Paper

Design of Memoryless Controllers for Robust Tracking and Model Following of Uncertain Systems with Multiple Time Delays

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The problem of robust tracking and model following is considered for a class of linear dynamical systems with multiple delayed state perturbations, time–varying uncertain parameters, and disturbance. A class of continuous memoryless state feedback controllers is proposed for robust tracking of dynamical signals. The proposed robust tracking controllers can guarantee that the tracking error decreases asymptotically to zero in the presence of multiple delayed state perturbations, time–varying uncertain parameters, and disturbance. A procedure for designing such a class of zero–error tracking state feedback controllers is also introduced. Finally, a numerical example is given to demonstrate the validity of the results.

Keywords: Robust tracking control, model following, uncertain systems, delayed state perturbations, memoryless state feedback, reference model.

1. Introduction

The robust tracking and model following problem for dynamical systems with significant uncertainties has been widely investigated over the last decades. Some approaches to tracking dynamical signals in such uncertain dynamical systems have been developed (see, e.g. Refs. (1)∼(3) and the references therein). In Ref. (1), for example, a nonlinear state feedback controller is proposed for robust tracking of dynamical signals. In Ref. (2), a class of uncertain linear dynamical systems are considered, and a class of linear state feedback controllers are given for robust tracking of dynamical signals. In Ref. (3), the problem of robust model following control is considered for a class of dynamical systems which contain uncertain nonlinear terms and bounded unknown disturbances, and a method of designing state feedback controllers is developed for robust tracking of dynamical signals. However, these robust state feedback tracking controllers do not produce asymptotic tracking; instead, the so–called practical tracking is achieved. That is, by employing the robust state feedback tracking controllers proposed in Refs. (1)∼(3), one cannot guarantee that the tracking error decreases asymptotically to zero.

On the other hand, except for significant uncertainties, a constant or time–varying delay is often encountered in various engineering systems to be controlled, such as chemical processes, hydraulic, and rolling mill systems, economic systems, and the existence of the delay is frequently a source of instability. Therefore, the problem of robust stabilization of uncertain dynamical systems with time delay has received considerable attention of many researchers, and many solution approaches have been developed (see, e.g. Refs. (4)∼(10)).

Particularly, in recent papers (see Ref. (11) and Ref. (12)), the problem of robust tracking and model following for uncertain time–delay systems is considered. In Ref. (11), a nonlinear switching controller is proposed to guarantee that the outputs of the controlled uncertain time–delay system track the outputs of the non–delay reference model. However, the switching controller is discontinuous, and such a discontinuous controller cannot be directly implemented. To avoid this, in Ref. (11) a continuous (nonlinear) controller proposed in Ref. (13) to take the place of the switching controller in the practical implementation for the control. Thus, one cannot practically guarantee that the tracking error decreases asymptotically to zero. In Ref. (12), a continuous robust tracking controller is proposed, and this controller can only guarantee the ultimate boundedness of tracking error.

In this paper, similar to Refs. (11) (12), we also consider the problem of robust tracking and model following for a class of linear dynamical systems with multiple delayed state perturbations, time–varying uncertain parameters, and disturbance. We propose another class of continuous memoryless state feedback controllers for robust tracking of dynamical signals. By using our robust tracking controller, we can guarantee that the tracking error decreases asymptotically to zero. That is, we can make it is possible that the outputs of the controlled uncertain time–delay system track exactly the outputs of the reference model without time–delay.

The paper is organized as follows. In Section 2, the model following problem to be tackled is stated and some standard assumptions are introduced. In Section 3, a class of continuous (nonlinear) state feedback controller is proposed for guaranteeing zero–error tracking error, and an algorithm to form such a controller is presented. In Section 4, a numerical example is given to illustrate the use of our results. The paper is concluded in Section 5 with a brief discussion of the results.

2. Problem Formulation and Assumptions

We consider a class of uncertain linear dynamical systems with multiple delayed state perturbations described by the following differential–difference equations:

$$\frac{dx(t)}{dt} = \left[\, A + \Delta A(\upsilon, t) \,\right] x(t) + \sum_{j=1}^{r} \Delta E_j(\zeta, t) x(t - h_j(t)) + \left[\, B + \Delta B(\nu, t) \,\right] u(t) + w(q, t) \quad \cdots\cdots (1a)$$

$$y(t) = Cx(t) \quad \cdots\cdots (1b)$$

where $x(t) \in R^n$ is the current value of the state, $u(t) \in R^m$ is the control function, $y(t) \in R^p$ is the output vector which is to track the reference output $y_m(t)$, $w(q,t) \in R^n$ is the external disturbance vector, $(\upsilon,\ \zeta,\ \nu,\ q) \in \Psi$ is the uncertain vector, $\Psi \subset R^L$ is a compact set, A, B, C are constant matrices of appropriate dimensions, and the matrices $\Delta A(\cdot)$, $\Delta B(\cdot)$, $\Delta E_j(\cdot)$, $j = 1, 2, \ldots, r$, represent the system uncertainties and are assumed to be continuous in all their arguments. In addition, for each $j \in \{1, 2, \ldots, r\}$, the time delay $h_j(t)$ is any bounded, and continuous function, i.e. $0 \le h_j(t) \le \bar{h}_j$ where $\bar{h}_j$ is any nonnegative constant.

The initial condition for system (1) is given by

$$x(t) = \varphi(t), \quad t \in [t_0 - \bar{h},\ t_0] \quad \cdots\cdots (2)$$

where

$$\bar{h} = \max_j \{\, \bar{h}_j,\ j = 1, 2, \ldots, r \,\}$$

and $\varphi(t)$ is a continuous function on $[t_0 - \bar{h},\ t_0]$.

In this paper, the reference output $y_m(t)$ is assumed to be the output of the reference model described by the differential equations of the form

$$\frac{dx_m(t)}{dt} = A_m x_m(t) \quad \cdots\cdots (3a)$$

$$y_m(t) = C_m x_m(t) \quad \cdots\cdots (3b)$$

where $x_m(t) \in R^{n_m}$ is the state vector of the reference model, and $y_m(t)$ has the same dimension as $y(t)$. Furthermore, we require that the model state must be bounded, i.e., there exists a finite positive constant M such that for all $t \ge t_0$,

$$\left\| x_m(t) \right\| \le M \quad \cdots\cdots (4)$$

As pointed out in Ref. (2), not all models of the form given in (3) can be tracked by a system given in (1) with a feedback controller. Similar to Refs. (2) (11) (12), in this paper, the requirement for the developed controller to track the model described by (3) is the existence of the matrices $G \in R^{n \times n_m}$ and $H \in R^{m \times n_m}$ such that the following matrix algebraic equation holds.

$$\begin{bmatrix} A & B \\ C & 0 \end{bmatrix} \begin{bmatrix} G \\ H \end{bmatrix} = \begin{bmatrix} GA_m \\ C_m \end{bmatrix} \quad \cdots\cdots (5)$$

If a solution cannot be found to satisfy this algebraic matrix equation, a different model or output matrix C must be chosen. In particular, the approach to finding the solution to (5) is also discussed in detail in Refs. (2) (11).

Provided that all states are available, the state feedback controller can be represented by a function:

$$u(t) = p(x(t), t) \quad \cdots\cdots (6)$$

Now, the problem is to find a state feedback controller such that it is possible for the system output $y(t)$ to follow the reference model output $y_m(t)$ in the presence of the uncertain $\Delta A(\cdot)$, $\Delta B(\cdot)$, $\Delta E_j(\cdot)$, $j = 1, 2, \ldots, r$, and the external disturbance $w(\cdot)$.

Remark 2.1. For such the model following problem of uncertain linear systems without delayed state perturbations, a class of robust linear feedback tracking controllers are presented in Ref. (2). In particular, as stated in Section 1, for the model following problem of uncertain linear systems with single delayed state perturbation, some memoryless state feedback tracking controllers are also proposed in recent control literature (see, e.g. Refs. (11) and (12)). It should be pointed out that the controllers proposed in Refs. (11) and (12) can only guarantee the ultimate boundedness of tracking error. That is, by employing their tracking controllers there exists always a tracking error in the model following control problem. In this paper, in order to obtain a more exact control result, we will propose another class of robust feedback tracking controllers. We will also show that the proposed tracking controller can guarantee that tracking error decreases asymptotically to zero in the presence of multiple delayed state perturbations, time–varying uncertain parameters and disturbance.

Before proposing our tracking controllers, similar to Refs. (2) (11) (12), we introduce for (1) the following standard assumptions.

Assumption 2.1. The pair $\{A,\ B\}$ given in (1) is completely controllable.

Assumption 2.2. For all $(\upsilon,\ \zeta,\ \nu,\ q) \in \Psi$, there exist some continuous and bounded matrix functions $N(\cdot)$, $D_j(\cdot)$, $E(\cdot)$, $\tilde{w}(\cdot)$ of appropriate dimensions such that

$$\begin{aligned} \Delta A(\upsilon,t) &= BN(\upsilon,t) \\ \Delta E_j(\zeta,t) &= BD_j(\zeta,t) \\ \Delta B(\nu,t) &= BE(\nu,t) \\ w(q,t) &= B\tilde{w}(q,t) \end{aligned}$$

where $j \in \{1,2,\ldots,r\}$.

Remark 2.2. It is obvious that Assumption 2.2 defines the matching condition about the uncertainties and disturbance, and is a rather standard assumption for robust control problem (see, e.g. Refs. (1) (2) (8) (11) (12), and the references therein). For a dynamical system with matched uncertainties, one can always design some types of state (or output) feedback controllers such that the stability of the system can be guaranteed. This assertion is not valid, however, for dynamical systems with unmatched uncertainties. For such uncertain systems, one must find some conditions such that some types of stability can be guaranteed (see, e.g. Refs. (14)∼(17)). Here, we should point out that, similar to Refs. (2) (15) (16), the method for designing a zero–error tracking controller may be extended to uncertain time–delay systems with mismatched uncertainties. In this case, a sufficient condition for robust tracking of the systems should be derived.

For convenience, we now introduce the following notations which represent the bounds of the uncertainties.

$$\begin{aligned} \rho_\upsilon(t) &:= \max_{\upsilon} \|N(\upsilon,t)\| \\ \rho_j(t) &:= \max_{\zeta} \|D_j(\zeta,t)\| \\ \rho_q(t) &:= \max_{q} \|\tilde{w}(q,t)\| \\ \mu(t) &:= \min_{\nu} \left[\frac{1}{2}\lambda_{\min}\left(E(\nu,t)+E^\top(\nu,t)\right)\right] \\ \sigma &:= \sqrt{\lambda_{\max}(P)/\lambda_{\min}(P)} \end{aligned}$$

where $j \in \{1,2,\ldots,r\}$, $\|\cdot\|$ is the spectral norm of a matrix, and $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ denote the minimum and maximum eigenvalues of the matrix, respectively. Here, the functions $\rho_\upsilon(t)$, $\rho_j(t)$, $\rho_q(t)$, $\mu(t)$ are assumed without loss of generality to be uniformly continuous with respect to time.

By employing the notations given above, we introduce for uncertain time–delay system (1) the following assumption [2] [11] [12].

Assumption 2.3. For every $t \geq t_0$, $\mu(t) > -1$.

On the other hand, it follows from Assumption 2.1 that for any given symmetric positive definite matrix $Q \in R^{n\times n}$, there exists an unique symmetric positive definite matrix $P \in R^{n\times n}$ as the solution of the algebraic Riccati equation of the form

$$A^\top P + PA - \eta PBB^\top P = -Q \quad \cdots\cdots\cdots\cdots (7)$$

where η is any given positive constant.

In the rest of this section, we introduce for time–delay system the following lemma (see, e.g. *Theorem 4.3* of [Ref. (18), Chapter 5]) which will be used in the subsequent sections.

Lemma 2.1. [4] [18] Consider the retarded functional differential equation

$$\frac{dx(t)}{dt} = f(t,\ x_t) \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (8)$$

with the initial condition

$$x(t) = \psi(t), \quad t \in [t_0 - \bar{h},\ t_0]$$

Suppose that the functions $\gamma_i(\cdot)$, $i = 1,2,3$, are of K–class. If there is a continuous function $V(\cdot) : [t_0 - \bar{h},\ \infty) \times R^n \to R^+$ such that

i) for any $t \in [t_0 - \bar{h},\ \infty)$ and $x \in R^n$,

$$\gamma_1(\|x\|) \leq V(t,x) \leq \gamma_2(\|x\|)$$

ii) there is a continuous non–decreasing function $p(s) > s$ for $s > 0$, such that

$$\frac{dV(t,x)}{dt} \leq -\gamma_3(\|x\|)$$

if for any $\xi \in [t - \bar{h},\ t]$ and $t \geq t_0$,

$$V(\xi, x(\xi)) < p[V(t, x(t)],$$

then the solutions of functional differential equation (8) are uniformly asymptotically stable.

3. Robust Tracking controllers

In this section, we propose a class of memoryless state feedback controllers which can guarantee that the output $y(t)$ of uncertain time–delay system (1) follows the output $y_m(t)$ of reference model (2) and that tracking error decreases asymptotically to zero. For this, let the tracking error be defined as

$$e(t) = y(t) - y_m(t) \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (9)$$

then the state feedback tracking control law can be constructed for system (1) as

$$u(t) \ = \ Hx_m(t) + \tilde{p}(t) \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots (10)$$

where $H \in R^{m\times n_m}$ is assumed to satisfy matrix algebraic equation (5) and $\tilde{p}(t)$ is an auxiliary control function which will be given later.

Here, we define a new auxiliary state vector $z(t)$ as follows.

$$z(t) \ = \ x(t) - Gx_m(t) \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots (11)$$

where $G \in R^{n\times n_m}$ is assumed to satisfy matrix algebraic equation (5).

From (5) and (11) we can obtain

$$e(t) \ = \ Cz(t) \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (12)$$

Then, from (12) we have

$$\|e(t)\| \ \le \ \|C\|\|z(t)\|$$

Since $\|C\| \le \infty$, it follows that

$$\|z(t)\| \to 0 \quad \text{implies} \quad \|e(t)\| \to 0$$

So it is sufficient to consider the stability of $\|z(t)\|$.

Applying (10) to (1) yields the auxiliary systems described by

$$\begin{aligned}\frac{dz(t)}{dt} = &\left[\, A + \Delta A(\upsilon,t) \,\right] z(t) \\ &+ \sum_{j=1}^{r} \Delta E_j(\zeta,t) z(t-h_j(t)) \\ &+ \left[\, B + \Delta B(\nu,t) \,\right] \tilde{p}(t) \\ &+ g(\upsilon,\zeta,\nu,q,x_m) \quad \cdots\cdots\cdots\cdots\cdots\cdots (13)\end{aligned}$$

where

$$\begin{aligned}g(\upsilon,\zeta,\nu,q,x_m) &:= \left[\Delta A(\upsilon,t)G + \Delta B(\nu,t)H\right]x_m(t) \\ &+ \sum_{j=1}^{r}\Delta E_j(\zeta,t)Gx_m(t-h_j(t)) + w(q,t) \quad \cdots\cdots (14)\end{aligned}$$

Then, by making use of the matching condition (see Assumption 2.2), (14) can be reduced to

$$g(\upsilon,\zeta,\nu,q,x_m) \ = \ BF(\upsilon,\zeta,\nu,q,x_m) \quad \cdots\cdots\cdots (15)$$

where

$$\begin{aligned}F(\upsilon,\zeta,\nu,q,x_m) &:= \left[N(\upsilon,t)G + E(\nu,t)H\right]x_m(t) \\ &+ \sum_{j=1}^{r} D_j(\zeta,t)Gx_m(t-h_j(t)) + \tilde{w}(q,t) \quad \cdots\cdots (16)\end{aligned}$$

Furthermore, we introduce for (16) the following notation.

$$\begin{aligned}\rho(t) \ := \ \max\Big\{&\|F(\upsilon,\zeta,\nu,q,x_m)\| \ : \\ &(\upsilon,\zeta,\nu,q) \in \Psi, \|x_m(t)\| \le M, t \in R\Big\} \quad \cdots\cdots (17)\end{aligned}$$

Here, function $\rho(t)$ is still assumed to be uniformly continuous with respect to time.

Now, we give the auxiliary control function $p(t)$ as follows.

$$\tilde{p}(t) \ = \ \tilde{p}_1(z(t),t) + \tilde{p}_2(z(t),t) \quad \cdots\cdots\cdots\cdots (18a)$$

where

$$\tilde{p}_1(z(t),t) = -\frac{1}{2}k_1(t)B^\top Pz(t) \quad \cdots\cdots\cdots\cdots (18b)$$

$$\tilde{p}_2(z(t),t) = -\frac{k_2(t)B^\top Pz(t)}{\|B^\top Pz(t)\|\,\rho(t) + \varepsilon\|z(t)\|^2} \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (18c)$$

and where the control gain functions $k_1(t)$ and $k_2(t)$ are given by

$$k_1(t) = \frac{\eta + \delta^2\rho_\upsilon^2(t) + \sum_{j=1}^{r}\delta_j^2\rho_j^2(t)}{1+\mu(t)} \quad \cdots\cdots (18d)$$

$$k_2(t) = \frac{\rho^2(t)}{1+\mu(t)} \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (18e)$$

where $\varepsilon,\ \delta,\ \delta_j,\ \ j = 1, 2, \ldots, r$, are positive constants, and $\delta,\ \delta_j,\ \ j = 1, 2, \ldots, r$, are selected such that the following conditions are satisfied.

$$\frac{1}{\delta^2} + \sum_{j=1}^{r}\frac{\sigma^2}{\delta_j^2} \ < \ \lambda_{\min}(Q) - 2\varepsilon \quad \cdots\cdots\cdots\cdots (18f)$$

Here, ε has been chosen such that $0 < 2\varepsilon < \lambda_{\min}(Q)$.

Remark 3.1. The memoryless state feedback controller described in (18) consists of two parts, $\tilde{p}_1(\cdot)$ and $\tilde{p}_2(\cdot)$. Here, $\tilde{p}_1(\cdot)$ is linear in the state, and $\tilde{p}_2(\cdot)$ is continuous (nonlinear) controller which is employed to compensate for the uncertain $F(\cdot)$ including external disturbance to produce an asymptotic stability results for tracking error $e(t)$ between uncertain time–delay system (1) and the reference model without time–delay.

Remark 3.2. In Assumption 2.2, it is assumed that the matrix functions $N(\cdot)$, $D_j(\cdot)$, $E(\cdot)$, $\tilde{w}(\cdot)$ are continuous in all their arguments. In addition, their bounds $\rho_\upsilon(t)$, $\rho_j(t)$, $\rho_q(t)$, $\mu(t)$, as well as $\rho(t)$ are also assumed to be uniformly continuous with respect to time. Thus, it is obvious that the nonlinear auxiliary control function $\tilde{p}_2(z(t),\, t)$ described by (18c) is uniformly continuous in with respect to time. Moreover, by noting that for any $(z,\, t) \in R^n \times R^+$,

$$\|B^\top Pz(t)\|\,\rho(t) \le \|B^\top Pz(t)\|\,\rho(t) + \varepsilon\|z(t)\|^2$$

we can obtain from (18c) that for any $(z,\, t) \in R^n \times R^+$,

$$\|\tilde{p}_2(z(t),\, t)\| \ \le \ \frac{\rho(t)}{1+\mu(t)}$$

which shows the boundedness of the nonlinear auxiliary control function $\tilde{p}_2(z,t)$. Therefore, it can be concluded that the auxiliary control function described by (18) is uniformly continuous with respect to time, and and uniformly bounded with respect to z.

Remark 3.3. It is worth pointing out that at the origin $z=0$, both numerator and denominator of the control described by (18c) vanish. This implies that the existence of the solutions to the closed–loop auxiliary system given by (13 and (18) may not be guaranteed at the origin $z=0$ in the usual sense. However, by employing the method similar to the one presented in Ref. (19), we can easily prove that the right–hand side of the closed–loop system given by (13) and (18) is upper semicontinuous on $(z,\,t)\in R^n\times R^+$. Therefore, as a generalized dynamical system (GDS), the existence of the solutions to the closed–loop system can be well guaranteed. This implies that the limit of control (18c) as z approaches 0 exists.

Thus, we can obtain the following theorem which shows that by employing the auxiliary controller described in (18), one can guarantee the uniform asymptotic stability of the auxiliary systems, described by (13), in the presence of the uncertain parameters and multiple delayed state perturbations.

Theorem 3.1. Consider the auxiliary systems, described in (13). Suppose that *Assumptions 2.1* to *2.3* are satisfied. Then, by employing the auxiliary state feedback controllers given in (18), one can guarantee the uniform asymptotic stability of the auxiliary system.

Proof: Applying the controller given in (18) to (13) yields the following closed–loop auxiliary systems.

$$\begin{aligned}\frac{dz(t)}{dt} &= \left[A-\frac{1}{2}k_1(t)BB^\top P\right]z(t)\\ &\quad+\Delta B(\nu,t)\tilde{p}_1(z(t),t)+\Delta A(\upsilon,t)z(t)\\ &\quad+\sum_{j=1}^{r}\Delta E_j(\zeta,t)z(t-h_j(t))\\ &\quad+\Big[B+\Delta B(\nu,t)\Big]\tilde{p}_2(z(t),t)\\ &\quad+g(r,s,q,x_m)\ \cdots\cdots\cdots\cdots\cdots\cdots\ (19)\end{aligned}$$

For a nominal system (i.e. the system in the absence of the uncertain parameters and delayed state perturbations) of auxiliary system (19), we first define a positive definite function of the form

$$V(z(t),t)\;=\;z^\top(t)Pz(t),\ \cdots\cdots\cdots\cdots\cdots\ (20)$$

where $P\in R^{n\times n}$ is the solution of algebraic Riccati equation (7).

Let $z(t)$ be the solution of the closed–loop auxiliary systems described by (19) for $t\geq t_0$; and let the Lyapunov function, described by (20), of the nominal system be a candidate of the Lyapunov function of systems (19). Then, we can obtain that for any $t\geq t_0$,

$$\begin{aligned}\frac{dV(z_t,t)}{dt} &= z^\top(t)\big[A^\top P+PA-k_1(t)PBB^\top P\big]z(t)\\ &\quad+2z^\top(t)P\Delta A(\upsilon,t)z(t)\\ &\quad+2z^\top(t)P\Delta B(\nu,t)\tilde{p}_1(z(t),t)\\ &\quad+2z^\top(t)P\sum_{j=1}^{r}\Delta E_j(\zeta,t)z(t-h_j(t))\\ &\quad+2z^\top(t)P\Big[B+\Delta B(\nu,t)\Big]\tilde{p}_2(z(t),t)\\ &\quad+2z^\top(t)Pg(\upsilon,\zeta,\nu,q,x_m)\ \cdots\cdots\ (21)\end{aligned}$$

In the light of (13) and Assumption 2.2, (21) can be rewritten as follows.

$$\begin{aligned}\frac{dV(z_t,t)}{dt} &= z^\top(t)\big[A^\top P+PA-k_1(t)PBB^\top P\big]z(t)\\ &\quad-k_1(t)z^\top(t)PB\left[\frac{1}{2}\left(E+E^\top\right)\right]B^\top Pz(t)\\ &\quad+2z^\top(t)PBN(\upsilon,t)z(t)\\ &\quad+2z^\top(t)PB\sum_{j=1}^{r}D_j(\zeta,t)z(t-h_j(t))\\ &\quad+2z^\top(t)PB\Big[I+E(\nu,t)\Big]\tilde{p}_2(z(t),t)\\ &\quad+2z^\top(t)PBF(\upsilon,\zeta,\nu,q,x_m)\\ &\leq z^\top(t)\big[A^\top P+PA-k_1(t)PBB^\top P\big]z(t)\\ &\quad-k_1(t)\mu(t)z^\top(t)PBB^\top Pz(t)\\ &\quad+2\left\|B^\top Pz(t)\right\|\|N(\upsilon,t)\|\,\|z(t)\|\\ &\quad+2\left\|B^\top Pz(t)\right\|\sum_{j=1}^{r}\|D_j(\zeta,t)\|\,\|z(t-h_j(t))\|\\ &\quad-\frac{2k_2(t)\big(1+\mu(t)\big)z^\top(t)PBB^\top Pz(t)}{\left\|B^\top Pz(t)\right\|\rho(t)\;+\;\varepsilon\|z(t)\|^2}\\ &\quad+2\left\|B^\top Pz(t)\right\|\|F(\upsilon,\zeta,\nu,q,x_m)\|\\ &\leq -z^\top(t)Qz(t)\\ &\quad-\Big(\delta^2\rho_\upsilon^2(t)+\sum_{j=1}^{r}\delta_j^2\rho_j^2(t)\Big)\left\|B^\top Pz(t)\right\|^2\\ &\quad+2\rho_\upsilon(t)\left\|B^\top Pz(t)\right\|\|z(t)\|\end{aligned}$$

$$+2\sum_{j=1}^{r}\rho_j(t)\left\|B^\top Pz(t)\right\|\|z(t-h_j(t))\|$$

$$-\frac{2\rho^2(t)\left\|B^\top Pz(t)\right\|^2}{\left\|B^\top Pz(t)\right\|\rho(t)\ +\ \varepsilon\|z(t)\|^2}$$

$$+2\rho(t)\left\|B^\top Pz(t)\right\| \cdots\cdots\cdots\cdots\cdots\cdots\cdots (22)$$

where $Q \in R^{n\times n}$ is given positive definite matrix.

Following the Razumikhin–type theorem (see, e.g. Lemma 2.1), we assume that for any positive number $\bar{q} > 1$, the following inequality holds.

$$V(z(\xi),\xi)\ <\ \bar{q}^2V(z(t),t),\qquad \xi\in[t-\bar{h},\ t]$$

Then, it follows from (20) and the property of the matrix P that

$$\|z(\xi)\|\ <\ \bar{q}\sigma\|z(t)\|,\qquad \xi\in[t-\bar{h},\ t]\ \cdots\cdots (23)$$

By substituting (23) into (22) we can obtain that for any $t \geq t_0$,

$$\begin{aligned}
\frac{dV(z_t,t)}{dt}\ \le\ & -z^\top(t)Qz(t)\\
&-\Big(\delta^2\rho_v^2(t)+\sum_{j=1}^{r}\delta_j^2\rho_j^2(t)\Big)\left\|B^\top Pz(t)\right\|^2\\
&+2\rho_v(t)\left\|B^\top Pz(t)\right\|\|z(t)\|\\
&+2\sum_{j=1}^{r}\bar{q}\sigma\rho_j(t)\left\|B^\top Pz(t)\right\|\|z(t)\|\\
&-\frac{2\rho^2(t)\left\|B^\top Pz(t)\right\|^2}{\left\|B^\top Pz(t)\right\|\rho(t)\ +\ \varepsilon\|z(t)\|^2}\\
&+2\rho(t)\left\|B^\top Pz(t)\right\|\\
=& -z^\top(t)Qz(t)-\delta^2\rho_v^2(t)\left\|B^\top Pz(t)\right\|^2\\
&+2\rho_v(t)\left\|B^\top Pz(t)\right\|\|z(t)\|\\
&-\sum_{j=1}^{r}\delta_j^2\rho_j^2(t)\left\|B^\top Pz(t)\right\|^2\\
&+2\sum_{j=1}^{r}\bar{q}\sigma\rho_j(t)\left\|B^\top Pz(t)\right\|\|z(t)\|\\
&-\frac{2\rho^2(t)\left\|B^\top Pz(t)\right\|^2}{\left\|B^\top Pz(t)\right\|\rho(t)\ +\ \varepsilon\|z(t)\|^2}\\
&+2\rho(t)\left\|B^\top Pz(t)\right\|\\
=& -z^\top(t)Qz(t)+(1\ /\ \delta^2)\|z(t)\|^2\\
&-\left[\delta\rho_v(t)\left\|B^\top Pz(t)\right\|-\frac{1}{\delta}\|z(t)\|\right]^2\\
&-\sum_{j=1}^{r}\left[\delta_j\rho_j(t)\left\|B^\top Pz(t)\right\|-\frac{\bar{q}\sigma}{\delta_j}\|z(t)\|\right]^2\\
&+\sum_{j=1}^{r}\frac{(\bar{q}\sigma)^2}{\delta_j^2}\|z(t)\|^2\\
&+\frac{2\left\|B^\top Pz(t)\right\|\rho(t)\ \cdot\ \varepsilon\|z(t)\|^2}{\left\|B^\top Pz(t)\right\|\rho(t)\ +\ \varepsilon\|z(t)\|^2}\ \cdots\cdots\cdots (24)
\end{aligned}$$

Therefore, it follows from (24) and from the inequality

$$0\ \le\ \frac{ab}{a+b}\ \le\ a,\quad \forall\, a,\, b\ >\ 0$$

that

$$\begin{aligned}
\frac{dV(z_t,t)}{dt}\le& -z^\top(t)Qz(t)+(1\ /\ \delta^2)\|z(t)\|^2\\
&+\sum_{j=1}^{r}\frac{(\bar{q}\sigma)^2}{\delta_j^2}\|z(t)\|^2+2\varepsilon\|z(t)\|^2\\
\le& -\gamma\,\|z(t)\|^2\ \cdots\cdots\cdots\cdots\cdots\cdots (25)
\end{aligned}$$

where γ is defined by

$$\gamma:=\lambda_{\min}(Q)-2\varepsilon-\left[\frac{1}{\delta^2}+\sum_{j=1}^{r}\frac{(\bar{q}\sigma)^2}{\delta_j^2}\right]\ \cdots\cdots (26)$$

If the control gain parameters δ and δ_j, $j = 1, 2, \ldots, r$, are selected such that (18f) is satisfied, then a sufficiently small $\bar{q} > 1$ exists such that $\gamma > 0$. Thus, according to [Ref. (4), *Theorem 4.2* or Ref. (18)], the closed–loop system, described by (19), is uniformly asymptotically stable. That is, the auxiliary state $z(t)$ tends asymptotically to zero. ■

Thus, from Theorem 3.1 we can obtain the following theorem which shows that by employing the memoryless state feedback controllers described in (10) with (18), one can guarantee the zero–error tracking errors between the uncertain time–delay systems and reference model without time–delay.

Theorem 3.2. Consider the model following problem of uncertain time–delay system (1) satisfying *Assumptions 2.1* to *2.3*. Then, by using the state feedback controllers $u(t)$ described in (10) with (18), one can guarantee that the tracking error $e(t)$ decreases asymptotically to zero.

Proof: From *Theorem 3.1*, we have shown that the closed–loop auxiliary system described by (13) and (18) is uniformly asymptotically stable. That is, for the auxiliary state $z(t)$, we can obtain that

$$\lim_{t\to\infty} z(t)\ =\ 0$$

Then, it can be obtained from the relationship between $e(t)$ and $z(t)$, i.e. $e(t) = Cz(t)$, that the tracking error $e(t)$ also decreases asymptotically to zero. ■

Remark 3.4. Similar to Ref. (2), we can give a procedure for constructing the memoryless state feedback controller described in (10) with (18) as follows.

(i) Find the solutions G and H of algebraic matrix equation (5). If no solution exists, then a different choice of the reference model or the output matrix C must be made.

(ii) Solve, for any given positive constant η and positive definite matrix Q, algebraic Riccati equation (7) for P.

(iii) Evaluate the bounds of the uncertain $N(\cdot)$, $D_j(\cdot)$, $j = 1, 2, \ldots, r$, $E(\cdot)$, and $F(\cdot)$, to obtain $\rho_v(\cdot)$, $\rho_j(\cdot)$, $j = 1, 2, \ldots, r$, $\mu(\cdot)$, and $\rho(\cdot)$.

(iv) Select a set of the control parameters ε, δ, and δ_j, $j = 1, 2, \ldots, r$, such that the inequality described by (18f) holds.

(v) Form the memoryless state feedback tracking controller described by (10) with (18).

4. An Illustrative Example

In this section, we give a numerical example to illustrate the procedure for designing the tracking controller proposed in the paper. Here, we consider an uncertain time–delay systems described by the following differential–difference equations.

$$\frac{dx(t)}{dt} = \left(\begin{bmatrix} -1 & 1 \\ 0 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \cos(t) & \sin(t) \end{bmatrix} \right) x(t)$$

$$+ \begin{bmatrix} 0 & 0 \\ 0.5\sin(t) & 0.5\cos(t) \end{bmatrix} x(t - h_1(t))$$

$$+ \begin{bmatrix} 0 & 0 \\ 0 & \sin(t) \end{bmatrix} x(t - h_2(t))$$

$$+ \left(\begin{bmatrix} 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 - 0.5\sin(t) \end{bmatrix} \right) u(t)$$

$$+ \begin{bmatrix} 0 \\ 0.5\cos(t) \end{bmatrix} \quad \cdots\cdots\cdots\cdots\cdots\cdots \quad (27a)$$

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(t) \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots \quad (27b)$$

The time–varying delays $h_1(t)$ and $h_2(t)$ are given in *Fig.1*, where $h_1(t) = 1 + 0.5\sin(\pi t)$ and $\bar{h} = 2$. It is worth noting that $h_1(t)$ and $h_2(t)$ are two continuous and bounded functions, and that $\dot{h}_2(t)$ is not defined at $t = 0.25n$, $n = 1, 2, \ldots$.

The problem is to determine a control law in the form (10) with (18), that will cause system (27) to follow the reference model described by

$$\frac{dx_m(t)}{dt} = \begin{bmatrix} -2 & 1 \\ 0 & -1 \end{bmatrix} x_m(t) \quad \cdots\cdots\cdots\cdots \quad (28a)$$

$$y_m(t) = \begin{bmatrix} 0 & 1 \end{bmatrix} x_m(t) \quad \cdots\cdots\cdots\cdots\cdots\cdots \quad (28b)$$

and can guarantee that the tracking error $e(t)$ decreases asymptotically to zero in the presence of the uncertain parameters.

For simulation, the following initial condition for (27) and (28) are given as follows.

$$x(t) = \begin{bmatrix} 2.0\cos(t) & 1.0\cos(t) \end{bmatrix}^\top, \quad t \in [-\bar{h},\ 0]$$

$$x_m(0) = \begin{bmatrix} 1.0 & 1.0 \end{bmatrix}^\top$$

Now, following the algorithm given in Remark 3.4, we will construct such a robust memoryless state feedback tracking controller.

(i) From algebraic matrix equation (5), it can be obtained that the solution of the matrices G and H is as follows.

$$G = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \qquad H = \begin{bmatrix} 0 & 0 \end{bmatrix}$$

(ii) For a given constant $\eta = 2$ and the positive definite matrix $Q = \mathrm{diag}\{5,\ 3\}$, it can be obtained from algebraic Riccati equation (7) that

$$P = \begin{bmatrix} 2.383 & 0.684 \\ 0.684 & 8.973 \end{bmatrix}$$

(iii) By evaluating we can obtain that

$$\rho_v(t) = 1.42, \qquad \rho_1(t) = 0.71$$
$$\rho_2(t) = |\sin(t)|, \qquad \rho_q(t) = 0.50$$
$$\mu(t) = 1 - 0.5\sin(t), \qquad \rho(t) = 3.855$$

(iv) Select a set of the control parameters ε, δ, δ_1, and δ_2, as follows.

$$\varepsilon = 0.5, \quad \delta = 2.0, \quad \delta_1 = \delta_2 = 3.0$$

(v) Then, from (10) with (18) we can obtain a robust memoryless state feedback tracking controller described by

$$u(t) = -\frac{1}{2}k_1(t)B^\top Pz(t)$$
$$- \frac{k_2(t)B^\top Pz(t)}{3.855\left\|B^\top Pz(t)\right\| + 0.5\|z(t)\|^2}$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \quad (29a)$$

where

$$k_1 = \frac{14.5 + 9\sin^2(t)}{2 - 0.5\sin(t)} \quad \cdots\cdots\cdots \quad (29b)$$

$$k_2 = \frac{14.861}{2 - 0.5\sin(t)} \quad \cdots\cdots\cdots\cdots \quad (29c)$$

$$B^\top Pz(t) = 0.684x_1(t) + 8.973x_2(t)$$
$$-0.684x_{2m}(t) \quad \cdots\cdots\cdots\cdots \quad (29d)$$

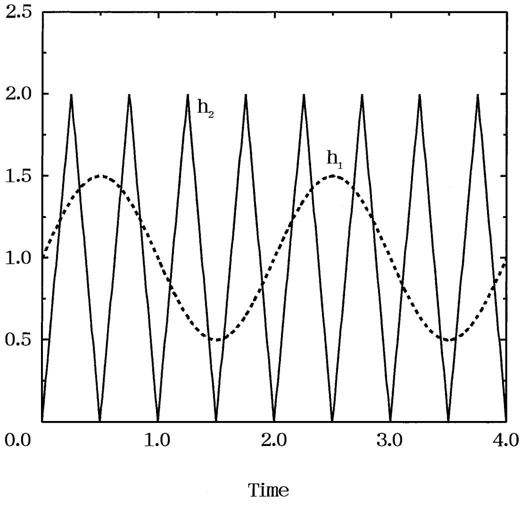

Fig. 1. Time delay history.

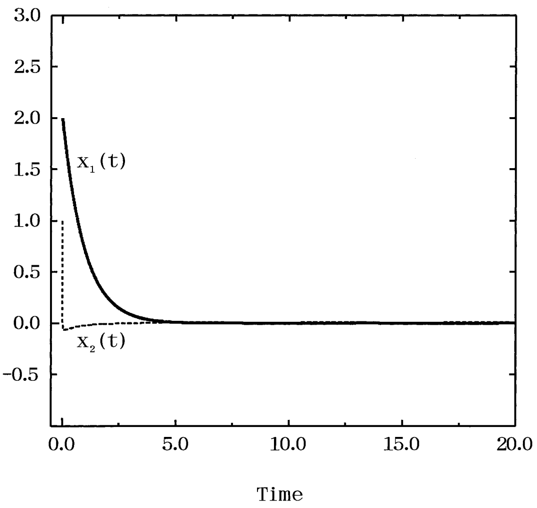

Fig. 2. Response of state variable $x(t)$.

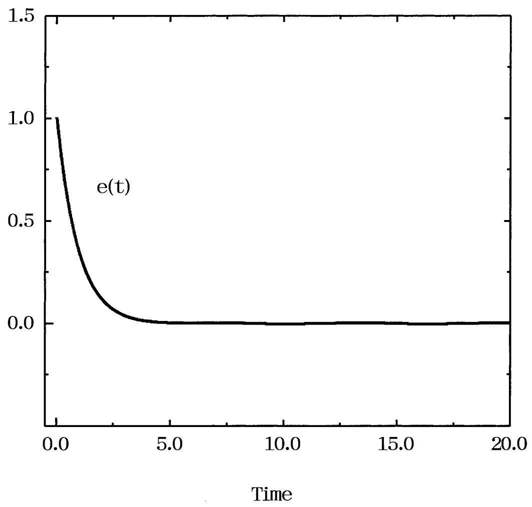

Fig. 3. Tracking error $e(t)$.

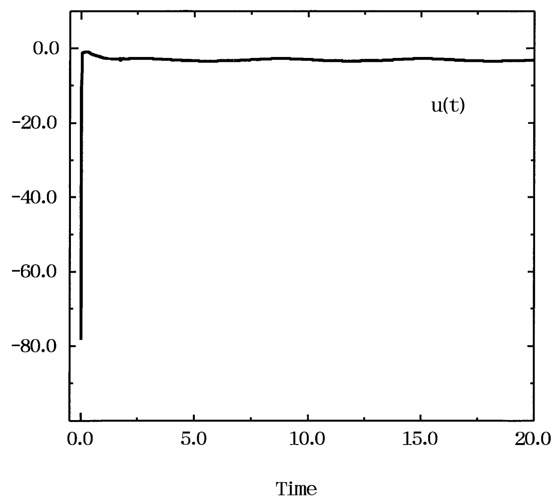

Fig. 4. Control input $u(t)$.

The results of the simulations of this numerical example are depicted in *Figs. 2* to *4*. It is shown from *Fig.3* that the tracking error $e(t)$ indeed decreases asymptotically to zero in the presence of uncertain parameters, delayed state perturbations, and disturbance.

5. Concluding Remarks

The problem of robust tracking and model following for a class of linear dynamical systems with multiple delayed state perturbations, time–varying uncertain parameters, and disturbance has been considered. A class of continuous memoryless state feedback controllers have been proposed for robust tracking of dynamical signals in such a class of uncertain time-delay systems. The proposed controller consists of two parts, i.e. linear and nonlinear. The nonlinear controller is continuous and bounded, and is used to compensate for the uncertainty including the external disturbance of the systems to produce an asymptotic tracking result. That is, the proposed tracking controller can guarantee a zero–tracking error, instead of the ε–tracking error reported in the control literature.

It is worth pointing out that the method for designing a zero–error tracking controller can easily be extended to systems with mismatched uncertainties. In this case, a sufficient condition for robust stability of the systems should be derived.

Finally, a numerical example is given to demonstrate the synthesis procedure for the proposed zero–error tracking controller. It is shown from the example and the results of its simulation that the results obtained in the paper are effective and feasible. Therefore, our results may be expected to have some applications to practical robust tracking and model following problems of uncertain time–delay systems.

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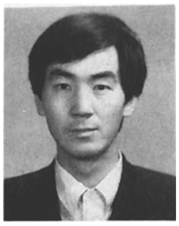

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