3-Dimensional Simulation on Electrical Conductivity of CuCr Contact Materials

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According to the phenomenal process of electrical conductance in metals and alloys, a 3-dimensional unit net model for calculating the electrical conductivity of shammed alloys is presented in this paper. By this model, the influence of the components and their distribution upon conductivity of CuCr serial alloys are simulated, as well as the gas content in the phase structure. The comparison of the simulation results with typical experimental values is carried out. The precision of the model is analyzed. The results approved the availability and effectiveness of this model.

Keywords : Unit net model, CuCr contact material, electrical conductivity, simulation

1. INTRODUCTION

It is well known that contact material is one of the most important factors to determine the performance of vacuum switches. This is due to the fact that the vacuum arc is a low-pressure plasma burning in the vapor of contact material. Because of its superiority, CuCr alloy is widely used in vacuum switches in recent years. As a basic parameter to demonstrate the features of contact materials, the electrical conductivity not only affects the current carrying capability of vacuum switches, also has an important influence on the current chopping property, electrical insulation strength, current interrupting ability and post-arc recrystallization characteristics of melted electrode surface [1-3].

However, as an apparent shammed alloy, i.e., the Cr can hardly be solvable in Cu under 500°C, and with only 1.28% solvability even when the temperature increases to 1080°C [4][5], it is difficult to calculate the electrical conductivity of CuCr alloys in theory till now. Of course, the conductivity of CuCr material is basically determined by its ingredients. But the exact value of the conductivity is characterized by construct sensitivity, that is to say, it is also influenced by the grain size, distribution and bond state of Cu and Cr atoms [2][3], as well as the content of inclusions and gases. It seems impossible to obtain the exact value of electrical conductivity of CuCr alloy in theory.

In practice, a statistical weighted average method is widely employed, i.e., the electrical conductivity of CuCr alloy is regarded as the weight averaged value according to the content of Cu and Cr,

$$\sigma = \sum_i C_i \sigma_i$$

where C_i is the volume ratio of element i, σ_i the

conductivity of element i. It is simple for this model in use, but the result is.

On the other hand, in phenomenon aspect, the electrical conduction can be described as a moving process of free electrons along the reverse electric field from one lattice to others. In physics, the formation of resistance in metals and alloys is due to the collision and scattering by grains (also the defects and inclusions) to the directed moving free electrons. In effect, each collision happens between the electron and the grain can be described by a little resistor. Then all of the collision happening in the bulk material can be collectively represented by a resistor network in phenomenon. It can be found, by this idea, the problem of calculating the electrical conductivity of contact material has been transformed into a problem of solution of a resistor network circuit, which appears simpler than the original one. If all of the element resistor of the network have been identified, an equivalent resistor stands for this network can be figured out. Because we only pay our attention upon a unit volume (1×1×1) of the bulk material, then the resistivity of the material is the same as the equivalent resistance of the network in value, and the conductivity is the reverse of the resistivity.

2. THE UNIT NET MODEL

Figure 1 is a 3-dimensional resistor network formed according to the idea mentioned above. It needs to point out that each appeared element resistor in a really constructed network is not an exact description of one collision, but a collective representation of a series of collision occur in a certain volume.

For a certain node n_i (i =1,2,3...) of the network, it usually has 6 adjacent nodes (forward and behind, left and

right, up and down, as shown in figure 1), which can be recorded as n_{ij}, (j =1,2,..., 6). There is an element resistor between the two adjacent nodes which can be recorded as $R_{(i,ij)}$, (j =1,2,...,6). When carrying out the simulation, a characteristic matrix $\left| N_{ij} \right|$ is used to describe the correlation of node n_i with its adjacent nodes n_{ij}. If there is no adjacent node in some directions (for boundary nodes), then filled with zero in the element of the characteristic matrix.
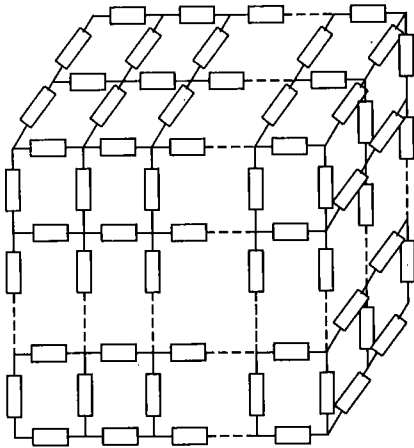

Fig.1   3-D resistor network model

For the purpose of identifying the element resistors of the network, a series of rules are proposed.

Supposing that every element of the selected unit bulk material has the same volume, the kind of its component is determined according to the component volume percent. Supposing that the weight percent of Cu and Cr in the alloy bulk is α and β, respectively, where $\alpha + \beta = 1.0$. The volume percent of gas content in the alloy is set to be θ. The volume percent of Cu and Cr are P_1 and P_2 respectively, where

$$P_1 = \frac{\dfrac{\alpha}{\rho_{m-Cu}}}{\dfrac{\alpha}{\rho_{m-Cu}} + \dfrac{\beta}{\rho_{m-Cr}}}(1-\theta) \tag{1-a}$$

$$P_2 = \frac{\dfrac{\beta}{\rho_{m-Cr}}}{\dfrac{\alpha}{\rho_{m-Cu}} + \dfrac{\beta}{\rho_{m-Cr}}}(1-\theta) \tag{1-b}$$

where ρ_{m-Cu}, ρ_{m-Cr} is the mass density of Cu and Cr, P_1 and P_2 the volume percentage of Cu and Cr in alloy, respectively.

Because the Cu and Cr grains are regarded to be randomly distributed in a bulk CuCr alloy, to determine the identity of an element, a number series W with equal

possibility between [0,1] is produced by computer itself firstly. It is simple to run a Monte-Caro algorithm to fulfill this task. For any element of this number series, it is a random number w. For a certain node n_i, to determine the values of the 6 element resistors between it and the adjacent nodes, we introduce a function $f[n_i]$ firstly, it can be defined that

(1) If $0.0 \le w < P_1$, then $f[n_i] = 1$

(2) If $P_1 \le w < (P_1 + P_2)$, then $f[n_i] = 2$

(3) If $(P_1 + P_2) \le w < 1.0$, then $f[n_i] = 0$

The identification of resistance distribution of the equivalent circuit is

(1) If $f[n_i] \; f[n_j] = 0$, then

$$R_{(i,j)} = R(n_i, n_j) = +\infty \;,$$

(2) If $f[n_i] \; f[n_j] = 1$, then

$$R_{(i,j)} = R(n_i, n_j) = \eta / \sigma_{Cu} \;,$$

(3) If $f[n_i] \; f[n_j] = 4$, then

$$R_{(i,j)} = R(n_i, n_j) = \eta / \sigma_{Cr} \;,$$

(4) If $f[n_i] \; f[n_j] = 2$, then

$$R_{(i,j)} = R(n_i, n_j) = 0.5(\eta / \sigma_{Cu} + \eta / \sigma_{Cr}) .$$

The parameters σ_{Cu} and σ_{Cr} are the conductivity of Cu and Cr component, η is a parameter which marks the size of element volume. $R_{(i,j)} = R(n_i, n_j)$ is the element resistor between the node n_i and node n_j. It is easy to realize the meaning of the mentioned above determination criterion, i.e., for case (1), it means that the element volume represented by element resistor $R_{(i,j)}$ is mainly consisted by electrically insulated gas void, and for case (2) by Cu, for case (3) by Cr, and for case (4) by equal mechanical mixture of Cu and Cr. This is due to the reason of the shammed alloy of CuCr series material, as mentioned before. It can be postulated that when the number of element volume increases high enough, this model can stand for the reality with high enough precision.

Because the characteristic matrix for node n_i has no greater than 6 non-zero elements which represent the directly adjacent nodes with n_i in various directions, it can ensure that only directly adjacent two nodes may have non-zero element resistor, which can also be noted as $R_{(i,ij)}$, (j=1, 2,..., 6). This is consistent with our net model shown in figure 1. Also it can be proved that this series of rules can ensure the consistent value of $R_{(i,j)} = R_{(j,i)}$.

Because we take only a unit volume of the bulk material into account, and all of the element volume represented by an element resistor are supposed to be equal, then it can be

realized that η is equal to the number of element resistor in one direction in value.

3. SIMULATION

From the viewpoint of electrical conduction, conductivity of contact material is determined by its component jointly. The current transmitted by every node is decided not only by its applied voltage but also by the distinctions of its adjacent nodes.

Based on the Ohm's law, the flowing current out from node n_i of the network circuit must content to the following equation,

$$\sum_{j=1}^{6} I_{(i,ij)} = 0 \qquad (2a)$$

where $I_{(i,ij)}$ is the current flowing from node n_i to adjacent n_{ij}, namely,

$$\sum_{j=1}^{6} \frac{\phi_i - \phi_{ij}}{R_{(i,ij)}} = 0 \qquad (2b)$$

then,

$$\left(\frac{1}{R_{(i,i1)}} + \frac{1}{R_{(i,i2)}} + \frac{1}{R_{(i,i3)}} + \frac{1}{R_{(i,i4)}} + \frac{1}{R_{(i,i5)}} + \frac{1}{R_{(i,i6)}}\right)\phi_i - \frac{1}{R_{(i,i1)}}\phi_{i1} - \frac{1}{R_{(i,i2)}}\phi_{i2} - \frac{1}{R_{(i,i3)}}\phi_{i3} - \frac{1}{R_{(i,i4)}}\phi_{i4} - \frac{1}{R_{(i,i5)}}\phi_{i5} - \frac{1}{R_{(i,i6)}}\phi_{i6} = 0 \qquad (3)$$

where ϕ_i (i =1,2,3,...) is the potential of node n_i, ϕ_{ij} (j =1,2, ...,6) represents the potential of the adjacent node n_{ij}.

For all of the nodes of the network circuit, an equation set is formed as following,

$$A\Phi = F \qquad (4)$$

where A is a $(N \times N \times N) \times (N \times N \times N)$ matrix and characterized by large scale, symmetrical, positive and sparse. Here N is the number of element resistor (element volume) in one direction. There has pointed out in section 2 that $\eta = N$ in value. The matrix A has 7 non-zero elements in every line in general situation (represent itself and the other 6 adjacent nodes, respectively), and can be memorized by one-dimensional extracted method in practice. Φ and F are all $(N \times N \times N) \times 1$ matrix. This equation set can be solved by Incomplete Cholesey Conjugate Gradient (ICCG) algorithm.

4. RESULTS

The physical parameters of Cu and Cr in 20°C are the following,

$$\rho_{m\text{-}Cu} = 8.96 \times 10^3 kg \cdot m^{-3},$$
$$\sigma_{Cu} = 59.9 \times 10^6 \Omega^{-1} \cdot m^{-1}$$
$$\rho_{m\text{-}Cr} = 7.19 \times 10^3 kg \cdot m^{-3},$$
$$\sigma_{Cr} = 6.7 \times 10^6 \Omega^{-1} \cdot m^{-1}$$

The electrical conductivity of gas void is regarded to be zero.

<4.1> Electrical conductivity of CuCr in dependence of the Cu component

Firstly, we simulate the influence of the composition of CuCr upon the conductivity. The obtained electrical conductivity of CuCr alloy without gas content (θ=0) in dependence of Cu component is illustrated in figure 2. It is nearly an exponential increase function. An approximate correlation between conductivity and the content of Cu can be figured out that

$$\sigma = \exp(1.75 + 2.28C_{Cu}) \qquad (5)$$

where C_{Cu} is the volume content of Cu element and σ the conductivity ($\times 10^6 \Omega^{-1} \cdot m^{-1}$). For comparison, the statistical averaged values and the experimental measurements of reference [2] also are plotted in the figure.

From this figure, it can be find that the results obtained from our simulation agree very well with the typical experimental data, as the statistical averaged values are overestimated.

<4.2> Electrical conductivity of Cu75Cr25 in dependence of gas content

In above calculations, the influence of gas content did not be taken into account. In fact, any contact materials produced by any preparations may contain a certain volume of micro gas voids in the bulk, and the real density of the bulk material is lower than the ideal value. It is apparent especially for CuCr, because the Cr is an active metal and easy to bond with oxygen [2]. It is certainly that the content of gas and its distribution have an influence on the conductivity. This can be taken into account by the parameter θ in equation (1).

Figure 3(a) depicts out the simulation results of the influence of non-conductive gas content on electrical conductivity for Cu75Cr25 alloy. From this result it can be found that the gas content has a considerable effect on the characteristics of electrical conductivity, i.e., the σ nearly linear decreases with the increase of gas content. When the volume content of gases reaches only 4%, the conductivity of Cu75Cr25 drastically decreases to less than 90% of the value of no gas be included ($31.8 \times 10^6 \Omega^{-1} \cdot m^{-1}$). It also can be found from this figure that, even when the gas content reaches about 41% but not
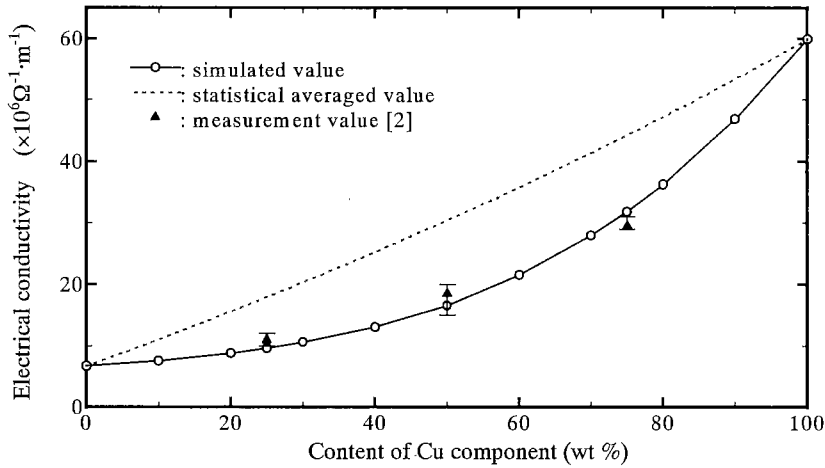
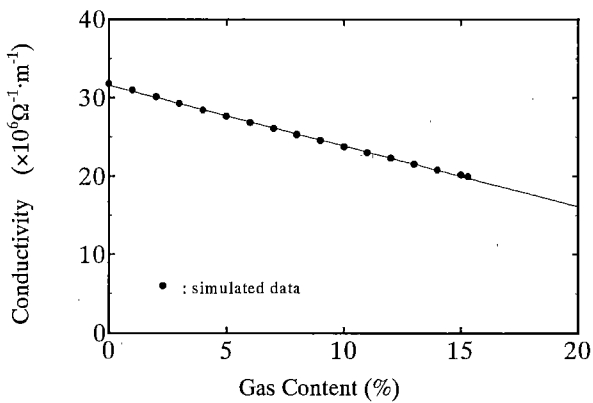Fig 2   Electrical conductivity of CuCr in dependence of Cu component

Fig.3(a)   Electrical conductivity of Cu75Cr25
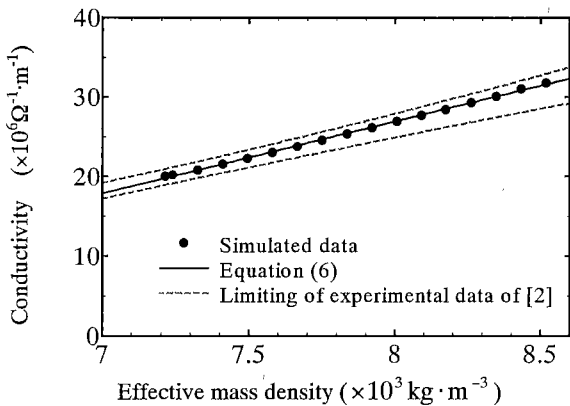in dependence of gas content

Fig.3(b)   Electrical conductivity of Cu75Cr25
in dependence of mass density

100%, the conductivity of the bulk material becomes 0. This represents that all of the conductive element of the bulk are isolated by insulated gas voids, and there is no a totally conductive branch occurring in the viewpoint of

circuit.

If we change the variable of the gas content in the bulk material into the effective mass density of Cu75Cr25, we can obtain figure 3(b). From this figure it can be found that our simulation results agrees very well with the experimental data [2]. An linear relation between the conductivity and the effective mass density can be summarized by the most fit method as

$$\sigma = 9.08\rho_{eff} - 45.72 \qquad (6)$$

where σ is the conductivity ($\times 10^6 \Omega^{-1} \cdot m^{-1}$), ρ_{eff} is the effective mass density ($\times 10^3 kg \cdot m^{-3}$) which can be calculated by

$$\rho_{eff} = \frac{1-\theta}{\dfrac{\alpha}{\rho_{m-Cu}} + \dfrac{\beta}{\rho_{m-Cr}}} \qquad (7)$$

This result implies the necessity of the emphasis on gas control in CuCr preparing, if we want to obtain a high performance alloy.
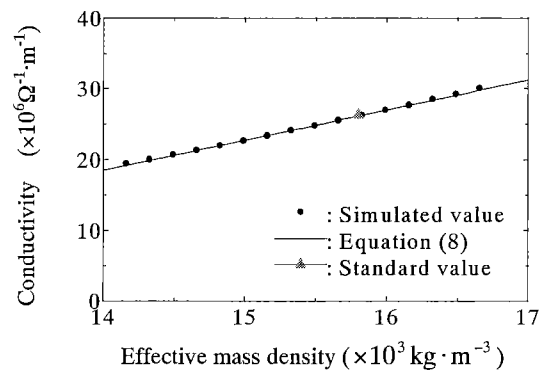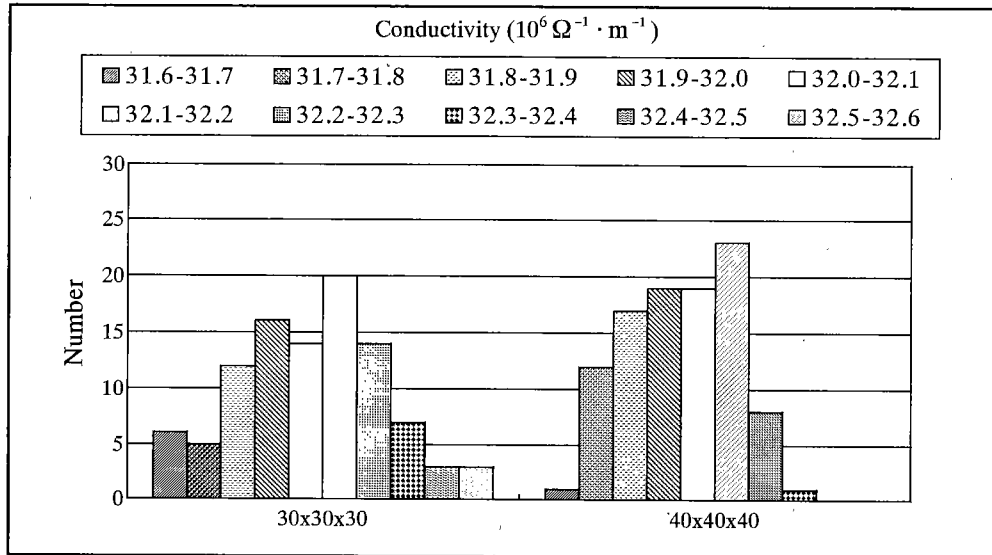

Fig.4   Electrical conductivity of Ag30W70
in dependence of the density of the material

Fig 5. Distribution of calculated values of 100 times

<4.3> Electrical conductivity of AgW contact material

The above results approved the success of our model for CuCr series alloys. To further affirm its availability and effectiveness, we employ this unit net model to another type of shammed alloy AgW, which is widely used in low over-voltage vacuum switches. The simulated result of Ag30W70 with 5% gas content is illustrated in figure 4. An linear relation between the conductivity and the mass density also can be figured out as

$$\sigma' = 4.25\rho'_{eff} - 40.98. \qquad (8)$$

where σ' is the electrical conductivity of Ag30W70 ($\times 10^6 \Omega^{-1} \cdot m^{-1}$), ρ'_{eff} the effective mass density ($\times 10^3 kg \cdot m^{-3}$).

According to the Standard, the resistivity of Ag30W70 alloy with 5% gas content in volume is $3.84 \times 10^{-8} \Omega \cdot m$, the accordingly conductivity can be calculated to be $26.0 \times 10^6 \Omega^{-1} \cdot m^{-1}$. On the other hand, the effective mass density of Ag30W70 with 5% gas content can be calculated by equation (7) to be $15.8 \times 10^3 kg \cdot m^{-3}$. Our simulation shown in figure 4 gives the result that, in this condition its conductivity is $26.3 \times 10^6 \Omega^{-1} \cdot m^{-1}$.

It is revealed that the simulation result agrees very well with the standard value.

5. ANALYSIS OF SIMULATION PRECISION

It is easy to realize that the precision of simulation by our model is determined by the choice of number of element resistor. However, because of the limitations of computer memory and running speed, we can simulate no more than 55×55×55 elements at one time. But how many elements are necessary to ensure a high enough precision? Here we give the analysis about the influence of the

number of element volume upon the precision of simulation result.

For this purpose, 100 times of calculation for Cu75Cr25 alloy (without gas content) in condition that the element of the network is selected as 30×30×30 and 40×40×40, respectively, have been carried out. The histogram to stand for the distribution of the obtained results is shown in figure 5.
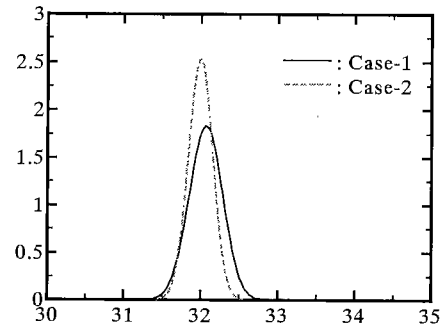

Fig. 6. Distribution function of calculated value

From this figure, we can find that the calculated results of 100 times distributed in a relatively narrow interval from about 31.6×10^6 to about $32.6 \times 10^6 (\Omega^{-1} \cdot m^{-1})$, the dissipation is less than 3% in each case. The distribution of the results can be described by a normal distribution law in statistics for each case, as shown in figure 6. The distribution density can be described by

$$f(\sigma) = \frac{1}{\chi \sqrt{2\pi}} \exp[-\frac{1}{2}(\frac{\sigma - \bar{\sigma}}{\chi})^2], \qquad (9)$$

where $\bar{\sigma}$ is the arithmetic average value, χ the standard deviation. The distribution parameters of these two cases are calculated [6] and given out in table 1.

Table 1 Characteristic parameters of the distribution function

	Case-1 30×30×30	Case-2 40×40×40
Arithmetic average value $\bar{\sigma}$ $(\times10^6 \Omega^{-1} \cdot m^{-1})$	32.066	31.994
Standard deviation χ	0.218	0.158
σ_{max} $(\times10^6 \Omega^{-1} \cdot m^{-1})$	32.530	32.303
σ_{min} $(\times10^6 \Omega^{-1} \cdot m^{-1})$	31.609	31.661
Confidence interval under 95% confidence coefficient $(\times10^6 \Omega^{-1} \cdot m^{-1})$	[32.023, 32.110]	[31.963, 32.026]

It can be found that there is a high enough precision when 40×40×40 elements is applied. If the element number is too small, because of the influence of boundary, the dissipation of the results is very obvious. In this paper, the results of figure 2, figure 3 and figure 4 all come from the simulations undertaken when 40×40×40 elements are adopted.

6. CONCLUSIONS

From the simulation results and the comparison with the measurements, the following conclusions can be drawn out.

(1) The electrical conductivity of CuCr alloy depends on its component and is characterized by construct sensitive. It can be calculated in certain accuracy effectively by our proposed model based on the analysis of electrical conductance in phenomenon aspect.

(2) Gas content has an important effect on electrical conductivity. The simulation reveals that the conductivity decreases linearly with the increase of the volume percentage of gas. This implies the necessity of the emphasis on gas control in CuCr alloy preparing.

(3) Because the physical mechanism of heat conductance in metals and alloys is the same as electrical conductance in original, it can be postulated that the model proposed at here also can be adopted in simulation on characteristics of heat conductance. However, preliminary research on using this model upon the problem of heat conductance reveals that the heat conductivity is more sensitive on construct than electrical conductivity.

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Reference

[1] Heitzinger F, et al., "Contact materials for vacuum switch devices", IEEE Trans. Plasma Science, Vol. 21, No. 5, pp447-453, 1993

[2] Rieder W F, et al., "The influence of composition and Cr size of CuCr contacts on chopping current, contact resistance and breakdown voltage in vacuum interrupters", IEEE Trans. Comp. Hybrid & Manufactory & Technology, Vol. 12, No. 2, pp273-283, 1989

[3] Ballat J, "Insulation characteristics and welding behavior of vacuum switch contacts made from CuCr alloys", IEEE Trans. Electrical. Insulation, Vol. 28, No. 5, pp628-633, 1993

[4] Frey P, et al., "Metallurgical aspects of contact materials for vacuum switch devices", IEEE Trans. Plasma Science, Vol. 17, No. 5, pp734-740, 1989

[5] Spaie S, et al., "Microstructure and properties of sintered Cu-25Cr alloy", Material Science & Technology, No. 5, pp1069-1073, 1989

[6] Yindou T, "Probability and Statistics", Tokyo: KORONASHA, 1974 (in Japanese)

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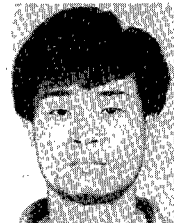
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