Paper

Analysis of diffracted waves from a grounded uniaxial chiral medium with a plane metallic grating

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A 4×4 matrix-based analysis of the electromagnetic waves diffracted by a grounded slab of uniaxial chiral medium with a plane metallic grating has been presented. In the analysis, the total fields are given by the superposition of the current-independent primary fields and the scattered fields depending on the currents induced on the grating. The scattered fields are expanded in terms of Floquet modes. The coupled mode equations which are derived for the uniaxial chiral media give the general solutions of fields as the superposition of left and right-handed elliptically-polarized eigen modes. The unknown currents on the grating are determined by applying Galerkin's method to the condition of the perfect conductivity on the grating. Both the direction of the optical axis of the uniaxial anisotropy and that of the incident wave vector are assumed to be perpendicular to the strip conductors of the grating. In the numerical computation, the polarization characteristics of diffracted waves are investigated.

Keywords: chiral, grating, electromagneic wave, diffraction

1. Introduction

Electrically small chiral objects distributed in a certain volume can macroscopically be treated as "chiral medium" where electromagnetic fields are governed by constitutive relations which have been known in (1)- (3) and Maxwell's equations. Chiral media have been considered to be novel type components of electromagnetic wave devices due to their optical activity involving the left and right-handed eigen modes with different wave numbers in such media [(3)-(6)]. These include research work on uniaxial chiral media as a kind of polarization transformers which can change the orientation, axial ratio and handedness of the polarization ellipse of waves [(6)]. As one of the attractive branches of applications, introducing chirality in periodic structures has gained much attention and has been reported so far [(7)]. Theoretical studies have been focused on isotropic [(8)] or uniaxial chiral media [(9)] with periodic shapes or with periodically-modulated chirality. These have been aiming at designing devices which can control both the polarization ellipse and the distribution of power of waves. Adding plane metallic gratings [(10)-(11)] to chiral media however can be an alternative to shaping the media periodically and are preferable with the advantages of the conventional printed devices such as FSS, microstrip antennas etc. in the frequency region of radio waves. The cases of isotropic chiral media with metallic gratings have been reported [(12), (13)] where the characteristics of scattered waves depending on the isotropic chirality and other structural parameters have been investigated. In case the chirality shows uniaxial anisotropy, the characteristics of polarization and power distribution will be influenced also by the direction of the optical axis of the uniaxial chirality, and then the freedom of design for this kind of structures will increase. In this paper, electromagnetic waves diffracted by a grounded slab of uniaxial chiral medium with a one-dimensional plane metallic grating attached on it are analyzed in 4×4 matrix form [(13), (14)]. For simplicity, the direction of the optical axis of the uniaxial anisotropy which has biased effect on each diffracted wave is assumed to be in the plane of incidence which is perpendicular to the conductors of the grating. In the analysis, the total fields are given by the superposition of the current-independent primary fields and the scattered fields depending on the currents on the grating. The scattered fields are expanded in terms of Floquet modes each of which obeys the coupled mode equation derived for the uniaxial chiral media. General solutions are given by the superposition of left and right-handed elliptically-polarized eigen modes. After the Green's functions in the spectral domain and the primary fields are computed from the boundary conditions, the Galerkin's method is applied to determine the unknown currents on the grating. In the numerical computations, the polarization characteristics of diffracted waves are investigated considering the design parameters.

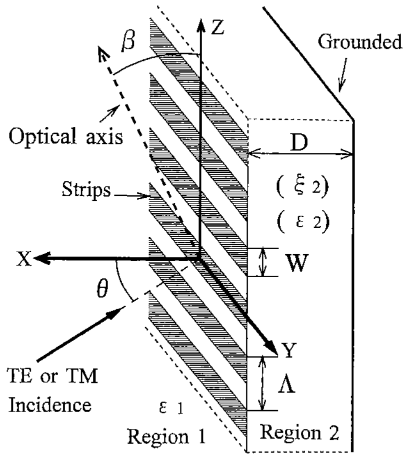

Fig. 1. Geometry of the problem

2. Description of the problem

The configuration of the problem being considered is shown in the Figure 1. A uniaxial chiral planar medium with thickness D (region 2) is placed adjacent to a semi-infinite isotropic achiral region 1 and is backed by a ground plane. All regions are lossless and have the same permeability μ_0. The optical axis of the uniaxial anisotropy in region 2 is assumed to make an angle β with Z axis in XZ plane as shown in the same figure. Considering that an isotropic achiral medium is a limited case of a uniaxial anisotropic chiral one, the tensors of chirality admittance and relative permittivity for region l, $(l = 1, 2)$ are generally expressed in equitorial form as

$$(\xi_l) = \begin{pmatrix} \xi_{l,XX} & 0 & \xi_{l,XZ} \\ 0 & \xi_{l,YY} & 0 \\ \xi_{l,ZX} & 0 & \xi_{l,ZZ} \end{pmatrix} \quad (l = 1, 2), \quad (1)$$

$$(\epsilon_l) = \begin{pmatrix} \epsilon_{l,XX} & 0 & \epsilon_{l,XZ} \\ 0 & \epsilon_{l,YY} & 0 \\ \epsilon_{l,ZX} & 0 & \epsilon_{l,ZZ} \end{pmatrix} \quad (l = 1, 2) \quad (2)$$

respectively. Each element in above tensors for region 1 comes to

$$(\xi_1) = 0,$$
$$\epsilon_{1,XX} = \epsilon_{1,YY} = \epsilon_{1,ZZ} = \epsilon_1,$$
$$\epsilon_{1,XZ} = \epsilon_{1,ZX} = 0. \quad \cdots\cdots\cdots\cdots\cdots (3)$$

As for the region 2, each element can be given as

$$\xi_{2,XX} = \xi_2 \cos^2\beta + \xi_{2e}\sin^2\beta,$$
$$\xi_{2,YY} = \xi_2,$$
$$\xi_{2,ZZ} = \xi_2 \sin^2\beta + \xi_{2e}\cos^2\beta,$$
$$\xi_{2,ZX} = \xi_{2,XZ} = (\xi_2 - \xi_{2e})\sin\beta\cos\beta,$$
$$\epsilon_{2,XX} = \epsilon_2 \cos^2\beta + \epsilon_{2e}\sin^2\beta,$$
$$\epsilon_{2,YY} = \epsilon_2,$$
$$\epsilon_{2,ZZ} = \epsilon_2 \sin^2\beta + \epsilon_{2e}\cos^2\beta,$$
$$\epsilon_{2,ZX} = \epsilon_{2,XZ} = (\epsilon_2 - \epsilon_{2e})\sin\beta\cos\beta \quad \cdots\cdots (4)$$

where ξ_{2e} and ϵ_{2e} are chirality admittance and relative permittivity for the direction of optical axis and ξ_2 and ϵ_2 are those for the directions perpendicular to the optical axis. An infinitely-extended thin metallic grating with period Λ and width W is placed at the boundary between the regions 1 and 2. The structure considered is assumed to be uniform in Y direction and have a plane of incidence in XZ plane where a TE or TM plane wave (magnetic and electric field vector is in XZ plane respectively) illuminates under the incidence angle θ. In the following analysis electromagnetic fields are assumed to have time harmonic dependence $\exp(i\omega t)$. Electromagnetic fields in each region l $(l = 1, 2)$ satisfy the Maxwell's equations and the Post-Jaggard type constitutive relations as follows [3]:

$$\boldsymbol{D} = \epsilon_0(\epsilon_l)\boldsymbol{E} - i(\xi_l)\boldsymbol{B}, \quad \boldsymbol{H} = -i(\xi_l)\boldsymbol{E} + \frac{1}{\mu_0}\boldsymbol{B}. \quad (5)$$

These equations enable us to derive the relations of electromagnetic fields:

$$\overline{\mathrm{curl}}\sqrt{Y_0}\boldsymbol{E} = -i\sqrt{Z_0}\boldsymbol{H} + (\tau_l)\sqrt{Y_0}\boldsymbol{E},$$
$$\overline{\mathrm{curl}}\sqrt{Z_0}\boldsymbol{H} = i((\epsilon_l) + (\tau_l)^2)\sqrt{Y_0}\boldsymbol{E} + (\tau_l)\sqrt{Z_0}\boldsymbol{H}, \quad (6)$$

$$Y_0 = \frac{1}{Z_0} = \sqrt{\frac{\epsilon_0}{\mu_0}}, \quad (\tau_l) = (\xi_l)\sqrt{\frac{\mu_0}{\epsilon_0}} \quad \cdots\cdots\cdots (7)$$

where the space variables expressed by capital letters X, Y and Z as used in the Figure 1 are normalized by wave number in vacuum $k_0 (= \omega\sqrt{\epsilon_0\mu_0})$ to be transformed into lower-case ones, putting $k_0X \to x$, $k_0Y \to y$ and $k_0Z \to z$. $\overline{\mathrm{curl}}$ and (τ_l) are the normalized operator curl and chiral admittance tensor respectively.

3. General solutions for fields

The total fields in each region are given by the superposition of the current-independent primary fields and the scattered fields depending on the currents induced on the grating. The conditions of phase matching at the boundaries for the incidence of a plane wave give the forms of the primary fields as

$$\sqrt{Y_0}E_t^p(x,z) = e_t^p(x)e^{-is_0z}, \quad \cdots\cdots\cdots\cdots\cdots (8)$$
$$\sqrt{Z_0}H_t^p(x,z) = h_t^p(x)e^{-is_0z}, \quad \cdots\cdots\cdots\cdots\cdots (9)$$
$$t = x, y, z, \quad s_0 = \sqrt{\epsilon_1}\sin\theta. \quad \cdots\cdots\cdots\cdots (10)$$

The scattered fields in each region can be expanded in terms of Floquet modes as follows:

$$\sqrt{Y_0}E_t^s(x,z) = \sum_{m=-\infty}^{\infty} e_{tm}^s(x)e^{-is_mz}, \quad \cdot\ (11)$$

$$\sqrt{Z_0}H_t^s(x,z) = \sum_{m=-\infty}^{\infty} h_{tm}^s(x)e^{-is_mz}, \quad \cdot\ (12)$$

$$t = x, y, z, \quad s_m = s_0 + \frac{2\pi m}{k_0\Lambda} = s_0 + \lambda\frac{m}{\Lambda} \quad \cdot\ (13)$$

where λ is the wavelength in air and the same phase matching as in Eq.(8) and Eq.(9) are considered. The coefficients $e_t^p(x)$, $h_t^p(x)$ in Eq.(8) and Eq.(9) and expansion coefficients $e_{tm}^s(x)$, $h_{tm}^s(x)$ in Eq.(11) and Eq.(12)

$(t = x, y, z)$ are expressed here in vector form as

$$\boldsymbol{f}^p(x) = \begin{pmatrix} e_y^p \\ e_z^p \\ h_y^p \\ h_z^p \end{pmatrix}, \quad \boldsymbol{f}_m^s(x) = \begin{pmatrix} e_{ym}^s \\ e_{zm}^s \\ h_{ym}^s \\ h_{zm}^s \end{pmatrix}, \quad (14)$$

$$\boldsymbol{f}^{pn}(x) = \begin{pmatrix} e_x^p \\ h_x^p \end{pmatrix}, \quad \boldsymbol{f}_m^{sn}(x) = \begin{pmatrix} e_{xm}^s \\ h_{xm}^s \end{pmatrix}. \quad (15)$$

Introducing the above scattered fields into Eq.(6) derives the following coupled wave equations in the form of matrix about the expansion coefficients of the mth Floquet mode in region l $(l = 1, 2)$ as follows:

$$\frac{d}{dx}\boldsymbol{f}_m^s = i(R_{l,m})\boldsymbol{f}_m^s, \quad \boldsymbol{f}_m^{sn} = (K_{l,m})\boldsymbol{f}_m^s \quad \cdots (16)$$

where $(R_{l,m})$ and $(K_{l,m})$ are 4×4 and 2×4 coupling matrices for region l $(l = 1, 2)$:

$$(R_{l,m}) = \begin{pmatrix} (R_e^e) & (R_h^e) \\ (R_e^h) & (R_h^h) \end{pmatrix}, \quad \cdots\cdots\cdots (17)$$

$$(K_{l,m}) = \begin{pmatrix} (K_e^e) & (K_h^e) \\ (K_e^h) & (K_h^h) \end{pmatrix} = \begin{pmatrix} d_{11} & d_{12} & d_{13} & d_{14} \\ d_{21} & d_{22} & d_{23} & d_{24} \end{pmatrix} \quad (18)$$

where

$$(R_e^e) = \begin{pmatrix} -i\tau_{zx}d_{11} & -i\tau_{zz} - i\tau_{zx}d_{12} \\ i\tau_{yy} - s_m d_{11} & -sd_{12} \end{pmatrix}, \quad (19)$$

$$(R_h^e) = \begin{pmatrix} -\tau_{zx}d_{13} & -1 - i\tau_{zx}d_{14} \\ 1 - s_m d_{13} & -s_m d_{14} \end{pmatrix}, \quad (20)$$

$$(R_e^h) = \left(\begin{matrix} (\epsilon_{zx} + (\tau_{xx} + \tau_{zz})\tau_{zx})d_{11} - i\tau_{zx}d_{21} \\ -\epsilon_{yy} - \tau_{yy}^2 - s_m d_{21} \end{matrix} \right. \quad (21)$$

$$\left. \begin{matrix} \epsilon_{zz} + \tau_{zz}^2 + \tau_{xz}\tau_{zx} + (\epsilon_{zx} + (\tau_{xx} + \tau_{zz})\tau_{zx})d_{12} - i\tau_{zx}d_{22} \\ -s_m d_{22} \end{matrix} \right),$$

$$(R_h^h) = \left(\begin{matrix} (\epsilon_{zx} + (\tau_{xx} + \tau_{zz})\tau_{zx})d_{13} - i\tau_{zx}d_{23} \\ i\tau_{yy} - s_m d_{23} \end{matrix} \right. \quad (22)$$

$$\left. \begin{matrix} -i\tau_{zz} + (\epsilon_{zx} + (\tau_{xx} + \tau_{zz})\tau_{zx})d_{14} - i\tau_{zx}d_{24} \\ -s_m d_{24} \end{matrix} \right),$$

$$(K_e^e) = \begin{pmatrix} \frac{-i\tau_{xx}s_m}{G_{xx}} & -\frac{\epsilon_{xz} + \tau_{zz}\tau_{xz}}{G_{xx}} \end{pmatrix}, \quad \cdots\cdots (23)$$

$$(K_h^e) = \begin{pmatrix} \frac{s_m}{G_{xx}} & -\frac{i\tau_{xz}}{G_{xx}} \end{pmatrix}, \quad \cdots\cdots (24)$$

$$(K_e^h) =$$

$$\begin{pmatrix} -s_m - \frac{s_m\tau_{xx}^2}{G_{xx}} & -i\tau_{xz} + \frac{i\tau_{xx}(\epsilon_{xz} + \tau_{zz}\tau_{xz})}{G_{xx}} \end{pmatrix}, \quad (25)$$

$$(K_h^h) = \begin{pmatrix} \frac{-i\tau_{xx}s_m}{G_{xx}} & -\frac{\tau_{xx}\tau_{xz}}{G_{xx}} \end{pmatrix} \quad \cdots\cdots\cdots (26)$$

$$\text{where} \quad G_{xx} = \epsilon_{xx} + \tau_{xz}\tau_{zx}. \quad \cdots\cdots\cdots (27)$$

The solutions of Eq.(16) in region l $(l = 1, 2)$ are given in matrix form [15] as

$$\boldsymbol{f}_m^s(x) = (U_{l,m}) \begin{pmatrix} (P_{l,m}^+(x - x_{g,l}^+))\boldsymbol{g}_{l,m}^{s+} \\ (P_{l,m}^-(x - x_{g,l}^-))\boldsymbol{g}_{l,m}^{s-} \end{pmatrix} \cdot (28)$$

where

$$(P_{l,m}^{\pm}(x)) = \begin{pmatrix} e^{i\kappa_{l,m}^{R\pm}x} & 0 \\ 0 & e^{i\kappa_{l,m}^{L\pm}x} \end{pmatrix}, \quad \cdots\cdots (29)$$

$$\boldsymbol{g}_{l,m}^{s\pm} = \begin{pmatrix} g_{l,m}^{s,R\pm} \\ g_{l,m}^{s,L\pm} \end{pmatrix}, \quad \cdots\cdots\cdots (30)$$

$$(U_{l,m}^{\pm}) = ((U_{l,m}^{+}) \ (U_{l,m}^{-})) = (\boldsymbol{v}_{l,m}^{R+} \ \boldsymbol{v}_{l,m}^{L+} \ \boldsymbol{v}_{l,m}^{R-} \ \boldsymbol{v}_{l,m}^{L-}), \quad \cdots\cdots (31)$$

$$\text{and} \quad x_{g,1}^{\pm} = x_{g,2}^{-} = 0, \quad x_{g,2}^{+} = -k_0 D. \quad \cdots (32)$$

$\kappa_{l,m}^{R+}$, $\kappa_{l,m}^{L+}$, $\kappa_{l,m}^{R-}$, $\kappa_{l,m}^{L-}$ and $\boldsymbol{v}_{l,m}^{R+}$, $\boldsymbol{v}_{l,m}^{L+}$, $\boldsymbol{v}_{l,m}^{R-}$, $\boldsymbol{v}_{l,m}^{L-}$ in the 4×4 and 4×2 matrices $(P_{l,m}^{\pm})$ and $(U_{l,m}^{\pm})$ are eigenvalues and eigenvectors of the matrix $(R_{l,m})$ respectively. The superscripts $R\pm$ and $L\pm$ denote right and left-handed elliptically-polarized eigenmodes propagating in the positive and the negative directions as for along x-axis respectively. These two kinds of eigenmodes have different wave numbers because of the chirality and anisotropy of the medium. Each $\boldsymbol{g}_{l,m}^{s\pm}$ is an unknown column vector and is defined only at $x = x_{g,l}^{\pm}$ in Eq.(28) where $\boldsymbol{g}_l^{p\pm}$ (appeared later) is also defined. The solutions of the primary field in region l $(l = 1, 2)$ are also derived in a similar mannar as

$$\boldsymbol{f}^p(x) = (U_{l,0}) \begin{pmatrix} (P_{l,0}^+(x - x_{g,l}^+))\boldsymbol{g}_l^{p+} \\ (P_{l,0}^-(x - x_{g,l}^-))\boldsymbol{g}_l^{p-} \end{pmatrix} \quad \cdots (33)$$

$$\text{where} \quad \boldsymbol{g}_l^{p\pm} = \begin{pmatrix} g_l^{p,R\pm} \\ g_l^{p,L\pm} \end{pmatrix}. \quad \cdots\cdots\cdots (34)$$

The eigenmode fields in an isotropic chiral region can be expressed in closed forms and are separated into right and left-handed circularly-polarized modes with different wave numbers. In an isotropic achiral region, separation of TE and TM linearly-polarized modes and that of modes with right and left-handed circular polarization are possible where every mode has an identical wave number [13]

4. Method of solution

The y and z components of the current-independent primary fields satisfy the conditions of continuity at $x = 0$ and the conditions of electric fields vanishing at $x = -k_0 D$. Considering these conditions in the form of Eq.(33) yields linear equations:

$$(U_{1,0}) \begin{pmatrix} \boldsymbol{g}_1^{p+} \\ \boldsymbol{g}_1^{p-} \end{pmatrix} = (U_{2,0}) \begin{pmatrix} (P_{2,0}^+(k_0 D))\boldsymbol{g}_2^{p+} \\ \boldsymbol{g}_2^{p-} \end{pmatrix}, \quad (35)$$

$$((E) \ (O)) \ (U_{2,0}) \begin{pmatrix} \boldsymbol{g}_2^{p+} \\ (P_{2,0}^-(-k_0 D))\boldsymbol{g}_2^{p-} \end{pmatrix} = (O) \quad (36)$$

by which the unknowns $\boldsymbol{g}_1^{p+}$, $\boldsymbol{g}_2^{p\pm}$ are determined assuming

$$\boldsymbol{g}_1^{p-} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad \cdots\cdots\cdots (37)$$

for TE or TM incidence respectively where (O) and (E) are 2×2 null and unit matrices respectively. The scattered fields satisfy the conditions of y and z components of electric fields being continuous and vanishing at $x = 0$ and $x = -k_0 D$ respectively and that of $-x$ components of wave number vectors in region 1 being null. These fields depend on surface currents $I_y(0, z)$ and $I_z(0, z)$ on the strips at $x = 0$ which can be expanded in terms of Floquet modes in the same way as in Eq.(11) and Eq.(12):

$$\sqrt{Z_0} I_t(0, z) = \sum_{m=-\infty}^{\infty} c_{tm}(0) e^{-is_m z}, (t = y, z). \quad (38)$$

The unknown coefficients $c_{ym}(0)$ and $c_{zm}(0)$ are related to those of magnetic fields $h^s_{ym}(0)$ and $h^s_{zm}(0)$ as

$$c_{ym}(0) = h^s_{zm}(0)_{(\text{region1})} - h^s_{zm}(0)_{(\text{region2})}, \quad (39)$$

$$c_{zm}(0) = h^s_{ym}(0)_{(\text{region1})} - h^s_{ym}(0)_{(\text{region2})} \quad (40)$$

because of the jump condition of magnetic fields. Linear equations are obtained from the conditions described above and Eq.(28) as

$$\left((U^+_{1,m}) \middle| \begin{matrix} (O) \\ (-E) \end{matrix} \right) \begin{pmatrix} \boldsymbol{g}^{s+}_{1,m} \\ c_{zm} \\ c_{ym} \end{pmatrix} = (U_{2,m}) \begin{pmatrix} (P^+_{2,m}(k_0 D)) \boldsymbol{g}^{s+}_{2,m} \\ \boldsymbol{g}^{s-}_{2,m} \end{pmatrix}, \cdots\cdots\cdots (41)$$

$$\big((E)\,(O)\big)\,(U_{2,m}) \begin{pmatrix} \boldsymbol{g}^{s+}_{2,m} \\ (P^-_{2,m}(-k_0 D)) \boldsymbol{g}^{s-}_{2,m} \end{pmatrix} = (O) \quad (42)$$

which have solutions $\boldsymbol{g}^{s\pm}_{l,m,y}$ or $\boldsymbol{g}^{s\pm}_{l,m,z}$ $(l = 1, 2.)$ corresponding the assumption that $(c_{ym}(0), c_{zm}(0)) = (1, 0)$ or that $(c_{ym}(0), c_{zm}(0)) = (0, 1)$. Then, the unknowns $\boldsymbol{g}^{s+}_{1,m}$, $\boldsymbol{g}^{s\pm}_{2,m}$ can be expressed as

$$\boldsymbol{g}^{s\pm}_{l,m} = \boldsymbol{g}^{s\pm}_{l,m,y} + \boldsymbol{g}^{s\pm}_{l,m,z} \qquad (l = 1, 2.). \cdots\cdots\cdots (43)$$

Combining Eq.(43) and Eq.(28) gives an expression of the scattered fields in region l $(l = 1, 2)$ as follows:

$$\boldsymbol{f}^s_m(x) = \begin{pmatrix} G^e_{l,ymy}(x) & G^e_{l,ymz}(x) \\ G^e_{l,zmy}(x) & G^e_{l,zmz}(x) \\ G^h_{l,ymy}(x) & G^h_{l,ymz}(x) \\ G^h_{l,zmy}(x) & G^h_{l,zmz}(x) \end{pmatrix} \begin{pmatrix} c_{ym}(0) \\ c_{zm}(0) \end{pmatrix}, \quad (44)$$

$$\begin{pmatrix} G^e_{l,ymt}(x) \\ G^e_{l,zmt}(x) \\ G^h_{l,ymt}(x) \\ G^h_{l,zmt}(x) \end{pmatrix} = (U_{l,m}) \begin{pmatrix} (P^+_{l,m}(x - x^+_{g,l})) \boldsymbol{g}^{s+}_{l,m,t} \\ (P^-_{l,m}(x - x^-_{g,l})) \boldsymbol{g}^{s-}_{l,m,t} \end{pmatrix} \quad (45)$$

where $l = 1, 2 and t = y, z$. The surface currents on the strips can be approximated by expansions in terms of a set of basis functions $\Phi_{tk^t}(z)$ $(k^t = 1, \ldots, N^t,\ t = y, z)$ as

$$\sqrt{Z_0} I_t(0, z) = \sum_{k^t=1}^{N^t} C_{tk^t} \Phi_{tk^t}(z) e^{-is_0 z} \quad (t = y, z). \quad (46)$$

The primary fields Eq.(33), the scattered fields Eq.(44) with the above approximated currents form the boundary condition of perfect conductor on the strips. Applying the Galerkin's Method to this condition in the spectral domain yields a system of linear equation to determine the unknown coefficients of currents C_{tk^t} $(k^t = 1, 2, \ldots, N^t\ t = y, z)$. in Eq.(46):

$$\sum_{t=y}^{z} \sum_{k^t=1}^{N^t} \Big(\sum_{m=-\infty}^{\infty} \phi^*_{uk^u m} G^e_{l,umt}(0) \phi_{tk^t m} \Big) C_{tk^t}$$

$$= -\phi^*_{uk^u 0} e^p_u(0) \quad (k^u = 1, 2, \ldots, N^u, \quad u = y, z) \quad (47)$$

where $\phi_{tk^t m}$ $(k^t = 1, \ldots, N^t,\ t = y, z)$ are the mth Floquet mode expansion coefficient of $\Phi_{tk^t}(z)$.

5. Numerical Results

The basis functions $\Phi_{tk^t}(z)$ $(k^t = 1, \ldots, N^t,\ t = y, z)$ in Eq.(46) and the mth Floquet mode expansion coefficient $\phi_{tk^t m}$ in Eq.(47) used in the following numerical calculations are expressed as follows [13]:

$$\Phi_{tk^t}(z) = \begin{cases} \left(1 - \frac{4z^2}{k_0^2 W^2}\right)^{p - \frac{1}{2}} \cos \frac{(k^t - 1)\pi z}{k_0 W} & \text{for odd } k^t \\ \left(1 - \frac{4z^2}{k_0^2 W^2}\right)^{p - \frac{1}{2}} \sin \frac{(k^t - 1)\pi z}{k_0 W} & \text{for even } k^t \end{cases}, \quad (48)$$

$$\phi_{tk^t m} = \frac{\pi W}{2\Lambda} \left\{ \frac{J_p(\zeta_1)}{\zeta_1^p} + (-1)^{k^t - 1} \frac{J_p(\zeta_2)}{\zeta_2^p} \right\}, \quad (49)$$

$$\zeta_q = \frac{\pi}{2} \left| k^t - 1 - (-1)^q \frac{2mW}{\Lambda} \right| \quad (q = 1, 2) \cdots (50)$$

$$(k^t = 1, \ldots, N^t,\ t = y, z)$$

where J_p is the pth-order Bessel function and

$$p = \begin{cases} 0 & \text{if } t = y \\ 1 & \text{if } t = z \end{cases}. \cdots\cdots\cdots\cdots\cdots\cdots (51)$$

The factors of edge singularity of the current are considered in these functions. The number of basis functions is chosen as $N^y = N^z = 30$ and Floquet mode expansion is truncated at $m = \pm 90$ where the solutions converge well enough to obtain graphical expression of data. Calculations are performed under the conditions that $\epsilon_1 = 1.0$, $\epsilon_2 = 1.2$, $\epsilon_{2e} = 1.35$, $\xi_1 = 0(S)$, $\xi_2 = 1.0 \times 10^{-4}(S)$, $\xi_{2e} = 7.0 \times 10^{-4}(S)$, $\Lambda = \lambda$ and $W = 0.3\Lambda$ where λ is the wave length in air. Polarization state can be expressed by using the ellipticity angles and the orientation angles of the polarization ellipses [8]. The positive and the negative values of ellipticity angles mean left and right-handed elliptically polarized waves and $\pm 45°$ and $0°$ of the values correspond to circular and linear polarization respectively. (The data of orientation angles are omitted here). Figures 2 and 3 show the ellipticity angles and diffraction efficiencies of the diffracted 0th and -1st Floquet modes versus the thickness of the uniaxial chiral region 2 respectively for TE incidence under the conditions that $\theta = 40°$ and $\beta = 20°$. It can be seen that the polarization state and diffraction efficiency are strongly influenced by the

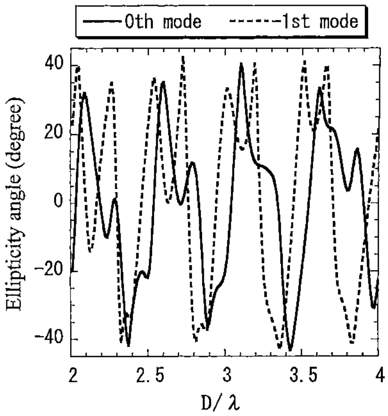

Fig. 2. Ellipticity angles of diffracted waves, $\theta = 40$(degree), $\beta = 20$(degree), TE incidence

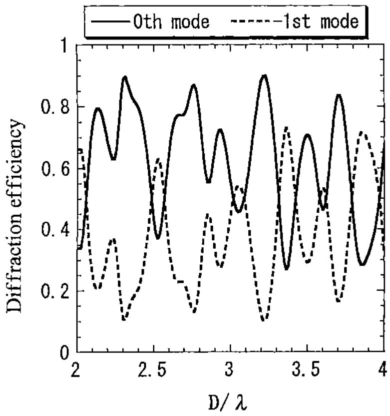

Fig. 3. Diffraction efficiencies, $\theta = 40$(degree), $\beta = 20$(degree), TE incidence

thickness of uniaxial chiral medium. It is found that at around $D/\lambda = 3.43$, the power of incident TE wave is almost evenly divided into 0th and -1st diffracted modes with almost right-handed circular and linear polarization respectively. The orientation angle of this linearly polarized wave is about 60°. This is a useful example of design. Other results under the conditions that $\theta = 25°$ and $\beta = 45°$ for TM incidence are shown in the Figures 4 and 5. These figures show that the incident power of TM wave is distributed to the same extent of powers of 0th and -1st modes with almost left and right-handed circular polarization respectively at around $D/\lambda = 2.7$, and to the powers of two linearly-polarized waves at around $D/\lambda = 3.75$. These two linear polarization have the orientation angles of about 140° to 150°. Figures 6 and 7 give the 3-dimensional expressions of the data of the same conditions as in the Figures 4 and 5 where the angle of the optical axis β (degree) additionally changes. It can be seen that the state of polarization is strongly influenced also by the angle of the optical axis of uniaxial chirality which has given design parameters in the previous figures.

6. Conclusion

The 4×4 matrix-based analysis of the electromagnetic waves diffracted by a grounded uniaxial chiral slab with a plane metallic grating has been presented. These

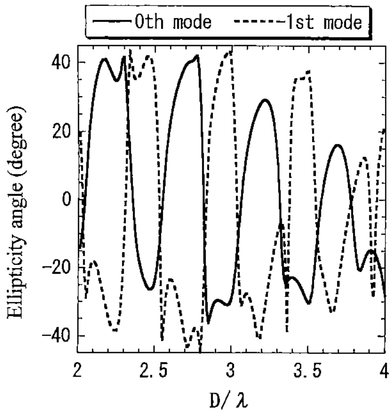

Fig. 4. Ellipticity angles of diffracted waves, $\theta = 25$(degree), $\beta = 45$(degree), TM incidence

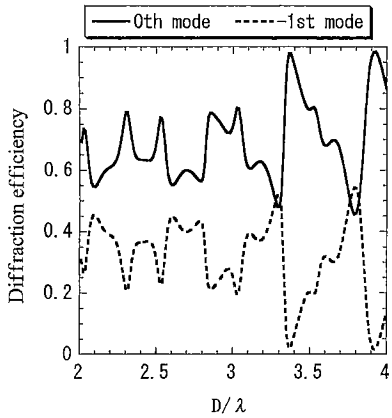

Fig. 5. Diffraction efficiencies, $\theta = 25$(degree), $\beta = 45$(degree), TM incidence

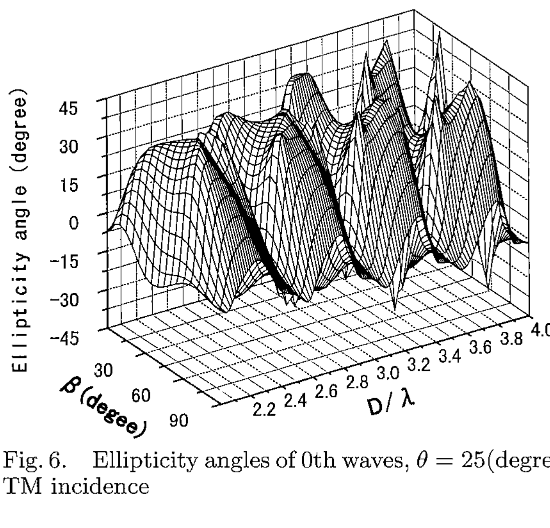

Fig. 6. Ellipticity angles of 0th waves, $\theta = 25$(degree), TM incidence

studies will increase the possibility of novel type devices which can transmit the incident signal to different communication devices such as antennas in local area or those on base stations, satellites etc. which individually have different availability for polarization. The present analysis can be extended for the cases of stratified strucures and of two-dimensional gratings which are now being considered.

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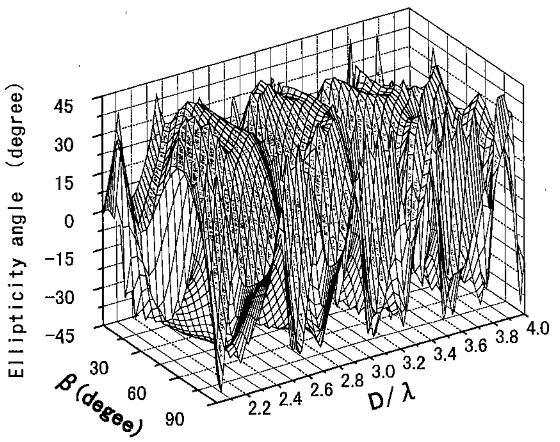

Fig. 7. Ellipticity angles of -1st waves, $\theta = 25$(degree), TM incidence

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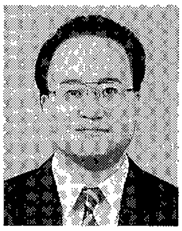

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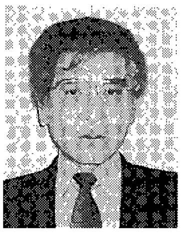

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