Paper

Tolerance Computation of a Power Electronic Circuit by Higher Order Sensitivity Analysis Method

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There is a tolerance between a designed and an actual characteristics when there are circuit parameter variations in a designed circuit. It is necessary to develop a tolerance analysis method which computes the tolerance region under circuit parameter variations. Interval mathematics is one of these analysis methods. However no practical method has been reported for transient or steady-state analysis because they are initial value or boundary value problems. This paper proposes an alternative method which efficiently computes a tolerance region with a parameter polynomial approximately and practically by using the Taylor series expansion with a higher order sensitivity analysis method.

Keywords: tolerance analysis, worst case, sensitivity analysis, interval analysis

1. Introduction

It is necessary to consider variations of circuit element values for a power electronic converter design to satisfy a given specification for a frequency characteristic or a dynamic transient response because the variations of element values generate those of the designed characteristic. Therefore element values and their variation ranges should be designed to give a satisfactory characteristic within an allowable specification range or a tolerance. This process needs to find relation between given parameter and its characteristic regions. This mapping computation is so-called tolerance analysis(1).

Interval analysis(2)−(4) is one of tolerance analysis methods and it expresses a tolerance as an "interval" which is a set of two numbers, the minimum and the maximum (min/max) values, and finds a characteristic region. This analysis is simple in its mathematical description. However it has two main disadvantages. First no practical method has been reported for transient or steady-state analysis because they are initial value or boundary value problems. Secondly it is true that it always includes a true solution within an interval, but it is too conservative and often includes excess regions especially for non-analytical functions. Its computation time is large and it grows exponentially with number of parameters. It is convenient to reduce such regions if possible. Therefore practical application classes are limited.

This paper proposes a new tolerance analysis method which converts the initial value min/max problem into a simple min/max problem by approximating the relation with a parameter polynomial by a parameter sensitivity technique(5)(6). A tolerance region for an objective characteristic is expressed with a Taylor expansion series with a parameter set vector at a center operating point. Its derivative values correspond to parameter sensitivities. Usually the expansion is truncated up to the first order term. However when a higher approximation level is necessary to analyze the characteristic more precisely, this truncation is not enough and higher order terms should be considered. A flexible adjustment technique for truncation orders is proposed by considering higher order parameter sensitivity values.

A computation algorithm for higher order parameter sensitivities is derived by using the direct differentiation method. Generally a k-th order sensitivity is computed from its $(k-1)$-th order sensitivity, and an arbitrary order sensitivity is computed one by one from its first order term. Then an algorithm which adjusts truncation order terms is proposed to pursuit necessary sensitivity terms systematically.

As an application example, a DC-DC converter circuit(4),(7) is taken up. Approximation levels for tolerances by the proposed method are investigated. Finally tolerances in a transient and a steady-state is computed and its accuracy is validated.

2. Tolerance Analysis of a Power Electronic Circuit

2.1 Tolerances due to parameter variations

Characteristics of a manufactured circuit should lie within a tolerance region to pass a specification test as shown in Fig.1(a). It is often necessary to know relation between a characteristic and a parameter regions in a design process. This computation is so-call tolerance analysis in which circuit element variations give a parameter region which gives a characteristic region.

A normal circuit simulator computes a circuit characteristic for designated parameters. However it is often necessary to compute a tolerance range of the circuit for a parameter variation region. Therefore it is needed to develop a tolerance simulator which computes a tolerance characteristic region, the min/max values, for a

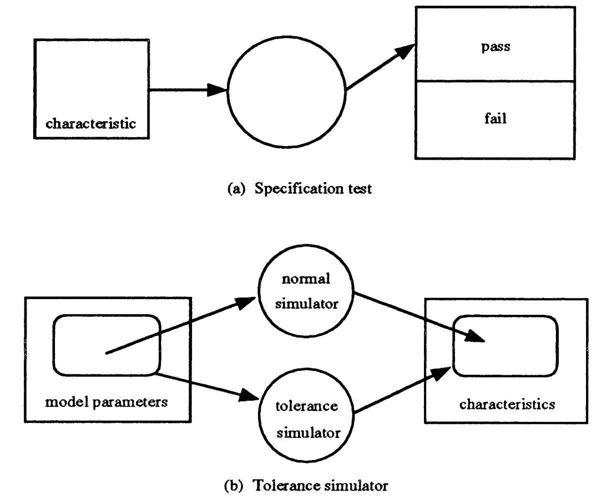


Fig. 1. Tolerance analysis.

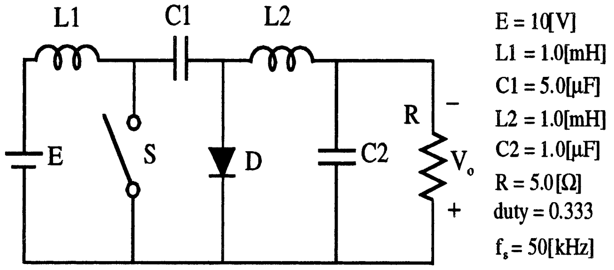


Fig. 2. Ćuk converter circuit example.

given parameter region as shown in Fig.1(b).

For example, a Ćuk converter in Fig.2 has two switch elements, one voltage source, and five passive circuit elements. The passive element parameters due to their production processes may have subtle variations which affect its output voltage. It is one of tolerance analysis examples to find an output voltage region for given element parameter variations.

This paper proposes to utilize parameter sensitivities for this tolerance analysis. Vectors of center parameter values and of its variations are denoted as $\mathbf{p}_c$ and $\Delta\mathbf{p}$ respectively. A characteristic value $f(\mathbf{p}_c + \Delta\mathbf{p})$ for a parameter value set $\mathbf{p}_c + \Delta\mathbf{p}$ is expressed with the following first order approximation[(5)]:

$$f(\mathbf{p}_c + \Delta\mathbf{p}) \simeq f(\mathbf{p}_c) + \frac{\partial f}{\partial \mathbf{p}}\Delta\mathbf{p} \quad (1)$$

where $\partial f/\partial \mathbf{p}$ is a parameter sensitivity vector for a characteristic function $f(\mathbf{p})$. However the equation does not consider its dependency on parameters themselves and their mutual effects. To increase the approximation level, higher order terms of sensitivities should be considered.

2.2 Consideration of higher order terms A characteristic value $f(\mathbf{p}_c + \Delta\mathbf{p})$ for a parameter value set $\mathbf{p}_c + \Delta\mathbf{p}$ is generally expressed with the following Taylor series expansion:

$$f(\mathbf{p}_c + \Delta\mathbf{p}) = f(\mathbf{p}_c) + \frac{\partial f}{\partial \mathbf{p}}\Delta\mathbf{p} + \frac{1}{2!}\Delta\mathbf{p}^T \frac{\partial^2 f}{\partial \mathbf{p}^T \partial \mathbf{p}}\Delta\mathbf{p} + \cdots \quad \cdots\cdots\cdots\cdots (2)$$

where ^{T} is transpose of a matrix.

The third term in the right hand side of (2) gives the second order sensitivities and expresses mutual dependencies between the parameters. Furthermore it is necessary to consider higher order terms to increase its accuracy. Steps of the proposed tolerance computation are the followings: first center parameter values $\mathbf{p}_c$ and their variations $\Delta\mathbf{p}$ are computed from a given parameter region. Then $f(\mathbf{p}_c)$, $\frac{\partial f}{\partial \mathbf{p}}$, $\frac{\partial^2 f}{\partial \mathbf{p}^T \partial \mathbf{p}}$, $\cdots$ are computed from the center values. In this process an efficient and automatic algorithm is proposed by controlling an truncation pursuit technique which is described in Section 4. Finally the two extreme values, the min/max values, are computed by (2) and they give its tolerance.

3. Parameter Sensitivity Computation by the Direct Differentiation Method

3.1 Basic idea of the direct differentiation method[(6)] For the proposed method, it is necessary to compute so-called sensitivity or a partial derivative of a circuit variable with respect to a parameter. In this paper the direct differentiation method[(6)] is adopted because it is simple and efficient. When a sensitivity with respect to a circuit parameter p_1 at a time t is computed, the derived sensitivity circuit is excited by responses of the original circuit. A computed voltage $\frac{\partial v}{\partial p_1}$ and a current $\frac{\partial i}{\partial p_1}$ in the sensitivity circuit are sensitivities of a voltage v and a current i with respect to the parameter p_1 in the original circuit. It is possible to compute any desired sensitivity by deriving a sensitivity circuit from the original circuit and by analyzing it.

As an example of circuit elements, sensitivity with respect to a parameter p_1 in a resistor R is derived as the following.

$$v(t) = Ri(t) \quad \cdots\cdots\cdots\cdots (3)$$

$$\frac{\partial v}{\partial p_1} = \frac{\partial R}{\partial p_1} i + R\frac{\partial i}{\partial p_1} \quad \cdots\cdots\cdots\cdots (4)$$

This is equivalent to the circuit of a resistor R and a voltage source $\frac{\partial R}{\partial p_1} i$ with a voltage $\frac{\partial v}{\partial p_1}$ and a current $\frac{\partial i}{\partial p_1}$ in Fig.3(a).

3.2 Computation of higher order sensitivities The second order sensitivity with respect to parameters p_1 and p_2 is the following.

$$\frac{\partial^2 v}{\partial p_1 \partial p_2} = \frac{\partial i}{\partial p_1}\frac{\partial R}{\partial p_2} + \frac{\partial i}{\partial p_2}\frac{\partial R}{\partial p_1} + R\frac{\partial^2 i}{\partial p_1 \partial p_2} \quad \cdots (5)$$

This is equivalent to the circuit of a resistor R and voltage sources $\frac{\partial i}{\partial p_1}\frac{\partial R}{\partial p_2}$ and $\frac{\partial i}{\partial p_2}\frac{\partial R}{\partial p_1}$ with a voltage and a current variables, $\frac{\partial^2 v}{\partial p_1 \partial p_2}$ and $\frac{\partial^2 i}{\partial p_1 \partial p_2}$ in Fig.3(b).

The third order sensitivity with respect to parameters p_1, p_2, and p_3 is the following.

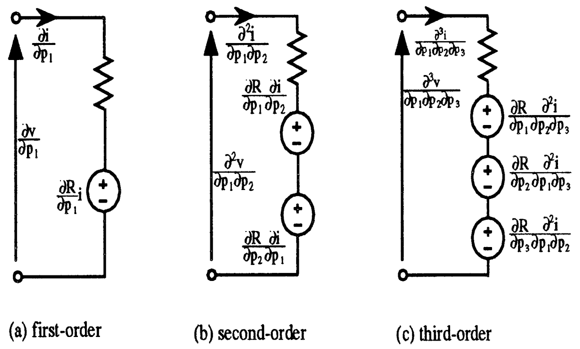


Fig. 3. Equivalent circuit expressions in sensitivity circuits.

$$\frac{\partial^3 v}{\partial p_1 \partial p_2 \partial p_3} = \frac{\partial^2 i}{\partial p_1 \partial p_2}\frac{\partial R}{\partial p_3} + \frac{\partial^2 i}{\partial p_2 \partial p_1}\frac{\partial R}{\partial p_3} + \frac{\partial^2 i}{\partial p_3 \partial p_1}\frac{\partial R}{\partial p_2} + R\frac{\partial^3 i}{\partial p_1 \partial p_2 \partial p_3} \cdots (6)$$

Similarly this is equivalent to the circuit of a resistor R and a voltage source in Fig.3(c).

It is possible to derive equivalent circuits similarly for higher order sensitivities and also for other types of elements. Computed examples of sensitivities are shown in Fig.4(a)-(c) for the Ćuk circuit from the first to the third orders.

4. Approximation Algorithm with Higher Order Polynomial by Parameter Sensitivity

4.1 Truncation pursuit algorithm of higher order sensitivities for polynomial approximation Parameter sensitivities are expressed by (2) with a polynomial equation where only selective terms practically contribute. Small components of $\frac{\partial f}{\partial p}\Delta p$ can be neglected. More generally, when a variation of $f(\mathbf{p})$ due to the $(k-1)$-th sensitivity is small enough, its k-th sensitivities can be assumed to be neglected because they express its dependence on parameter values. All necessary sensitivities which can not be neglected have to be computed. Then tolerance value (2) is evaluated by using them.

This paper proposes an efficient algorithm which computes all necessary sensitivities by the following truncation pursuit technique. First all sensitivities of the first order terms and second order terms are computed. Then higher order terms are computed selectively which satisfies the following condition. A critical value D is used to judge whether a k-th order sensitivity term can be neglected by the following equation.

$$D > \frac{\partial^k f}{\partial^{i_1} p_1 \partial^{i_2} p_2 \cdots \partial^{i_n} p_n} \prod_{j=1}^{n} \Delta p_j^{i_j} \quad \cdots\cdots\cdots (7)$$

where $\quad \sum_{j=1}^{n} i_j = k$

This equation computes a truncation level by estimating how much the term contribute to the tolerance value. This estimation is applied to the circuit in Fig.2 for the second order terms. Only three terms for L_1L_1, L_1C_1, and C_1C_1 are selected for 1% of the maximum center value, $D = 0.0644$[V], with rigid horizontal lines in Fig.5.

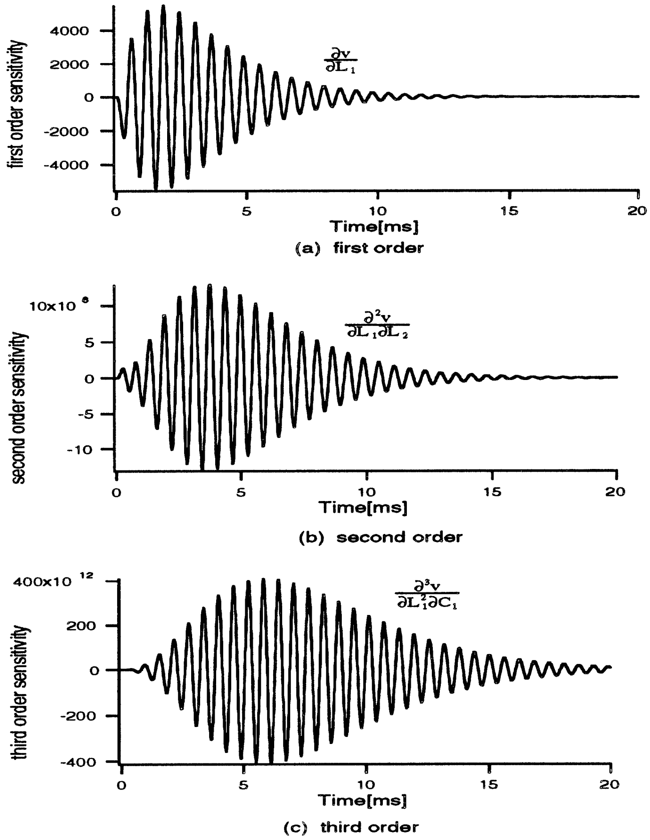


Fig. 4. Computed sensitivity examples.

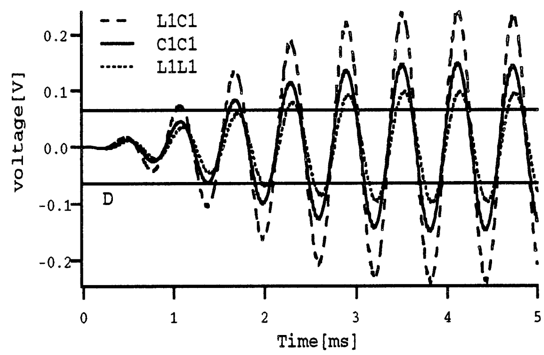


Fig. 5. Parameter selection of necessary sensitivities by the critical value D.

Step 1 Computation of the first order sensitivity.
Step 2 Computation of the second order sensitivity analysis.
For each set of two parameters p_i and p_j, selection of truncation terms by D criterion.
Step 3 Increment of order k by one.
Step 4 For each set of two parameters p_i and p_j, selection of truncation terms by D criterion.
Step 5 If all parameter sets are truncated, go to end.
Otherwise, go to Step 3.

Fig. 6. Truncation pursuit algorithm of the proposed method.

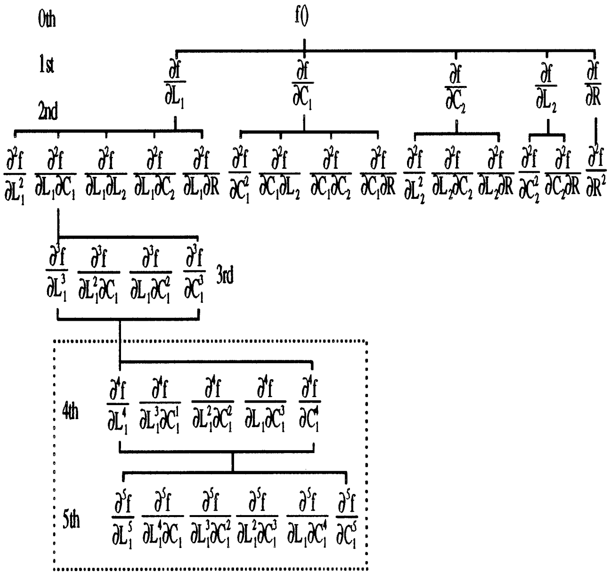


Fig. 7. Tree diagram by the truncation pursuit algorithm.

The proposed method is an approximate computation technique which can control its approximation level. It is possible to increase its approximation level by spending more computation time. On the contrary, it is possible to save computation time by permitting coarser approximation level. It has one degree of freedom for a sort of balance between them and it is not possible to determine all parameters in advance. However the truncation criterion value D can be a parameter for this balance. The proposed method is not one which assures its computation accuracy. The value D is given for a sensitivity term based on a ratio of the maximum center value.

The proposed algorithm uses the above criterion from the second order terms for each set of two parameters which include the same ones. So there are two types. One is, for example, when parameters L_1L_1 are selected in the second order, then only a term for $L_1L_1L_1$ in the third order is checked. The other is, when parameters L_1C_1 are selected, then four terms for $L_1L_1L_1$, $L_1L_1C_1$, $L_1C_1C_1$, and $C_1C_1C_1$ in the third order are checked. This process is repeated for each set, and for each order until all terms are truncated. Computation steps of the proposed algorithm are shown in Fig.6.

This algorithm is applied to the same example for a transient analysis from $t = 0$ to $t = 5$[ms]. The selected sensitivity terms are shown in Fig.7. At the truncation process in the second order, only one term $\frac{\partial^2 f}{\partial L_1 \partial C_1}$ for a set of parameters $L1$ and $C1$ is selected and all the other terms are truncated for further pursuit. With $D = 0.193$[V], 3% of the maximum center value, four terms $\frac{\partial^3 f}{\partial L_1^3}$, $\frac{\partial^3 f}{\partial L_1^2 \partial C_1}$, $\frac{\partial^3 f}{\partial L_1 \partial C_1^2}$ $\frac{\partial^3 f}{\partial C_1^3}$ in the third order are generated and then truncated. With $D = 0.0644$[V], 1% of the maximum center value, five and six terms in the fourth and fifth orders inside the dashed square are generated furthermore and then truncated automatically.

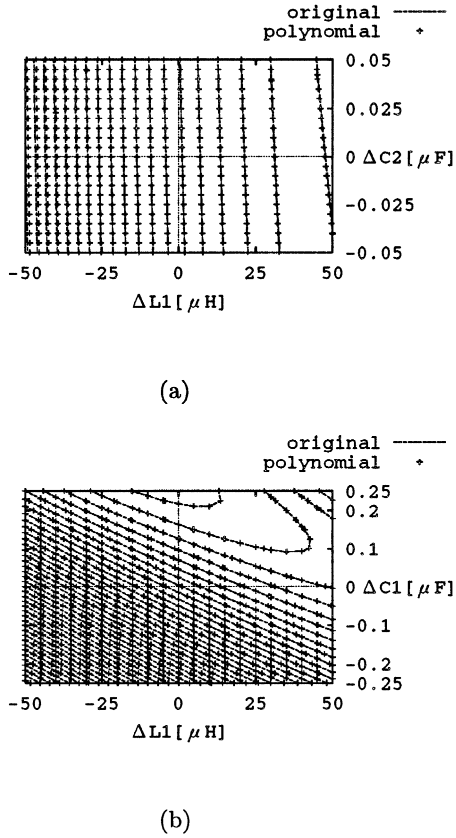


Fig. 8. Approximation levels of mutual dependence between two parameters.

4.2 Approximation level of the proposed method Voltage variation values computed due to parameter variations by the proposed method are shown in Fig.8 for the Ćuk converter case. One in the figure (a) is those for variations of parameters L_1 and C_1. The reference tolerance values, which are denoted as rigid contour curves in 0.02[V] steps are computed by mesh division of the parameter region. They decrease from the left side to the right side. Polynomial approximation values by the proposed method are denoted with the symbol + and they are exactly on the reference curves. This may be partly because the curves are almost linear and the second order polynomial is enough for this approximation. A more complex case in the figure (b) is further investigated. The variation values decrease from the left bottom to the right top. This case is approximated by a fifth order polynomial and computed values by the proposed method are still on the reference curves. These validate approximation levels of the proposed method.

Selection of D is important to control accuracy of results. The smaller D is, the higher their accuracy and the truncation orders are. Basically 1% or 0.1% value of an objective variable is reasonable and empirically 1% value is enough for most cases. Especially for steady-state analysis, either 1% or 0.1% value gives the same results because sensitivity terms are truncated up to the second order for most cases.

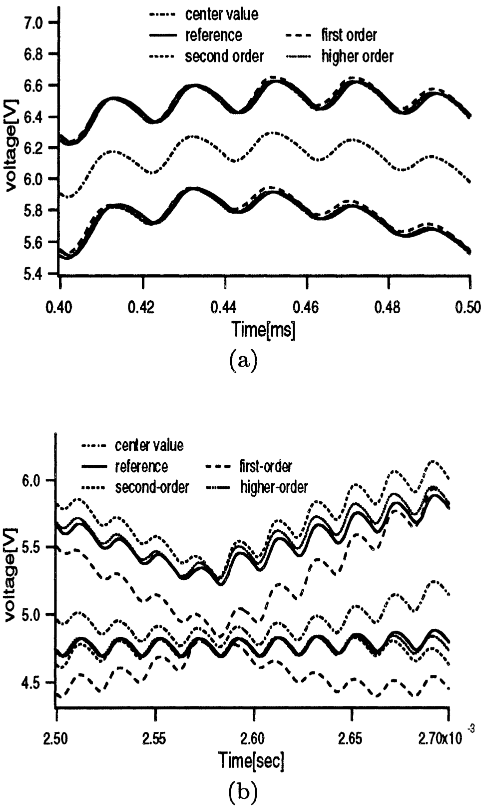


Fig. 9. Computed result in a transient state.

5. Computation Examples of Tolerance Analysis

5.1 Transient example As the first application example of the proposed method, the Ćuk converter circuit in Fig.2 is taken up for a transient analysis with zero initial values from $t = 0$ to $t = 2.8$[ms]. Transient waveforms for the center parameter values are shown with one-dotted curves in Fig.9 for two periods (a) $t = 0.4 \sim 0.5$[ms] and (b) $t = 2.5 \sim 2.7$[ms]. Their 5% values are set to be a parameter variation range. A reference tolerance, which is denoted as a dashed curve, is computed by mesh division of the parameter region. Other tolerances, which are denoted as long and short dashed curves and computed for a polynomial by the first and second order approximations, have deviations which grow with time from the reference. However a modified tolerance, which is denoted as dotted curves and computed by the proposed method, is almost coincide with the reference. This validates increase of approximation level and effectiveness of the proposed method, which can control accuracies by adjusting necessary orders.

The value $D = 0.0644$[V] is set to be the 1% value of the maximum value of the center waveform The maximum error is 0.1099[V], the number of computed sensitivity circuits is 35, and the maximum truncation order is fifth. The error is a little larger than D because it is an accumulated value of all truncated sensitivity terms. Furthermore relations between D and truncation information are investigated in Fig.10(a)-(c). The smaller D is, the higher the accuracy is by increasing the truncation order and sensitivity terms. Practically 1% value is enough in this case.

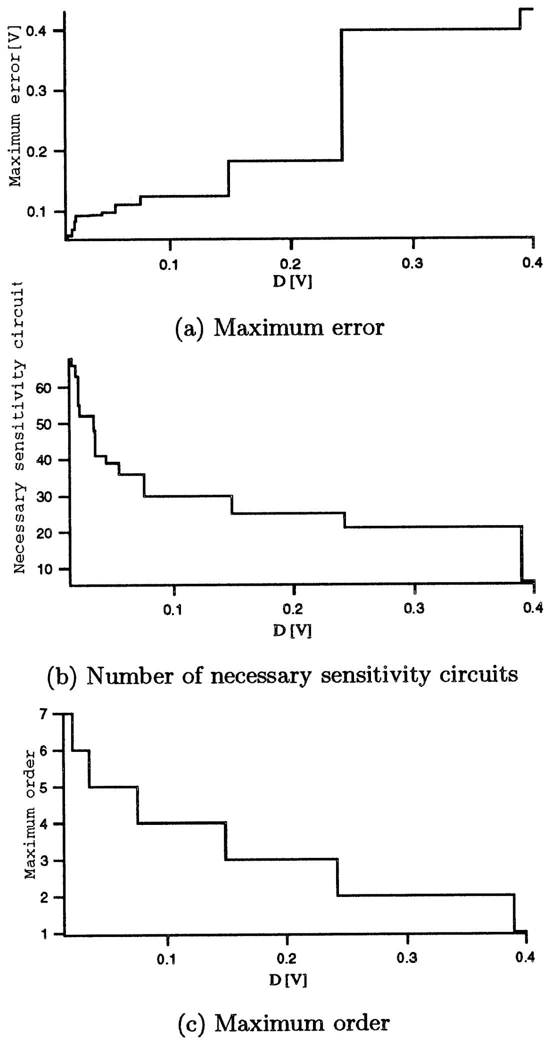
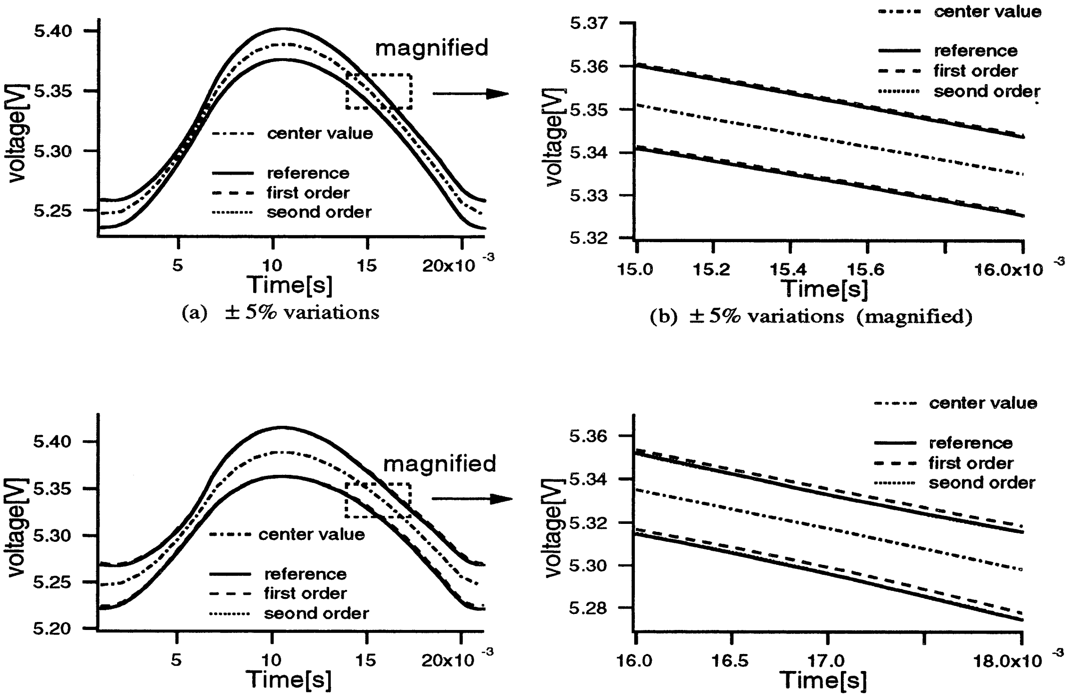


Fig. 10. Truncation Parameter variations due to variations of the critical value D.

5.2 Steady-state example The proposed method gives a good approximation level which adapts truncation orders flexibly for a computation time period. Conversely it is especially suitable, for example, for a steady-state analysis which can be approximated by a low order polynomial because a considered time period is relatively short. As the second application example, the same Ćuk converter circuit is taken up again for a steady-state analysis. Computed results by the reference mesh, the first order, and the proposed methods are shown in Fig.11 by using the same type notations. The figures (a) and (b) show a case with 5% parameter variations. The result by the first order method has a slight difference from the reference and that by the proposed method has very little difference because it is approximated with the second order polynomial. The figures (c) and (d) show a case with 10% parameter variations. The result by the first order has a larger difference from the reference and that by the proposed method still has very little difference because it is approximated also with the second order polynomial. This



(a) ± 5% variations (b) ± 5% variations (magnified)

(c) ± 10% variations (d) ± 10% variations (magnified)

Fig. 11. Computed result in a steady state.

validates effectiveness of the proposed method. In this case D is set to be 1% value. However either 1% or 0.1% value gives the same results because sensitivity terms are truncated up to the second order. The proposed method is effective for a steady-state analysis which has a good approximation with such lower order terms.

6. Conclusions

This paper proposed an efficient analysis method which approximates a tolerance region with parameter variations in a power electronic circuit. It is based on an adaptive polynomial approximation with parameter sensitivities by the Taylor series expansion. The approximation consists of truncation pursuit algorithm for each set of two parameter combinations in higher order term computations. The method has a good approximation level because it adapts truncation orders based on a truncation parameter D which can be set to 1% value of an objective variable practically. A necessary truncation order depends on an analysis time periods and a longer period needs higher order terms. The method is especially effective, for example, for a steady-state analysis which has a good approximation only with lower order terms.

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