac{d}{dt}(Li) \end{aligned} \quad \cdots\cdots (1)$$

where the coil inductance is given as

$$L(x) = \frac{Q}{X_\infty + x} + L_\infty \quad \cdots\cdots (2)$$

and, x: air gap (vertical position) of the steel ball; i: coil current; g: gravity acceleration; M: mass of the steel ball; R: electrical resistance; u: voltage control input applied to the system; L_∞, Q and X_∞: positive constants determined by the characteristics of the coil, magnetic core and steel ball.

Defining the state variable vector as $\boldsymbol{x} = [x_1, x_2, x_3]^T = [x, \dot{x}, i]^T$ and rewriting equation (1), we have the following nonlinear state space model.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} x_2 \\ \alpha(\boldsymbol{x}) \\ \beta(\boldsymbol{x}) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \gamma(\boldsymbol{x}) \end{bmatrix} u \quad \cdots\cdots (3)$$

where

$$\begin{aligned} \alpha(\boldsymbol{x}) &= g - \frac{Qx_3^2}{2M(X_\infty + x_1)^2} \\ \beta(\boldsymbol{x}) &= \frac{x_3\{Qx_2 - R(X_\infty + x_1)^2\}}{Q(X_\infty + x_1) + L_\infty(X_\infty + x_1)^2} \\ \gamma(\boldsymbol{x}) &= \frac{X_\infty + x_1}{Q + L_\infty(X_\infty + x_1)} \end{aligned} \quad \cdots (4)$$

Denote the nominal values of the physical parameters as g_0, M_0, R_0, $L_{\infty 0}$, Q_0 and $X_{\infty 0}$. It is assumed here these nominal parameters which are only rough estimates of their exact values are known *a priori*. Then the system model can be rewritten as

$$\begin{aligned} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} &= \boldsymbol{f}_0(\boldsymbol{x}) + \Delta_{\boldsymbol{f}}(\boldsymbol{x}) + \boldsymbol{g}_0(\boldsymbol{x})u + \Delta_{\boldsymbol{g}}(\boldsymbol{x})u \\ &= \begin{bmatrix} x_2 \\ \alpha_0(\boldsymbol{x}) + \Delta_\alpha(\boldsymbol{x}) \\ \beta_0(\boldsymbol{x}) + \Delta_\beta(\boldsymbol{x}) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \gamma_0(\boldsymbol{x}) + \Delta_\gamma(\boldsymbol{x}) \end{bmatrix} u \end{aligned} \quad (5)$$

†In this paper, both Input-to-State Stability and Input-to-State Stable will be denoted as ISS for convenience.

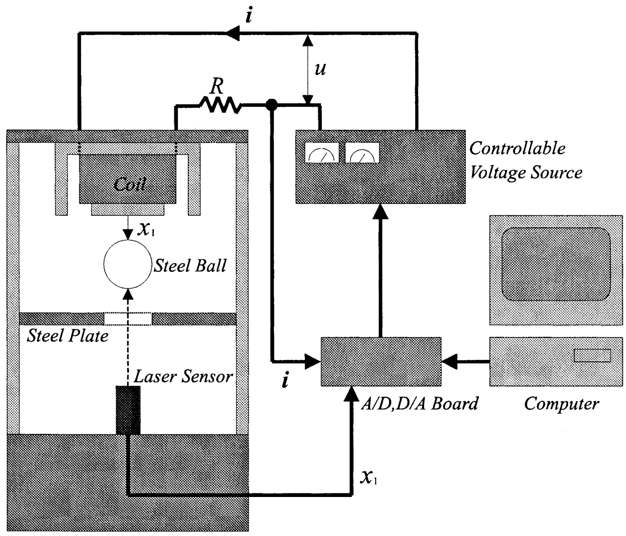


Fig. 1. Diagram of the magnetic levitation system.

where

$$\begin{aligned} \alpha_0(\boldsymbol{x}) &= g_0 - \frac{Q_0x_3^2}{2M_0(X_{\infty 0} + x_1)^2} \\ \beta_0(\boldsymbol{x}) &= \frac{x_3\{Q_0x_2 - R_0(X_{\infty 0} + x_1)^2\}}{Q_0(X_{\infty 0} + x_1) + L_{\infty 0}(X_{\infty 0} + x_1)^2} \\ \gamma_0(\boldsymbol{x}) &= \frac{X_{\infty 0} + x_1}{Q_0 + L_{\infty 0}(X_{\infty 0} + x_1)} \end{aligned} \quad (6)$$

are the nominal nonlinear functions, and

$$\begin{aligned} \Delta_\alpha(\boldsymbol{x}) &= \alpha(\boldsymbol{x}) - \alpha_0(\boldsymbol{x}) \\ \Delta_\beta(\boldsymbol{x}) &= \beta(\boldsymbol{x}) - \beta_0(\boldsymbol{x}) \\ \Delta_\gamma(\boldsymbol{x}) &= \gamma(\boldsymbol{x}) - \gamma_0(\boldsymbol{x}) \end{aligned} \quad \cdots\cdots (7)$$

are the modelling errors due to parametric uncertainties.

Remark 1: An external constant mechanical disturbance can be viewed as a biased error of g equivalently, i.e., the gravity acceleration is biased equivalently. Therefore we will not treat such a disturbance explicitly for simplicity.

3. Coordinate transformation

A general objective in the synthesis of feedback linearizing controllers is the derivation of coordinate transformations which convert the original nonlinear system into a system that is simpler in the sense that controller synthesis is more straightforward. For theoretical background, the readers are referred to references (6), (8) and (10). Our task here is to seek a local diffeomorphism $\boldsymbol{\xi} = \boldsymbol{T}(\boldsymbol{x})$ which transforms the nominal part in equation (5) into a canonical form [3] [4] [11] [19] [20].

Define

$$\phi(\boldsymbol{x}) = x_1 \quad \cdots\cdots (8)$$

Then through straightforward calculations, we have the following nonlinear coordinate transformation.

$$\boldsymbol{\xi} = [\xi_1,\ \xi_2,\ \xi_3]^T = \left[\phi(\boldsymbol{x}),\ L_{\boldsymbol{f}_0}\phi(\boldsymbol{x}),\ L^2_{\boldsymbol{f}_0}\phi(\boldsymbol{x})\right]^T = [x_1, x_2, \alpha_0(\boldsymbol{x})]^T \quad (9)$$

Notice that the coordinate transformation employed here has transparent physical meaning, i.e., states ξ_1 and ξ_2 are simply the original position and velocity, while state ξ_3 is the nominal acceleration applied to the levitated ball.

Remark 2: The diffeomorphim $\boldsymbol{\xi} = \boldsymbol{T}(\boldsymbol{x})$ is only locally defined in $\Omega = \{\boldsymbol{x}|0 < x_1 \leq x_{1M},\ x_3 > 0\} \subset R^3$. The restriction $0 < x_1 \leq x_{1M}$ is due to physically allowable operating region of x_1 (see Fig. 1), and the restriction $x_3 > 0$ is in order to avoid a singular point of the control input u, as will be seen later in equations (13) and (16).

The derivatives of the new states can be simply obtained as follows.

$$\frac{d\xi_1}{dt} = \dot{x}_1 \quad (10)$$

$$\frac{d\xi_2}{dt} = \dot{x}_2 = \alpha_0(\boldsymbol{x}) + \Delta_\alpha(\boldsymbol{x}) = \xi_3 + \Delta_\alpha(\boldsymbol{x}) \quad (11)$$

$$\frac{d\xi_3}{dt} = \frac{d\alpha_0(\boldsymbol{x})}{dt} = \frac{\partial\alpha_0}{\partial x_1}\frac{dx_1}{dt} + \frac{\partial\alpha_0}{\partial x_3}\frac{dx_3}{dt} \quad (12)$$

Hence the nonlinear state space model (5) is transformed into

$$\begin{aligned}\dot{\xi}_1 &= \xi_2\\ \dot{\xi}_2 &= \Delta_\alpha\left(\boldsymbol{T}^{-1}(\boldsymbol{\xi})\right) + \xi_3\\ \dot{\xi}_3 &= F_1\left(\boldsymbol{T}^{-1}(\boldsymbol{\xi})\right) + F_0\left(\boldsymbol{T}^{-1}(\boldsymbol{\xi})\right) + \Delta_F\left(\boldsymbol{T}^{-1}(\boldsymbol{\xi})\right)\\ &\quad + G_0\left(\boldsymbol{T}^{-1}(\boldsymbol{\xi})\right)u + \Delta_G\left(\boldsymbol{T}^{-1}(\boldsymbol{\xi})\right)u\end{aligned} \quad (13)$$

where

$$F_1\left(\boldsymbol{T}^{-1}(\boldsymbol{\xi})\right) = \frac{\partial\alpha_0}{\partial x_1}x_2 = \frac{Q_0x_3^2}{M_0(X_{\infty0}+x_1)^3}x_2 \quad (14)$$

$$F_0\left(\boldsymbol{T}^{-1}(\boldsymbol{\xi})\right) = \frac{\partial\alpha_0}{\partial x_3}\beta_0(\boldsymbol{x}) = -\frac{Q_0x_3^2\{Q_0x_2 - R_0(X_{\infty0}+x_1)^2\}}{M_0(X_{\infty0}+x_1)^3\{Q_0 + L_{\infty0}(X_{\infty0}+x_1)\}} \quad (15)$$

$$G_0\left(\boldsymbol{T}^{-1}(\boldsymbol{\xi})\right) = \frac{\partial\alpha_0}{\partial x_3}\gamma_0(\boldsymbol{x}) = -\frac{Q_0x_3}{M_0(X_{\infty0}+x_1)\{Q_0 + L_{\infty0}(X_{\infty0}+x_1)\}} \quad (16)$$

$$\Delta_F\left(\boldsymbol{T}^{-1}(\boldsymbol{\xi})\right) = \frac{\partial\alpha_0}{\partial x_3}\Delta_\beta(\boldsymbol{x}) \quad (17)$$

$$\Delta_G\left(\boldsymbol{T}^{-1}(\boldsymbol{\xi})\right) = \frac{\partial\alpha_0}{\partial x_3}\Delta_\gamma(\boldsymbol{x}) \quad (18)$$

$$\Delta_\alpha\left(\boldsymbol{T}^{-1}(\boldsymbol{\xi})\right) = \Delta_\alpha(\boldsymbol{x}) \quad (19)$$

Inspection of equation (13) indicates that if the parametric uncertainties do not exist such that the transformed perturbations $\Delta_\alpha\left(\boldsymbol{T}^{-1}(\boldsymbol{\xi})\right)$, $\Delta_F\left(\boldsymbol{T}^{-1}(\boldsymbol{\xi})\right)$ and $\Delta_G\left(\boldsymbol{T}^{-1}(\boldsymbol{\xi})\right)$ disappear, then the transformed system model can be exactly linearized by a simple state feedback controller. However, in the presence of parametric uncertainties, robustness of a feedback linearization controller is a very important topic, since modelling errors usually preclude exact cancellation of nonlinear terms. Fortunately, the structure of equation (13) while not being in a strict feedback form (12), still facilitates the backstepping design technique, which can incorporate nonlinear damping terms flexibly to suppress the effects of the modelling errors which may cause instability.

4. Backstepping design of the robust nonlinear controller

In this section, we show the design procedure of the robust nonlinear controller via the backstepping approach. It is assumed here that the reference position y_r of the steel ball and its first, second and third derivatives, i.e., $\dot{y}_r$, $\ddot{y}_r$ and $y_r^{(3)}$ are continuous, uniformly bounded, and available.

The concrete design procedure is given as follows.

Step 1:

Define the error signals of position ξ_1 and velocity ξ_2 as

$$\begin{aligned}z_1 &= \xi_1 - y_r\\ z_2 &= \xi_2 - \alpha_1\end{aligned} \quad (20)$$

where α_1 is a virtual input to stabilize z_1. Substituting $\xi_2 = z_2 + \alpha_1$ into the first row of equation (13), we have subsystem $\mathcal{S}1$ as the following.

$$\dot{z}_1 = \alpha_1 + z_2 - \dot{y}_r \quad (21)$$

The virtual input α_1 is designed here based on the common PI control technique, to stabilize subsystem $\mathcal{S}1$ and to remove the offset of z_1 due to z_2.

$$\alpha_1 = -c_{1p}z_1 - c_{1i}\int_0^t z_1\ dt + \dot{y}_r \quad (22)$$

where $c_{1p} > 0,\ c_{1i} > 0$.

Denote the Laplace operator as s. Then subsystem $\mathcal{S}1$ controlled by α_1 can be expressed as

$$z_1 = \frac{sz_2}{s^2 + c_{1p}s + c_{1i}} \quad (23)$$

Let h be the impulse response of transfer function $s/(s^2 + c_{1p}s + c_{1i})$. Then according to Theorem B.2 in reference (12), we have

$$\| z_1 \|_\infty \leq \| h \|_1 \| z_2 \|_\infty \quad (24)$$

Therefore, if the velocity error z_2 is stabilized to a neighbourhood of the origin, $|z_1|$ can be made sufficiently small by a suitably designed $\| h \|_1$, and the offset of z_1 can be removed by the integrator.

Equation (23) can be put into the following state-space model.

$$\dot{\boldsymbol{z}}_{1a} = \boldsymbol{A}\, \boldsymbol{z}_{1a} + \boldsymbol{B}\, z_2 \quad \cdots\cdots\cdots\cdots\cdots\cdots (25)$$

where

$$\begin{aligned} \boldsymbol{z}_{1a} &= [\, z_{1i} \quad z_1\,]^T \\ \boldsymbol{A} &= \begin{bmatrix} 0 & 1 \\ -c_{1i} & -c_{1p} \end{bmatrix} \\ \boldsymbol{B} &= [\,0 \quad 1\,]^T \end{aligned} \quad \cdots\cdots\cdots\cdots\cdots\cdots (26)$$

As a preparation for the ISS analysis (the definition of ISS can be found in Appendix C of reference (12)) of the overall error system discussed later, we have the ISS of subsystem $\mathcal{S}1$ with respect to z_2 by lemma 1 [(22)].

Lemma 1 If the virtual input α_1 is applied to subsystem $\mathcal{S}1$, and if z_2 is continuous and uniformly bounded, then $\mathcal{S}1$ is ISS, i.e., for $^\exists\lambda_0 > 0, {}^\exists\alpha_0 > 0$ and $^\exists M > 0$,

$$|\boldsymbol{z}_{1a}| \leq \lambda_0 e^{-\alpha_0 t}|\boldsymbol{z}_{1a}(0)| + M \left[\sup_{0\leq\tau\leq t} |z_2(\tau)|\right]$$

Step 2:

Define the error signal of the nominal acceleration as

$$z_3 = \xi_3 - \alpha_2 \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (27)$$

where α_2 is a virtual input to stabilize z_2.

Then equations (11) and (22) lead to subsystem $\mathcal{S}2$ as

$$\begin{aligned} \dot{z}_2 &= \dot{\xi}_2 - \dot{\alpha}_1 \\ &= -\dot{\alpha}_1 + \Delta_\alpha + \xi_3 \\ &= c_{1p}\dot{z}_1 + c_{1i}z_1 - \ddot{y}_r + \Delta_\alpha + \alpha_2 + z_3 \end{aligned} \quad \cdots (28)$$

It is known that in a nonlinear control system, neglecting the effects of uncertainties may degenerate the control performance or even destroy the stability of the system [(12)(13)]. Introducing a nonlinear damping term to the controller is an effective approach to counteract the uncertainties [(12)(13)]. Motivated by the works of (12) and (13), we design the virtual input α_2 as the following to stabilize subsystem $\mathcal{S}2$.

$$\begin{aligned} \alpha_2 &= \alpha_{20} - \alpha_{21} \\ \alpha_{20} &= -c_2z_2 - c_{1p}(\xi_2 - \dot{y}_r) - c_{1i}z_1 + \ddot{y}_r \\ \alpha_{21} &= \kappa_2\alpha_d z_2 \end{aligned} \quad \cdots (29)$$

where $c_2 > 0,\ \kappa_2 > 0$, and

$$\alpha_d = \frac{Q_0 x_3^2}{2M_0(X_{\infty 0} + x_1)^2} \quad \cdots\cdots\cdots\cdots\cdots\cdots (30)$$

Here α_{20} is a nominal controller to control the nominal system model, while α_{21} is a nonlinear damping term to counteract Δ_α.

Applying α_2 to subsystem $\mathcal{S}2$, we have

$$\dot{z}_2 = -c_2z_2 + \Delta_\alpha - \kappa_2\alpha_d z_2 + z_3 \quad \cdots\cdots\cdots\cdots (31)$$

The ISS of subsystem $\mathcal{S}2$ is shown in the following lemma.

Lemma 2 If the virtual input α_2 is applied to subsystem $\mathcal{S}2$, and if z_3 is continuous and uniformly bounded, then $\mathcal{S}2$ is ISS such that

$$|z_2(t)| \leq |z_2(0)|e^{-c_2t/2} + \sup_{0\leq\tau\leq t} \mu_2(\tau)$$

with respect to the following continuous and uniformly bounded function.

$$\mu_2(t) = \frac{|\Delta_\alpha| + |z_3|}{\dfrac{c_2}{2} + \kappa_2\alpha_d}$$

Proof: Based on equation (31), we have

$$\begin{aligned} &\frac{d}{dt}\left(\frac{z_2^2}{2}\right) \\ &= -c_2z_2^2 + \Delta_\alpha z_2 - \kappa_2\alpha_d z_2^2 + z_2z_3 \\ &\leq -\frac{c_2}{2}z_2^2 - \left[\frac{c_2}{2} + \kappa_2\alpha_d\right]|z_2|^2 + |\Delta_\alpha||z_2| + |z_2||z_3| \\ &= -\frac{c_2}{2}z_2^2 - \left[\frac{c_2}{2} + \kappa_2\alpha_d\right]|z_2| \left[|z_2| - \frac{|\Delta_\alpha| + |z_3|}{\dfrac{c_2}{2} + \kappa_2\alpha_d}\right] \end{aligned} \quad (32)$$

Define

$$\mu_2(t) = \frac{|\Delta_\alpha| + |z_3|}{\dfrac{c_2}{2} + \kappa_2\alpha_d} \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots (33)$$

It is trivial to verify that the first term of the right hand side of equation (33) is uniformly bounded since the denominator grows as the same order as the numerator grows. Therefore $\mu_2(t)$ is continuous and uniformly bounded if $z_3(t)$ is continuous and uniformly bounded. And hence we have [(13)]

$$|z_2(t)| \geq \mu_2(t) \Rightarrow \frac{d}{dt}\left(z_2^2\right) \leq -c_2z_2^2 \quad \cdots\cdots (34)$$

Finally, we have the following result [(13)].

$$|z_2(t)| \leq |z_2(0)|e^{-c_2t/2} + \sup_{0\leq\tau\leq t} \mu_2(\tau) \quad \cdots\cdots (35)$$

□

Remark 3: From the result of lemma 2, one may conclude that if the values of κ_2 and c_2 are sufficiently large, $|z_2|$ converges to an arbitrarily small neighbourhood of the origin exponentially. However, as will be found later in equations (39) and (40), it is required to let $(1 - \kappa_2z_2) > 0$, in order to avoid a singular point of the control input u. Therefore, what we can expect at this step is to make z_2 uniformly bounded by choosing a modest κ_2, rather than to suppress it to an arbitrarily small neighbourhood of the origin. The position error z_1 which is our final control task can be suppressed to a very small neighbourhood of the origin by choosing relatively large c_{1p} and c_{1i} even when $|z_2|$ is not very small, according to equations (23) and (24).

Remark 4: It should be commented here that $(1 - \kappa_2z_2) > 0$ is not a serious constraint in generic cases. From equations (33) and (35), we have

$$|\kappa_2 z_2(t)| \leq \kappa_2 |z_2(0)| e^{-c_2 t/2} + \sup_{0\leq\tau\leq t} [\kappa_2 \mu_2(\tau)] \quad (36)$$

and

$$\kappa_2 \mu_2(t) = \frac{|\Delta_\alpha|}{\dfrac{c_2}{2\kappa_2} + \alpha_d} + \frac{|z_3|}{\dfrac{c_2}{2\kappa_2} + \alpha_d} \quad \cdots\cdots\cdots (37)$$

By a suitable reference trajectory initialization, $\kappa_2 |z_2(0)| e^{-c_2 t/2}$ in equation (36) can be set to zero [12]. Also, by a relatively large c_2, the second term of the right hand side of equation (37) becomes small, if $|z_3|$ is made relatively small. Finally, the numerator in the first term of the right hand side of equation (37) is in fact the difference between the actual acceleration applied to the system and its nominal value, which is usually small compared to the denominator, if the parametric errors are not extremely large. Therefore it is possible to make $|\kappa_2 z_2| \ll 1$ in generic cases. Through extensive studies by numerical simulations and experiments on the system under study we have verified that the constraint $(1 - \kappa_2 z_2) > 0$ is not violated in generic cases, i.e., we have not been troubled by this problem in practice when the value of κ_2 is not chosen extremely large.

Step 3:

Through straightforward but tedious calculations based on some equations that appeared so far, we have

$$\begin{aligned}\dot{\alpha}_2 =& \kappa_2 z_2 \dot{\xi}_3 + (c_2 + \kappa_2 \alpha_d)^2 z_2 \\ & -(c_2 + \kappa_2 \alpha_d) z_3 - c_{1i}\xi_2 - c_{1p}\xi_3 \\ & + c_{1i}\dot{y}_r + c_{1p}\ddot{y}_r + y_r^{(3)} - (c_2 + \kappa_2 \alpha_d + c_{1p})\Delta_\alpha\end{aligned} \quad (38)$$

Then the dynamics of subsystem $\mathcal{S}3$ can be obtained as follows.

$$\begin{aligned}\dot{z}_3 &= \dot{\xi}_3 - \dot{\alpha}_2 \\ &= \Psi_0 + \Delta_\Psi + G_0 \mathcal{U} + \Delta_G \mathcal{U}\end{aligned} \quad \cdots\cdots\cdots\cdots\cdots (39)$$

where

$$\begin{aligned}\Psi_0 =& (1 - \kappa_2 z_2)(F_1 + F_0) - (c_2 + \kappa_2 \alpha_d)^2 z_2 + c_2 z_3 \\ & + \kappa_2 \alpha_d z_3 + c_{1i}\xi_2 + c_{1p}\xi_3 - c_{1i}\dot{y}_r - c_{1p}\ddot{y}_r - y_r^{(3)} \\ \Delta_\Psi =& (1 - \kappa_2 z_2)\Delta_F + (c_2 + \kappa_2 \alpha_d + c_{1p})\Delta_\alpha \\ \mathcal{U} =& (1 - \kappa_2 z_2) u\end{aligned} \quad (40)$$

If the augmented control input $\mathcal{U}$ is determined, then the actual voltage input u can be generated as $u = \mathcal{U}/(1 - \kappa_2 z_2)$ if $(1 - \kappa_2 z_2) > 0$.

Similar to the design technique in step 2, the augmented control input $\mathcal{U}$ is degined as

$$\begin{aligned}\mathcal{U} =& \frac{\alpha_{30} - \alpha_{31} - \alpha_{32} - \alpha_{33}}{G_0} \\ \alpha_{30} =& -c_3 z_3 - \Psi_0 \\ \alpha_{31} =& \kappa_{31}\left(1 - 0.5e^{-\lambda_1 |z_3|}\right)|1 - \kappa_2 z_2| F_d z_3 \\ \alpha_{32} =& \kappa_{32}\left(1 - 0.5e^{-\lambda_2 |z_3|}\right)|c_2 + \kappa_2 \alpha_d + c_{1p}|\alpha_d z_3 \\ \alpha_{33} =& \kappa_{33}\left(1 - 0.5e^{-\lambda_3 |z_3|}\right)|\alpha_{30}| z_3\end{aligned} \quad (41)$$

where $c_3 > 0,\ \kappa_{31} > 0,\ \kappa_{32} > 0,\ \kappa_{33} > 0$, and

$$F_d = \frac{Q_0 x_3^2 \{Q_0 |x_2| + R_0 (X_{\infty 0} + x_1)^2\}}{M_0 (X_{\infty 0} + x_1)^3 \{Q_0 + L_{\infty 0}(X_{\infty 0} + x_1)\}} \quad (42)$$

Here, α_{30} is a nominal controller, and α_{31}, α_{32} and α_{33} are nonlinear damping terms employed to counteract respectively Δ_F, Δ_α and Δ_G that appear in equations (39) and (40). Also, notice that $(1 - 0.5e^{-\lambda_i |z_3|})$, $i = 1, 2, 3$ are introduced to reduce control efforts due to the nonlinear damping terms when $|z_3|$ is relatively small, i.e., the amplitude of $(1 - 0.5e^{-\lambda_i |z_3|})$ is reduced to 50% when $|z_3|$ approaches zero.

When the designed $\mathcal{U}$ is applied to subsystem $\mathcal{S}3$, its dynamics becomes

$$\begin{aligned}\dot{z}_3 =& -c_3 z_3 + \Delta_\Psi - \alpha_{31} - \alpha_{32} - \alpha_{33} \\ & + \frac{\Delta_G}{G_0}(\alpha_{30} - \alpha_{31} - \alpha_{32} - \alpha_{33})\end{aligned} \quad \cdots\cdots\cdots (43)$$

The ISS of subsystem $\mathcal{S}3$ is shown in the following lemma.

Lemma 3 If the augmented input $\mathcal{U}$ is applied to subsystem $\mathcal{S}3$, then $\mathcal{S}3$ is ISS such that

$$|z_3(t)| \leq |z_3(0)| e^{-c_3 t/2} + \sup_{0\leq\tau\leq t} \mu_3(\tau)$$

with respect to the following continuous and uniformly bounded function.

$$\begin{aligned}\mu_3(t) =& \frac{|(1 - \kappa_2 z_2)\Delta_F|}{\dfrac{c_3}{6} + \dfrac{\kappa_{31}\gamma(\boldsymbol{x})}{2\gamma_0(\boldsymbol{x})}|1 - \kappa_2 z_2| F_d} \\ & + \frac{|(c_2 + \kappa_2 \alpha_d + c_{1p})\Delta_\alpha|}{\dfrac{c_3}{6} + \dfrac{\kappa_{32}\gamma(\boldsymbol{x})}{2\gamma_0(\boldsymbol{x})}|c_2 + \kappa_2 \alpha_d + c_{1p}|\alpha_d} \\ & + \frac{\left|\dfrac{\Delta_G}{G_0}\alpha_{30}\right|}{\dfrac{c_3}{6} + \dfrac{\kappa_{33}\gamma(\boldsymbol{x})}{2\gamma_0(\boldsymbol{x})}|\alpha_{30}|}\end{aligned}$$

Proof: Based on equation (43), we have

$$\begin{aligned}&\frac{d}{dt}\left(\frac{z_3^2}{2}\right) \\ &= -c_3 z_3^2 \\ &\quad -\kappa_{31}\left(1 - 0.5e^{-\lambda_1 |z_3|}\right)|1 - \kappa_2 z_2| F_d \left(1 + \frac{\Delta_G}{G_0}\right) z_3^2 \\ &\quad +(1 - \kappa_2 z_2)\Delta_F z_3 \\ &\quad -\kappa_{32}\left(1 - 0.5e^{-\lambda_2 |z_3|}\right)|c_2 + \kappa_2 \alpha_d + c_{1p}|\alpha_d \left(1 + \frac{\Delta_G}{G_0}\right) z_3^2 \\ &\quad +(c_2 + \kappa_2 \alpha_d + c_{1p})\Delta_\alpha z_3 \\ &\quad -\kappa_{33}\left(1 - 0.5e^{-\lambda_3 |z_3|}\right)|\alpha_{30}|\left(1 + \frac{\Delta_G}{G_0}\right) z_3^2 + \frac{\Delta_G}{G_0}\alpha_{30} z_3\end{aligned} \quad (44)$$

And from equations (16) and (18) we have

$$1 + \frac{\Delta_G}{G_0} = \frac{\gamma(\boldsymbol{x})}{\gamma_0(\boldsymbol{x})} > 0 \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (45)$$

Then we can derive the following result.

$$
\begin{aligned}
&\frac{d}{dt}\left(\frac{z_3^2}{2}\right)\\
&\le -\frac{c_3}{2}z_3^2-\frac{c_3}{6}z_3^2-\frac{\kappa_{31}}{2}|1-\kappa_2 z_2|F_d\frac{\gamma(\boldsymbol{x})}{\gamma_0(\boldsymbol{x})}z_3^2\\
&\quad+(1-\kappa_2 z_2)\Delta_F z_3\\
&\quad-\frac{c_3}{6}z_3^2-\frac{\kappa_{32}}{2}|c_2+\kappa_2\alpha_d+c_{1p}|\alpha_d\frac{\gamma(\boldsymbol{x})}{\gamma_0(\boldsymbol{x})}z_3^2\\
&\quad+(c_2+\kappa_2\alpha_d+c_{1p})\Delta_\alpha z_3\\
&\quad-\frac{c_3}{6}z_3^2-\frac{\kappa_{33}}{2}|\alpha_{30}|\frac{\gamma(\boldsymbol{x})}{\gamma_0(\boldsymbol{x})}z_3^2+\frac{\Delta_G}{G_0}\alpha_{30}z_3\\
&\le -\frac{c_3}{2}z_3^2-\left[\frac{c_3}{6}+\frac{\kappa_{31}}{2}|1-\kappa_2 z_2|F_d\frac{\gamma(\boldsymbol{x})}{\gamma_0(\boldsymbol{x})}\right]|z_3|\\
&\quad\times\left[|z_3|-\frac{|(1-\kappa_2 z_2)\Delta_F|}{\frac{c_3}{6}+\frac{\kappa_{31}}{2}|1-\kappa_2 z_2|F_d\frac{\gamma(\boldsymbol{x})}{\gamma_0(\boldsymbol{x})}}\right] \qquad \cdots\cdots (46)\\
&\quad-\left[\frac{c_3}{6}+\frac{\kappa_{32}}{2}|c_2+\kappa_2\alpha_d+c_{1p}|\alpha_d\frac{\gamma(\boldsymbol{x})}{\gamma_0(\boldsymbol{x})}\right]|z_3|\\
&\quad\times\left[|z_3|-\frac{|(c_2+\kappa_2\alpha_d+c_{1p})\Delta_\alpha|}{\frac{c_3}{6}+\frac{\kappa_{32}}{2}|c_2+\kappa_2\alpha_d+c_{1p}|\alpha_d\frac{\gamma(\boldsymbol{x})}{\gamma_0(\boldsymbol{x})}}\right]\\
&\quad-\left[\frac{c_3}{6}+\frac{\kappa_{33}}{2}|\alpha_{30}|\frac{\gamma(\boldsymbol{x})}{\gamma_0(\boldsymbol{x})}\right]|z_3|\\
&\quad\times\left[|z_3|-\frac{\left|\frac{\Delta_G}{G_0}\alpha_{30}\right|}{\frac{c_3}{6}+\frac{\kappa_{33}}{2}|\alpha_{30}|\frac{\gamma(\boldsymbol{x})}{\gamma_0(\boldsymbol{x})}}\right]
\end{aligned}
$$

Define

$$
\begin{aligned}
\mu_3(t)=&\frac{|(1-\kappa_2 z_2)\Delta_F|}{\frac{c_3}{6}+\frac{\kappa_{31}\gamma(\boldsymbol{x})}{2\gamma_0(\boldsymbol{x})}|1-\kappa_2 z_2|F_d}\\
&+\frac{|(c_2+\kappa_2\alpha_d+c_{1p})\Delta_\alpha|}{\frac{c_3}{6}+\frac{\kappa_{32}\gamma(\boldsymbol{x})}{2\gamma_0(\boldsymbol{x})}|c_2+\kappa_2\alpha_d+c_{1p}|\alpha_d} \qquad \cdots\cdots (47)\\
&+\frac{\left|\frac{\Delta_G}{G_0}\alpha_{30}\right|}{\frac{c_3}{6}+\frac{\kappa_{33}\gamma(\boldsymbol{x})}{2\gamma_0(\boldsymbol{x})}|\alpha_{30}|}
\end{aligned}
$$

It is trivial to verify that Δ_F/F_d, Δ_α/α_d and $(\Delta_G/G_0)/(\gamma(\boldsymbol{x})/\gamma_0(\boldsymbol{x}))$ are uniformly bounded, since in each the denominator grows as the same order as the numerator grows. This straightforwardly implies that $\mu_3(t)$ is continuous and uniformly bounded. And hence we have [13]

$$|z_3(t)|\ge\mu_3(t)\Rightarrow\frac{d}{dt}\left(z_3^2\right)\le -c_3 z_3^2 \qquad \cdots\cdots (48)$$

Finally, we have the following result [13].

$$|z_3(t)|\le|z_3(0)|e^{-c_3 t/2}+\sup_{0\le\tau\le t}\mu_3(\tau) \qquad \cdots\cdots (49)$$

□

5. Stability analysis of the overall error system

Combining equations (25), (31) and (43), we have the overall error system as

$$
\begin{bmatrix}\dot z_{1i}\\ \dot z_1\\ \dot z_2\\ \dot z_3\end{bmatrix}=\mathcal{A}\begin{bmatrix}z_{1i}\\ z_1\\ z_2\\ z_3\end{bmatrix}+\begin{bmatrix}0\\0\\ \Delta_\alpha\\ \Delta_\Psi+\Delta_G\mathcal{U}\end{bmatrix} \qquad \cdots\cdots (50)
$$

where

$$
\mathcal{A}=\begin{bmatrix}0&1&0&0\\ -c_{1i}&-c_{1p}&1&0\\ 0&0&-c_2-\kappa_2\alpha_d&1\\ 0&0&0&-\mathcal{A}_{44}\end{bmatrix} \qquad \cdots (51)
$$

and

$$
\begin{aligned}
\mathcal{A}_{44}=&c_3+\kappa_{31}\left(1-0.5e^{-\lambda_1|z_3|}\right)|1-\kappa_2 z_2|F_d\\
&+\kappa_{32}\left(1-0.5e^{-\lambda_2|z_3|}\right)|c_2+\kappa_2\alpha_d+c_{1p}|\alpha_d \qquad (52)\\
&+\kappa_{33}\left(1-0.5e^{-\lambda_3|z_3|}\right)|\alpha_{30}|
\end{aligned}
$$

Notice that if there is no parametric uncertainty such that the perturbations disappear and such that the nonlinear damping terms are not used, then the overall error system is completely linearized.

Since the overall error system is a cascade of the three ISS subsystems characterized by lemmas 1 ∼ 3 respectively, we can conclude based on lemma C.4 in reference (12) that the overall error system is ISS. It should be commented however, the results obtained here are valid only locally, since the coordinate transformation and controller are only feasible in a feasible region $\mathcal{F}_{\boldsymbol{x}\times\boldsymbol{z}}=\{(\boldsymbol{x},\boldsymbol{z})|0<x_1\le x_{1M},\ x_3>0,\ (1-\kappa_2 z_2)>0\}\subset R^3\times R^3$, according to remarks 2 ∼ 4.

To ensure the feedback linearization controller feasible, we should verify if there is a positively invariant set $\mathcal{D}_{\boldsymbol{x}\times\boldsymbol{z}}\subset\mathcal{F}_{\boldsymbol{x}\times\boldsymbol{z}}$ such that $(\boldsymbol{x},\boldsymbol{z})\in\mathcal{D}_{\boldsymbol{x}\times\boldsymbol{z}}$ for all $(\boldsymbol{x}(0),\boldsymbol{z}(0))\in\mathcal{D}_{\boldsymbol{x}\times\boldsymbol{z}}$. According to lemmas 1 ∼ 3, if the reference position y_r, the initial values of error signal vector $\boldsymbol{z}=[z_1,\ z_2,\ z_3]^T$ and the design parameters are chosen appropriately, we have a positively invariant set $\mathcal{D}_{\boldsymbol{z}}=\{\boldsymbol{z}|\,|z_1|\le\bar z_1,\ |z_2|\le\bar z_2,\ |z_3|\le\bar z_3,\ \exists\bar z_1,\exists\bar z_2,\exists\bar z_3>0\}\subset R^3$ such that $\boldsymbol{z}\in\mathcal{D}_{\boldsymbol{z}}$ for all $\boldsymbol{z}(0)\in\mathcal{D}_{\boldsymbol{z}}$. In this case, we can readily ensure that $0<x_1\le x_{1M}$, i.e., the levitated steel ball will not hit the lower or upper bound of the allowable operating region. Also we can ensure $(1-\kappa_2 z_2)>0$, according to remarks 3 and 4. Finally, as long as $\boldsymbol{z}\in\mathcal{D}_{\boldsymbol{z}}$, i.e., the steel ball is levitated and tracks a smooth reference trajectory with acceptable accuracy, we can conclude that the electromagnet is exerting an attractive force to counteract the gravity, i.e., $x_3>0$ is ensured in generic cases as long as $\boldsymbol{z}\in\mathcal{D}_{\boldsymbol{z}}$.

Based on the above discussions, we have the following result.

Theorem 1 There is a positively invariant set $\mathcal{D}_{\boldsymbol{x}\times\boldsymbol{z}}\subset\mathcal{F}_{\boldsymbol{x}\times\boldsymbol{z}}=\{(\boldsymbol{x},\boldsymbol{z})|0<x_1\le x_{1M},\ x_3>0,\ (1-\kappa_2 z_2)>0\}\subset R^3\times R^3$ such that for all $(\boldsymbol{x}(0),\boldsymbol{z}(0))\in\mathcal{D}_{\boldsymbol{x}\times\boldsymbol{z}}$, the coordinate transformation and robust feedback linearization controller are feasible such that the overall error system (50) is ISS.

Remark 5: Theorem 1 and equations (23) and (24) imply that all the internal signals of the nonlinear control system can be made uniformly bounded and the position tracking error $|z_1|$ can be made very small with zero offset.

Remark 6: Through extensive numerical simulation and experimental studies, we found that the restriction due to the feasible region is not a serious problem, i.e., we have not been troubled by it. Rather than this problem, we found especially in the presence of measurement noise of considerably high level, the nonlinear damping terms often make the voltage control input u quite noisy. As can be seen in (29) and (41), the time-varying feedback gains in the nonlinear damping terms include noisy measurements or error signals. Therefore even when the values of coefficients $\kappa_2,\ \kappa_{31},\ \kappa_{32},\ \kappa_{33}$ are not very large, the control input often become noisy and its amplitude may become relatively large especially during the transient phase. So far, in most of the theoretical papers on robust backstepping design with nonlinear damping terms reported in the literature, numerical simulation studies are only performed on continuous-time models without taking any measurement noise or sampling error into account. Some of these methods reported in the literature while being proven effective theoretically and numerically, may suffer from measurement noise and sampling error which often lead to noisy control efforts.

Remark 7: The guidelines for design of the controller parameters can be drawn here based on the above discussions. It is recommendable to choose modest $\kappa_2,\ \kappa_{31},\ \kappa_{32},\ \kappa_{33}$ to avoid noisy or large control efforts. In contrast, the parameters of the nominal controller $c_{1p},\ c_{1i},\ c_2,\ c_3$ which also contribute to achieve fast transient phase and small error signals can be chosen relatively large, without causing large amplitude of the control input. Hence the tracking error $|z_1|$ which is our final control task can be suppressed to a very small neighbourhood of the origin even when $|z_2|$ and $|z_3|$ are not made very small, according to equations (23) and (24).

Remark 8: lemmas 1 $\sim$ 3 imply that the initial conditions of the error signals $z_1(0),\ z_2(0),\ z_3(0)$ can influence the transient performance significantly. It is however, possible to improve the transient performance by an appropriate reference trajectory initialization [12]. Suppose the steel ball is initially at rest with $x_1(0) = x_{1M}$ and $x_2(0) = 0$, i.e., the steel ball is held on the steel plate shown in Fig. 1 before the feedback controller's start. Then if we choose the initial conditions of the reference trajectory such that $y_r(0) = x_1(0)$ and $\dot{y}_r(0) = \ddot{y}_r(0) = 0$, we have from equations (20) and (22) $z_1(0) = z_2(0) = 0$. And also from equations (6), (27) and (29) we have

$$z_3(0) = g_0 - \frac{Q_0 x_3^2(0)}{2M_0\{X_{\infty 0} + x_1(0)\}^2} \quad \cdots\cdots (53)$$

Thus $z_3(0)$ can be made relatively small if we set the initial coil current signal $x_3(0)$ to an appropriate value by a suitable step voltage input u, before we start to levitate the steel ball by the designed feedback controller. It should be noticed that initialization is an important issue here since in a typical magnetic levitation system, the operating region of the levitated object is quite narrow so that attention should be paid not to let the steel ball hit the lower or upper bound of the allowable operating region especially during the transient phase.

6. Experimental results

To verify the performance of the proposed robust nonlinear controller and our claims raised in the previous sections, experimental studies have been carried out on the magnetic levitation system shown in Fig. 1, whose physical parameters are given in Table 1. The physically allowable operating region of the steel ball shown in Fig. 1 is limited to $0[\mathrm{m}] < x_1 \leq 0.013[\mathrm{m}]$. The output of the controllable voltage source is limited to $-60.0[V] \leq u \leq 60.0[V]$. The velocity x_2 is measured by pseudo-differentiation of the measured position x_1 as $sx_1/(0.004s+1)$. The sampling interval is chosen as $T = 0.5[\mathrm{ms}]$. The resolution accuracy of the laser distance sensor is $\pm 0.00018[\mathrm{m}]$, which is considered to be relatively noisy for the system under study.

Table 1. Physical parameters of the magnetic levitation system

M	0.54	[kg]
g	9.8	[m/s^2]
X_∞	0.00789	[m]
Q	0.001599	[Hm]
L_∞	0.8052	[H]
R	11.58	[Ω]

The following nominal system parameters with considerable errors are used for experimental studies, to verify the robust performance of our proposed robust nonlinear controller in the presence of parametric uncertainties.

$$\begin{gathered} M_0 = 0.80[\mathrm{kg}],\ g_0 = 9.0[\mathrm{m/s^2}] \\ X_{\infty 0} = 0.0050[\mathrm{m}],\ Q_0 = 0.0010[\mathrm{Hm}] \\ L_{\infty 0} = 0.50[\mathrm{H}],\ R_0 = 10.0[\Omega] \end{gathered} \quad \cdots\cdots (54)$$

Experiments are performed for the following controllers, along the guidelines of controller design drawn in remark 7.

Controller 1: A nominal controller without nonlinear damping terms.

$$\begin{gathered} c_{1p} = 40,\ c_{1i} = 20^2,\ c_2 = 40,\ c_3 = 20 \\ \kappa_2 = 0,\ \kappa_{31} = 0,\ \kappa_{32} = 0, \kappa_{33} = 0 \end{gathered} \quad \cdots (55)$$

Controller 2: A robust controller with nonlinear damping terms.

$$\begin{gathered} c_{1p} = 40,\ c_{1i} = 20^2,\ c_2 = 40,\ c_3 = 20 \\ \kappa_2 = 2,\ \kappa_{31} = 0.5,\ \kappa_{32} = 0.5, \kappa_{33} = 0.5 \\ \lambda_1 = 0.1,\ \lambda_2 = 0.1,\ \lambda_3 = 0.1 \end{gathered} \quad \cdots (56)$$

According to remark 8, to begin with a suitable reference trajectory initialization, the reference trajectory is initialized based on the initial conditions of the steel ball ($x_1(0) = 0.013[\mathrm{m}]$ and $x_2(0) = 0[\mathrm{m/s}]$). Before the feedback controller's start, a step input $u = 15.0[\mathrm{V}]$

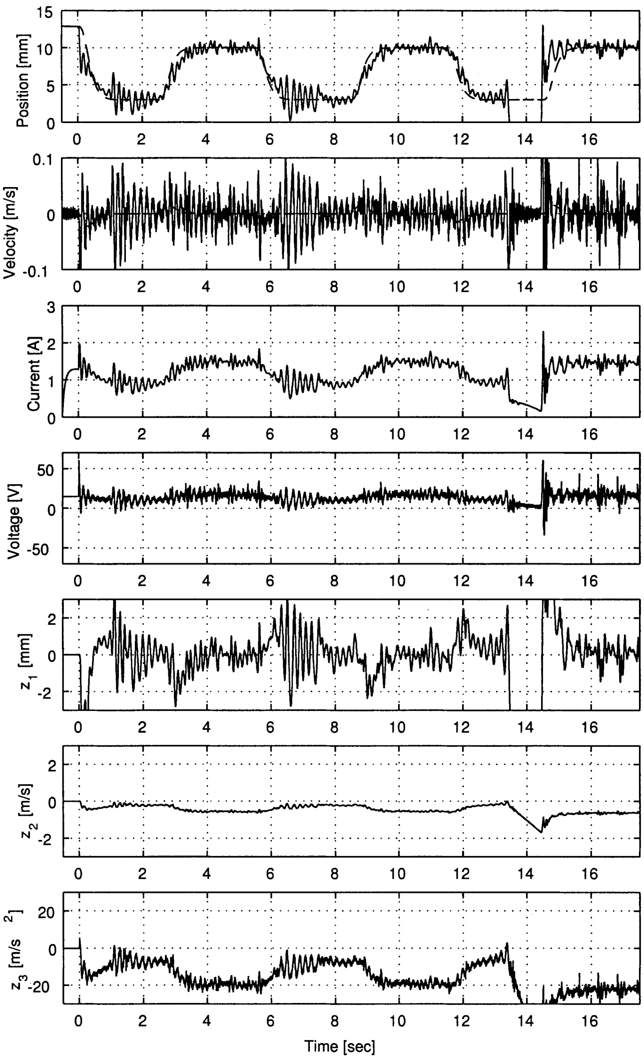


Fig. 2. Experimental results of the nominal controller.

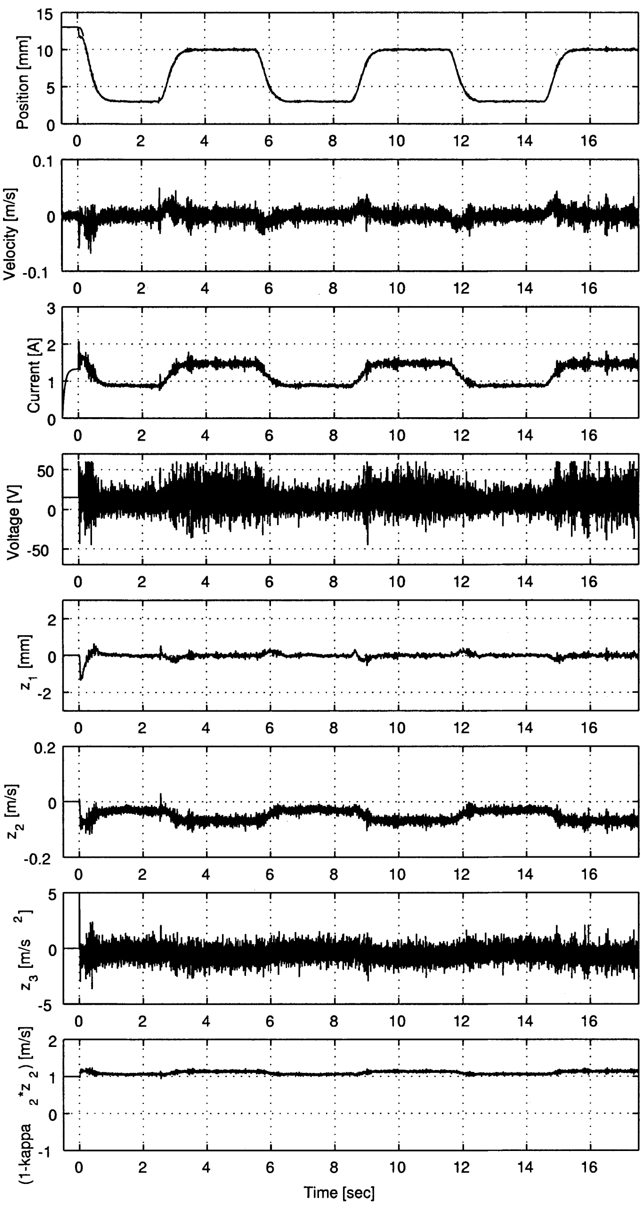


Fig. 3. Experimental results of the robust controller.

is applied to the system during $-0.5[\text{sec}] \leq t < 0[\text{sec}]$ in order to produce an appropriate initial coil current $x_3(0)$, as mentioned in remark 8.

At first, the nominal controller without any consideration of robustness is applied. The results are shown in Fig. 2. It can be seen in Fig. 2 that the nominal feedback linearization controller without nonlinear damping terms can not guarantee closed-loop stability in the presence of parametric errors. The position of the steel ball oscillates roughly and the steel ball hits the electromagnet (when the position x_1 becomes zero, the ball hits the electromagnet). This fact has also been verified through experiments with various combinations of the nominal system parameters and controller parameters. We have found that the control performance is not satisfactory in most cases. And sometimes we had to stop the controller to avoid destroying the apparatus.

Next, the robust controller with nonlinear damping terms is applied and the results are shown in Fig. 3. It can be verified in Fig. 3 that the robust controller indicates excellent performance. Also, it can be found that x_3 and $(1-\kappa_2 z_2)$ never approach zero, i.e., the controllers do not suffer from the singular points. Additionally, although $|z_2|$ and $|z_3|$ are not made very small here since modest κ_2, κ_{31}, κ_{32}, κ_{33} are chosen according to the design guidelines drawn in remark 7, the tracking error $|z_1|$ which is our final control task can be suppressed to a very small neighbourhood of the origin as claimed previously in remark 5. We have verified that our proposed robust nonlinear controller is quite robust against parametric uncertainties through not only the results shown here, but also extensive experiments with various nominal system parameters and controller parameters.

7. Conclusions

In this paper, we proposed a robust nonlinear controller in the presence of parametric uncertainties for control of a magnetic levitation system. The design procedure is carried out in a backstepping design manner, by employing nonlinear damping terms to suppress the effects of the parametric uncertainties which may cause instability. ISS property of the control system is analyzed. Some practical issues concerning the guidelines of the controller design and initialization technique are discussed. And finally experimental results are included to show the excellent position tracking performance of the designed control system.

(Manuscript received May 25, 2000)

References

(1) A. M. Annaswamy, C. Thanomsat, N. Mehta and A. P. Loh, Applications of adaptive controllers to systems with nonlinear parametrization, ASME Journal of Dynamic Systems, Measurement, and Control, Vol.120, 477/487, 1998.

(2) B. R. Barmish, New Tools for Robustness of Linear Systems, new york: Macmillan, 1994.

(3) W. Barie and J. Chiasson, Linear and nonlinear state-space controllers for magnetic levitation, International Journal of Systems Science, Vol.27, 1153/1163, 1996.

(4) A. Charara, J. D. Miras and B. Caron, Nonlinear control of a magnetic levitation system without premagnetization, IEEE Trans. Control Systems Technology, **4**/5, 513/523, 1996.

(5) D. Cho, Y. Kato and D. Silman, Sliding mode and classical control for magnetic levitation systems, IEEE Control System Magazine, 43/48, 1994.

(6) F. J. Doyle and M. A. Henson, Nonlinear systems theory, in Nonlinear Process Control, edited by M. A. Henson and D. E. Seborg, Prentice Hall, 111/148, 1997.

(7) S. A. Green and K. C. Craig, Robust, digital, nonlinear control of magnetic-levitation systems, ASME Journal of Dynamic Systems, Measurement, and Control, **120**, 488/495, 1998.

(8) M. A. Henson and D. E. Seborg, Feedback linearizing control, in Nonlinear Process Control, edited by M. A. Henson and D. E. Seborg, 149/232, 1997.

(9) P. A. Ioannou and J. Sun, Robust Adaptive Control, Prentice-Hall, 1996.

(10) A. Isidori, Nonlinear Control systems, Third Edition, Springer, 1995.

(11) A. J. Joo and J. H. Seo, Design and analysis of the nonlinear feedback linearizing control for an electromagnetic suspension system, IEEE Trans. Control Systems Technology, Vol.5, 135/144, 1997.

(12) M. L. Krstic, Kanellakopoulos, P. Kokotovic, Nonlinear and Adaptive Control Design, John Wiley & Sons, Inc., 1995.

(13) M.Krstic, J. Sun and P. Kokotovic, Robust control of nonlinear systems with input unmodeled dynamics, IEEE Trans. Automatic Control, vol. 41, 913/920, 1996.

(14) L. C. Lin and T. B. Gau, Feedback linearization and fuzzy control for conical magnetic bearings, IEEE Trans. Control Systems Technology, Vol.5, 417/426, 1997.

(15) Y. S. Lu and J. S. Chen, A self-organizing fuzzy sliding-mode controller design for a class of nonlinear servo systems, IEEE Trans. Industrial Electronics, Vol.41, 492/502, 1994.

(16) Y. S. Lu and J. S. Chen, Design of perturbation estimator using the theory of variable-structure systems and its application to magnetic levitation systems, IEEE Trans. Industrial Electronics, Vol.42, 281/289, 1995.

(17) M. S. De Queiroz and D. M. Dawson, Nonlinear control of active magnetic bearings: a backstepping approach, IEEE Trans. Control Systems Technology, Vol.4, 545/552, 1996.

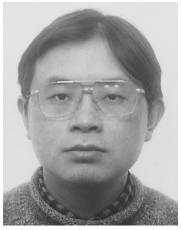
(18) P. K. Sinha, Electomagnetic Suspension: Dynamics & Control, IEE Contr. Eng. Series, vol. 30, 1987,

(19) T. Sugie, K. Shimizu and J. I. Imura, H_∞ control with exact linearization and its application to magnetic levitation systems, Transactions of the Institute of Systems, Control and Information Engineers, 57/63, 1993 (in Japanese).

(20) D. L. Trumpler, S. M. Olson and P. K. Subrahmanyan, Linearizing control of magnetic suspension systems, IEEE Trans. Control Systems Technology, Vol.5, 427/437, 1997.

(21) T. J. Yeh and K. Youcef-Toumi, Adaptive control of nonlinear uncertain systems using local function estimation, ASME Journal of Dynamic Systems, Measurement, and Control, Vol.120, 429/438, 1998.

(22) Z. J. Yang and M. Tateishi: Robust nonlinear control of a magnetic levitation system via backstepping design approach, Transactions of the Society of Instrument and Control Engineers, Vol.35, 85/90, 1999 (in Japanese).



Zi-Jiang Yang (Member) was born in 1964. He received his Dr. of Eng. degree in 1992 from Kyushu University. From 1996 to 2000, he was an Associate Professor in the Faculty of Computer Engineering and System Science, Kyushu Institute of Technology. He is currently an Associate Professor in the Department of Electrical and Electronic Systems Engineering, Graduate School of Information Science and Electrical Engineering, Kyushu University. His research interests include system identification, and related signal processing, soft computing and motion control, etc.

Masayuki Minashima (Non-member) was born in 1975. He graduated from the Faculty of Computer Engineering and System Science, Kyushu Institute of Technology in 1997. He was a student of master course, in the Department of Information System Engineering, Graduate School of Computer Engineering and System Science, Kyushu Institute of Technology. He was engaged in research on magnetic levitation system control. Since 2000,he has been with Fujitsu Business Systems (Ltd.).