Paper

Robust Performance Analysis of Magnetic Bearings

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This paper deals with uncertain model structures, model validation and robust performance analysis of active magnetic bearings. The dynamics of active magnetic bearing systems are characterized by their instability and complex dynamics of rotor and electromagnets. The feedback control is indispensable to stabilize the system, further the closed-loop systems of magnetic bearings should have robustness for stability and performance against model uncertainties. First we derive a nominal mathematical model of AMBs as a linear state-space model under some assumption and idealization, then we consider the discrepancy between the real physical systems and the obtained nominal design model. This discrepancy can be expressed as the structured uncertainties by Linear Fractional Transformation. These uncertainties include linearization error, parametric uncertainties, unmodeled dynamics, and gyroscopic effect. Then we set the interconnection structure which contains the above structurally represented uncertainties. Next we design a robust controller which achieves robust performance condition. Finally, we validate the interconnection structure with the nominal model and uncertainties, and analyze the robustness of stability and performance of the closed-loop system via the mixed structured singular value.

Keywords: Magnetic Bearings, Uncertain Model, Linear Fractional Transformation, Robust Control, μ-Analysis and Synthesis

1. Introduction

Active magnetic bearing(AMB)s allow contact-free suspension. They do not suffer from friction nor wear, and this is the most important advantage of these systems [1]. Since an active controlled-type magnetic bearing is inherently unstable, feedback control is indispensable to stabilize the system. A conventional PID controller is often employed as a feedback compensator, and this method often yields enough stability and performance, but owing to model uncertainties and changes of the state variables, the entire system sometimes becomes unstable. The ordinary approach to avoid this problem is an application of robust control methodologies. It is well known this is one of the effective control techniques for unstable systems. On the control of AMB, one of the most critical problems is a description of a complex behavior of the dynamics of electromagnets and their forces. Some approximations and assumptions must be employed, and consequently the discrepancy between the real physical system and the design model cannot be avoided. This discrepancy of AMB systems is a serious problem.

One of the first multivariable MIMO model of AMB systems was derived by Matsumura [2] via applying aircraft dynamical model, but no uncertainties nor disturbances were considered and described. In references (3) and (4), the multiplicative unstructured uncertainty description was introduced and the robust performance and robust stability of the closed-loop system was improved via an H_∞ control, but the problem was the conservative robustness analysis. The theoretically guaranteed perturbation for robust performance was much lesser than the real robust performance with the real physical systems. Note that "Robust Performance" means here that the system satisfies the performance specifications for all perturbed plants about the nominal model up to the worst-case model uncertainty.

By the way, the state-space control theory of uncertain system with Linear Fractional Transformation(LFT) has almost been settled up for practical use [5] recently. LFT and H_∞/μ control theory have come to play an important role in control system design and provide a uniform framework for realization, analysis and synthesis for uncertain systems. Application of LFT uncertainty modeling and H_∞/μ control methods to Active Magnetic Bearings has been reported in references (3), (4), (6), (7), (9) and (10).

References (4) and (6) were results of the application of μ synthesis to AMBs in the initial stage, and the quantities of the uncertainties were partially considered, but structures of the uncertainties were not dealt with. Reference (4) was an extended results of the reference (3). In reference (7), the quantities and also structure of the uncertainties were considered, but the set of uncertainties were in a class of linear time-varying systems, and the constant scaling matrices were employed. This method can not avoid a conservative robustness analysis.

Reference (8) modeled the structured uncertainties of the whole plant including digital controller, filter, sensor, amplifier and AMB, and mixed μ test were performed for the robustness analysis. But the most important electromagnetic uncertainties and the gyroscopic effects were not considered. And Reference (9) introduced the rotor model based on FEM and considered many uncertainty factors of the plant, however it treated only the complex μ test.

These efforts reveal that such a modeling and design methodology is quite promising, and further research is needed in this direction for the evaluation of the robust control of magnetic bearings. Therefore, in this paper we apply this approach to an active magnetic bearing system, and derive an uncertain magnetic bearing model which contains less conservativeness for robust performance analysis. The novelty of this paper is a comprehensive approach for robust AMB control to reduce conservativeness of the robustness analysis. The characterictics are as below.

- The smallest destabilizing perturbation with an appropiate block structure for a magnetic bearing is derived in LFT and μ framework. (15% parametric uncertainties.)
- The structured uncertainties of the whole plant including electro-magnetic dynamics(linearization error, unmodeled dynamics) parametric uncertainties, and gyroscopic effect of the rotating rotor, are modeled.
- In order to reduce conservativeness of the robustness analysis, the uncertain model structure is derived and the mixed structured singular value μ test [5] is performed.
- The proposed structured uncertainty model concerned to the robust performance by comparing with the unstructured uncertainty model is evaluated.

Note that the magnetic bearings are Multi-Input and Multi-Output and complex electro-mechanical unstable systems. We derived a novel unified structure of the model of a magnetic bearing with uncertainties in this paper. This is the most important contribution in the related fields.

2. Magnetic Bearing System and Its Model

2.1 System Construction The magnetic bearing system employed in this research is a 4-axis controlled horizontal shaft magnetic bearing with symmetrical structure. The axial motion is not controlled actively. The diagram of the experimental machine is shown schematically in Fig.1. A three-phase induction motor is located at the center of the rotor. Around the rotor, four pairs of electromagnets are arranged radially, and pairs of eddy-current type gap sensors are located on outside of the electromagnets. A tachometer is arranged on the side of the rotor to measure the rotational speed.

2.2 Nominal Mathematical Model At first, we choose a simplified linear time-invariant nominal model of this system. The final mathematical model will be described by a set using LFT form, therefore this nominal model is a central plant model of a model set. In order to derive a nominal model of the system, the following assumptions are introduced.

(1) The rotor is rigid and has no unbalance.
(2) All electromagnets are identical.
(3) Attractive force of an electromagnet is in proportion to (electric current / gap length)^2.
(4) The resistance and the inductance of the electromagnet coil are constant and independent of the gap length.
(5) Small deviations from the equilibrium point are treated.

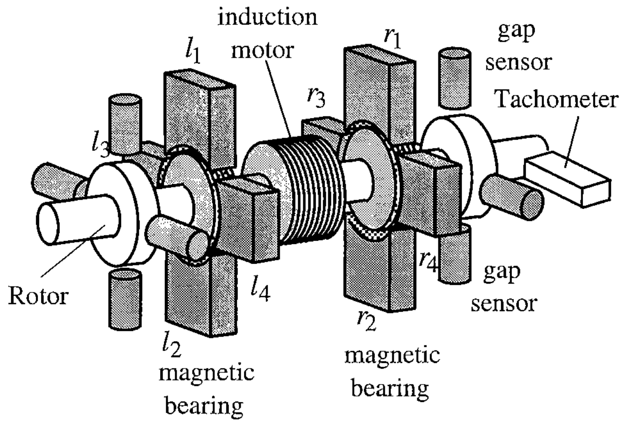


Fig. 1. 4-Axis Controlled-type Magnetic Bearings

These assumptions are not strong and suitable around the steady state operation, but if the rotor spins at super-high speed, these assumption will be failed. Based on the above assumptions, a four-input, four-output, 12-state nominal model has been derived as follows [2][3]. Further, the nominal block diagram of the plant is shown in Fig.2, and it clearly shows the structure of the plant. Hence we design a controller based on Fig.2.

$$\dot{x} = Ax + Bu, \qquad y = Cx \quad \cdots\cdots\cdots\cdots\cdots\cdots (1)$$

where

$$x := [g^T \quad \dot{g}^T \quad i^T]^T,$$
$$g := [\, g_{l1} \;\; g_{r1} \;\; g_{l3} \;\; g_{r3} \,]^T,$$
$$i := [\, i_{l1} \; i_{r1} \; i_{l3} \; i_{r3} \,]^T,$$
$$u := [\, e_{l1} \; e_{r1} \; e_{l3} \; e_{r3} \,]^T,$$
$$A := \begin{bmatrix} 0 & I & 0 \\ C_1B_1C_2C_1^{-1} & C_1A_1(p)C_1^{-1} & C_1B_1C_3 \\ 0 & 0 & -(R/L)I \end{bmatrix},$$
$$B := [\, 0 \;\; 0 \;\; (1/L)I \,]^T, \quad C := [\, I \;\; 0 \;\; 0 \,],$$

$$A_1(p) := \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -pJ_y/J_x \\ 0 & 0 & pJ_y/J_x & 0 \end{bmatrix},$$
$$B_1 := \begin{bmatrix} 0 & 0 & 1/m & 1/m \\ -1/m & -1/m & 0 & 0 \\ l_l/J_y & -l_r/J_y & 0 & 0 \\ 0 & 0 & l_l/J_y & -l_r/J_y \end{bmatrix},$$
$$C_1 := \begin{bmatrix} 0 & 1 & -l_l & 0 \\ 0 & 1 & l_r & 0 \\ -1 & 0 & 0 & -l_l \\ -1 & 0 & 0 & l_r \end{bmatrix},$$

$$C_2 := \frac{2}{W}\begin{bmatrix} 0 & -(F_{l1}+F_{l2}) & l_l(F_{l1}+F_{l2}) & 0 \\ 0 & -(F_{r1}+F_{r2}) & -l_r(F_{r1}+F_{r2}) & 0 \\ (F_{l3}+F_{l4}) & 0 & 0 & l_l(F_{l3}+F_{l4}) \\ (F_{r3}+F_{r4}) & 0 & 0 & -l_r(F_{r3}+F_{r4}) \end{bmatrix},$$

$$C_3 := 2\cdot \mathrm{diag}\cdot\left[\ \frac{F_{l1}}{I_{l1}}+\frac{F_{l2}}{I_{l2}} \quad \frac{F_{r1}}{I_{r1}}+\frac{F_{r2}}{I_{r2}} \quad \frac{F_{l3}}{I_{l3}}+\frac{F_{l4}}{I_{l4}} \quad \frac{F_{r3}}{I_{r3}}+\frac{F_{r4}}{I_{r4}}\ \right],$$

$g_k(t)$: gap length, $i_k(t)$: current, $e_k(t)$: input voltage, m: mass of the rotor, $J_x(J_y)$: moment of inertia about $X(Y)$ axis, l_l, l_r: distance between center of mass and left(right) electromagnet, W: steady state gap, F_k: steady attractive force, I_k: steady current, R: resistance, L: inductance, p: rotor speed, $(k = l1, l3, r1, r3)$.

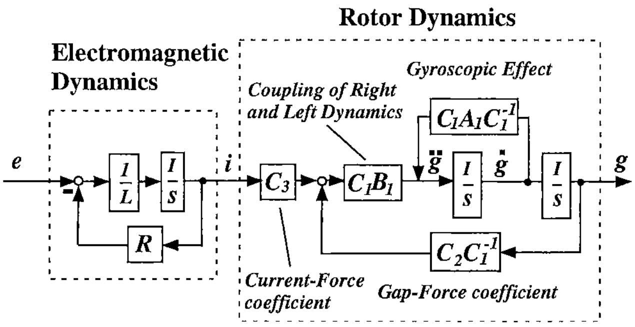


Fig. 2. Nominal linear model G_{nom}

3. Structured Uncertainties

The derived nominal model G_{nom} in Fig.2 works fairly well around the steady state operational point. However, if the state of the system deviates from the nominal operating point, the model no longer suitably describes the physical system. We treat this discrepancy as a model uncertainty, and make a new extended set of plant models, that is constructed with the nominal model and model uncertainties. This set of models can cover the relatively wider behaviors of the real plant, but still not globally. The following four items are well known to be the most general and serious uncertainties in magnetic bearings:

- Linearization Error
- Parametric Uncertainty
- Unmodeled Dynamics
- Uncertainty in Gyroscopic Effect

3.1 Linearization Error There should be model uncertainties caused by linearization of the electromagnetic force. Here we employ sector bounds to account for the linearization error, and describe as

$$C_3 := \underline{C_3} + k_i\delta_i, \quad \delta_i \in \mathcal{R}, \quad |\delta_i| \le 1 \quad \cdots\cdots (2)$$

$$C_2C_1^{-1} := \underline{C_2C_1}^{-1} + k_x\delta_x, \quad \delta_x \in \mathcal{R}, \quad |\delta_x| \le 1 \quad \cdots\cdots (3)$$

3.2 Unmodeled Dynamics Nominally the dynamics of electromagnets is expressed by a very simple transfer function $G_{EM}(s) := \frac{1}{Ls+R}$. It is well known that an inductance L and a resistance R of the electromagnet have frequency varying and gap(x)-dependent characteristics. Further, these parameters are very sensitive to be measured. Moreover the unmodeled dynamics of the eddy-current should be considered. The transfer functions of the electromagnet including the uncertainty caused by eddy-current are distributed in a frequency dependent belt. We describe this belt as an unstructured uncertainty as follows.

$$\frac{I}{Ls+R} := \frac{I}{\underline{L}s+\underline{R}} + W_{RL}(s)\Delta_{RL}(s), \quad \Delta_{RL} \in \mathcal{C}, \quad |\Delta_{RL}(j\omega)| \le 1. \quad \cdots\cdots (4)$$

3.3 Parametric Uncertainties The first request for the control system is robust stability against unexpected exogenous force disturbances. Another general demand in practical use of the magnetic bearing system is a flexible change of the mass of rotors. These two specifications can be described by a parametric perturbation of a C_1B_1 as

$$C_1B_1 := \underline{C_1B_1} + k_f\Delta_f, \quad \Delta_f \in \mathcal{C}^{4\times 4}, \quad ||\Delta_f||_\infty \le 1. \quad \cdots\cdots (5)$$

Note that $||\cdot||_\infty$ shows an $\mathcal{H}_\infty$ norm which is defined as, $||G(s)||_\infty := \max_\omega \bar{\sigma}(G(j\omega))$, where $\sigma(\cdot)$ is the singular value.

The uncertainty of the rotor flexibility can be included in k_f.

3.4 Uncertainties in the Gyroscopic Effect The gyroscopic effect is well-known as one of the most serious problems of magnetic bearings, which is a dynamic coupling of vertical and horizontal motions of the rotational rotors. Our demand is to guarantee stability and performance against changing rotational speeds. This behavior can be written as

$$C_1^{-1}A_1(p)C_1 := \underline{C_1}^{-1}\underline{A_1}(p)\underline{C_1} + k_p\Delta_p, \quad \Delta_p \in \mathcal{C}^{4\times 4}, \quad ||\Delta_p||_\infty \le 1 \quad \cdots\cdots (6)$$

3.5 Final Mathematical Model of the Plant Up to now, four types of uncertainties have been considered. Now, we combine them and the nominal model and construct the final mathematical model of the plant, which is a set model of this plant which expresses the relatively wider behaviors of the real plant.

The final mathematical model of the plant $\tilde{G}(s)$ is shown in Fig.3, which includes the above nominal model, linearization error, parametric uncertainties, unmodeled dynamics and gyroscopic effects. The order of this system is 12 plus the order of $W_{RL}(s)$.

Here, the uncertainties in current-force coefficient, gap-force coefficient and inductance are certainly somewhat interrelated. Hence Δ_{RL}, δ_i and δ_x are correlation uncertainties. If these three types of uncertainties are lumped in a complex partial differential equation, however, the obtained model have not been appropriate for the model of the control system design. Therefore these three blocks was divided, but this uncertainty partition brings the conservativeness of the robustness analysis.

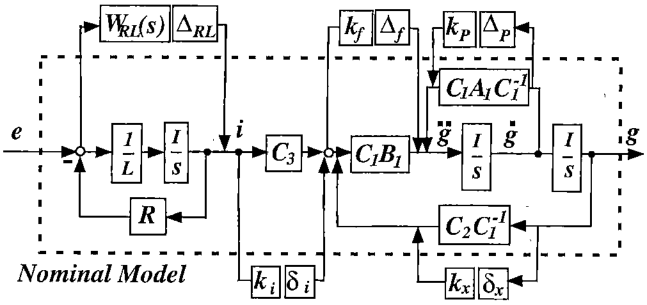


Fig. 3. Final Mathematical Model $\tilde{G}(s)$ of Magnetic Bearings

4. μ-Analysis and Synthesis

In this section, we design a robust controller for the plant model $\tilde{G}(s)$.

4.1 Quantification of Uncertainties Here we assume that the quantities of perturbation of all system matrices are 15 % of nominal values as

$$k_i := 0.15 \cdot \underline{C_3}, \qquad k_x := 0.15 \cdot \underline{C_2 C_1^{-1}},$$
$$k_f := 0.15 \cdot \underline{C_1 B_1}. \quad \cdots\cdots\cdots\cdots \quad (7)$$

The weighting function W_{RL} to cover the 15 % perturbation of L and R is chosen as

$$W_{RL}(s) := 2.0 \times 10^{-2} \frac{\frac{s}{2\pi \times 160} + 1}{\frac{s}{6.00} + 1} I_4. \quad \cdots\cdots \quad (8)$$

Here the frequency response of $W_{RL}(s)$ is shown in Fig.4, where solid line shows the $W_{RL}(s)$, and the dashed line indicates the upper bound of the uncertainty caused by the perturbation of L and R. The gain of $W_{RL}(s)$ is remarkably higher than that of the upper bound of the uncertainty at the high frequency, which strengthen the robustness of the control system against the unmodel dynamics as eddy current dynamics at this frequency.

The 15% uncertainty of the rotational speed against 10000[rpm] is described as

$$k_p := 0.15 \cdot \underline{C_1^{-1} A_1(p) C_1}, \quad p = 10000[\mathrm{rpm}]. \cdot (9)$$

4.2 Control System Design Utilizing the structured singular value μ [5], we design the controller which achieves robust performance against various types of uncertainties.

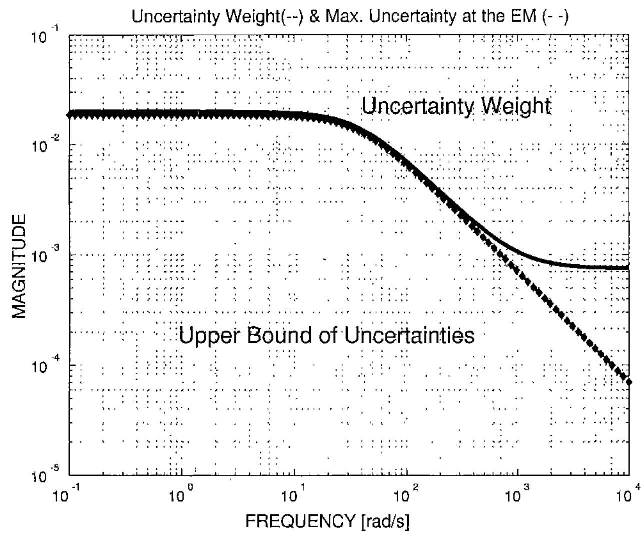


Fig. 4. Uncertainty Weight $W_{RL}(s)$ and the Upper bound of Uncertainty at the Electromagnets

The structured singular value $\mu_{\mathbf{\Delta}}(M)$ is defined for matrices $M \in \mathcal{C}^{n \times n}$ with the block structure $\mathbf{\Delta}$ as

$$\mu_{\mathbf{\Delta}}(M) := \frac{1}{\min\{\bar{\sigma}(\Delta) : \Delta \in \mathbf{\Delta}, \det(I - M\Delta) = 0\}} \quad (10)$$

unless no $\Delta \in \mathbf{\Delta}$ makes $(I - M\Delta)$ singular, in which case $\mu_{\mathbf{\Delta}}(M) := 0$. μ shows the size of the smallest destabilizing perturbation with an appropriate block structure in general.

4.2.1 Interconnection structure We construct an interconnection structure by LFT representation in Fig.5, where $W_{perf}(s)$ is a performance specification and also is a weight for a sensitivity function $S := (I + G_{nom}K)^{-1}$. The system should maintain the control bandwidth more than 10 [rad/s], and the steady state error with respect to a step input should be less than 1.00%. In order to achieve this specification, $W_{perf}(s)$ was chosen as the following function.

$$W_{perf}(s) = \frac{150}{\frac{s}{2\pi \times 0.01} + 1} I_4. \quad \cdots\cdots\cdots\cdots \quad (11)$$

The frequency response of this weighting function $W_{perf}(s)$ is shown in Fig.6.

4.2.2 Control Problem Next, for the robust performance synthesis, we define the block structure $\mathbf{\Delta}$ as

$$\begin{aligned} \mathbf{\Delta} := \{&\mathrm{diag}[\delta_i I_4, \delta_x I_4, \Delta_f, \Delta_p, \Delta_{RL} I_4, \Delta_{perf} I_4] : \\ &\delta_{\{i,x\}} \in \mathcal{R}, \ \ \Delta_{\{f,p\}} \in \mathcal{C}^{4\times 4} \\ &\Delta_{\{RL,perf\}} \in \mathcal{C}\}, \\ &|\delta_{\{i,x\}}| < 1, ||\Delta_{\{f,p\}}||_\infty < 1, \\ &|\Delta_{\{RL,perf\}}(j\omega)| < 1, \forall \omega. \quad \cdots\cdots\cdots \quad (12) \end{aligned}$$

Then, the control problem is to find the controller $K(s)$

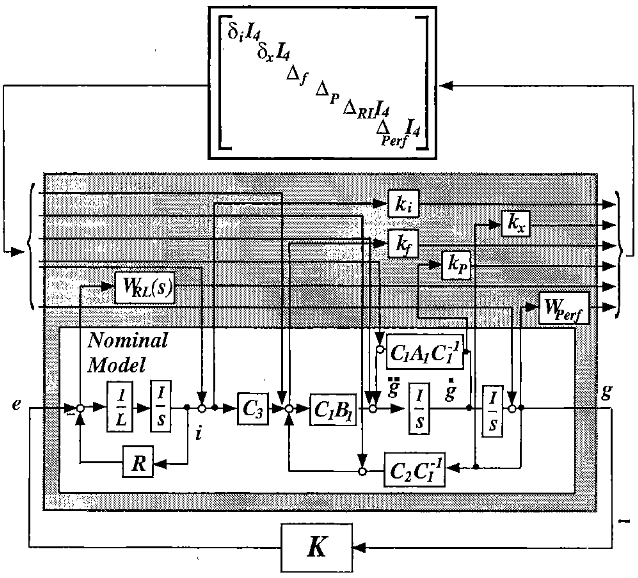


Fig. 5. Interconnection Structure

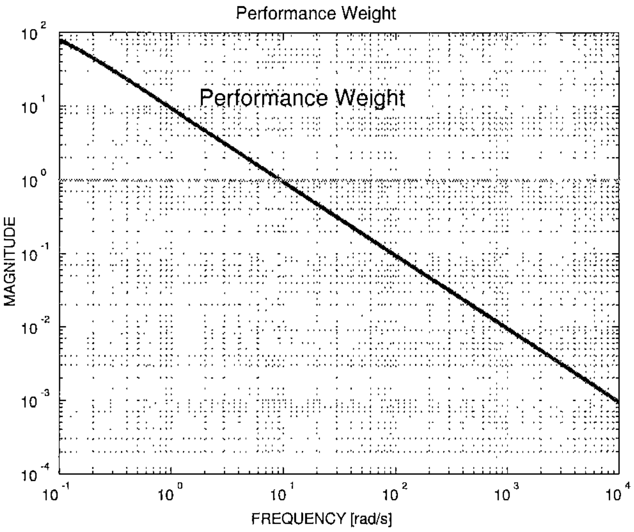


Fig. 6. Performance Weight $W_{perf}(s)$

which achieves the following robust performance condition [5], where $P(s)$ is the generalized plant which is expressed by the gray rectangle box in Fig.5.

$$\sup_{\omega\in\mathcal{R}} \mu_{\mathbf{\Delta}}[F_l(P(j\omega), K(j\omega))(j\omega)] < \gamma. \cdots\cdots\cdot (13)$$

Note that $F_l(P, K)$ is the lower linear fractional transformation (LFT) of P with K, and is defined as

$$F_l(P, K) := P_{11} + P_{12}K(I - P_{22}K)^{-1}P_{21}. \cdot (14)$$

We apply the standard $D - K$ iteration [5] to find the sub-optimal μ controller for the system, where $\gamma = 1$. After the 2nd iteration, we obtained a controller $K(s)$, where the supremum of $\mu_{\mathbf{\Delta}}[F_l(P, K)]$ is 0.973. Final scaling matrix $D(s)$ has 36 states, then $K(s)$ has 92 states.

Note that the mixed μ analysis was employed for magnetic bearings in reference(9) and (10), and succeeded to reduce conservativeness of the robustness analysis. The mixed μ synthesis was proposed by Young et. al. [11][12], however it seems to be difficult to implement this synthesis method because of numerical calculation problems.

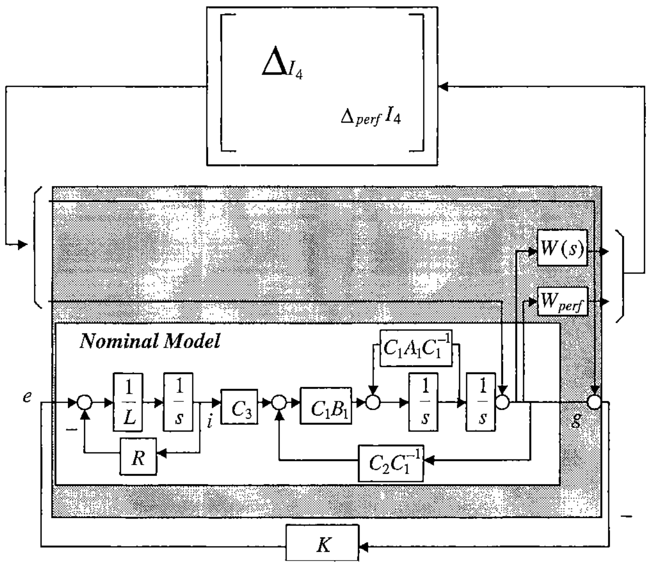


Fig. 7. Interconnection Structure 2 with unstructured uncertainty

4.3 Unstructured Uncertainty Description

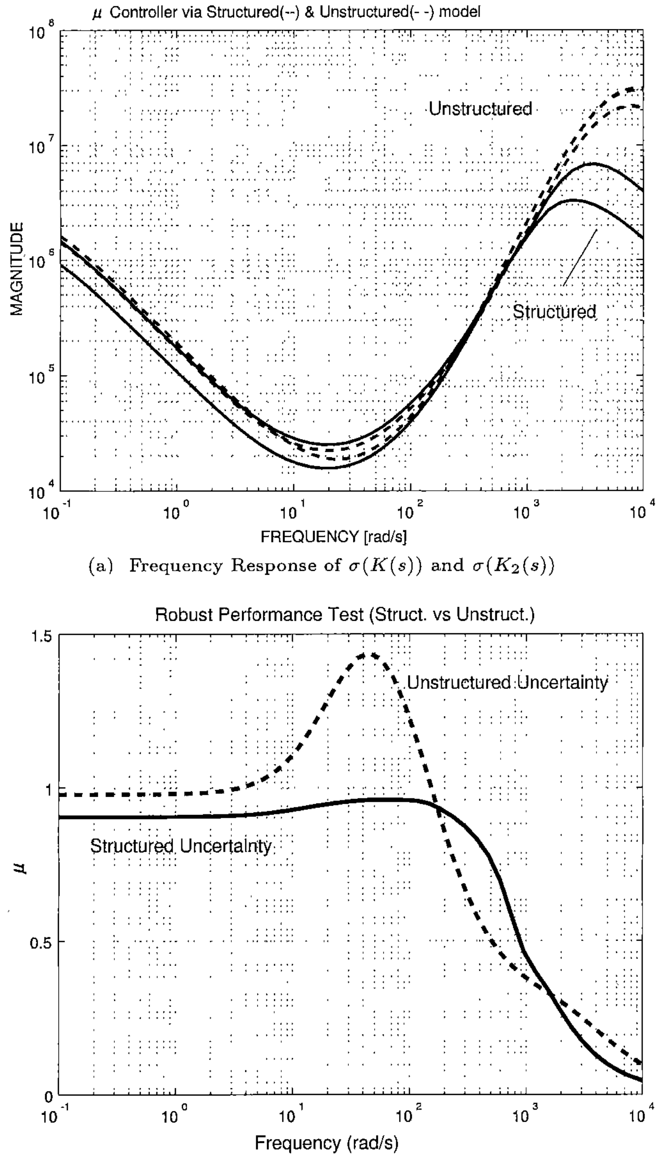
For the comparison of the conservativeness of robust performance, we here introduce the LFT-based unstructured uncertainty description, and its interconnection structure shown in Fig.7. The block structure of $\mathbf{\Delta}$ in Fig.7 is simple and different from (12) in Fig.5. Further the generalized plant differ from $P(s)$, hence we define the block structure, the generalized plant, and the derived robust controller as $\mathbf{\Delta_2}$, $P_2(s)$, $K_2(s)$, respectively. In $P_2(s)$, the weighting function $W(s)$ expresses the uncertainty weighting and here the 15% uncertainty is involved in it, and $W_{perf}(s)$ is same with (11).

4.4 Comparison of Robust Performance Analysis

The frequency response of $\sigma(K(s))$ and $\sigma(K_2(s))$ are shown in Fig.8(a), where the two solid lines respectively show $\sigma(K(s))$ and the dashed line shows $\sigma(K_2(s))$. These two controller are similar each other except at the high frequency. By using the $K(s)$, the upper bounds of $\mu_{\mathbf{\Delta}}[F_l(P, K)]$ and $\mu_{\mathbf{\Delta_2}}[F_l(P_2, K)]$ can be calculated and the results are shown in Fig.8(b). Since the peak value of the upper bound of $\mu_{\mathbf{\Delta}}[F_l(P, K)]$ is less than that of $\mu_{\mathbf{\Delta_2}}[F_l(P_2, K)]$, the conservativeness of robust performance analysis has been reduced by $\mathbf{\Delta}$ and $P(s)$ in comparison with $\mathbf{\Delta_2}$ and $P_2(s)$.

5. Evaluation of Robust Performance Analysis

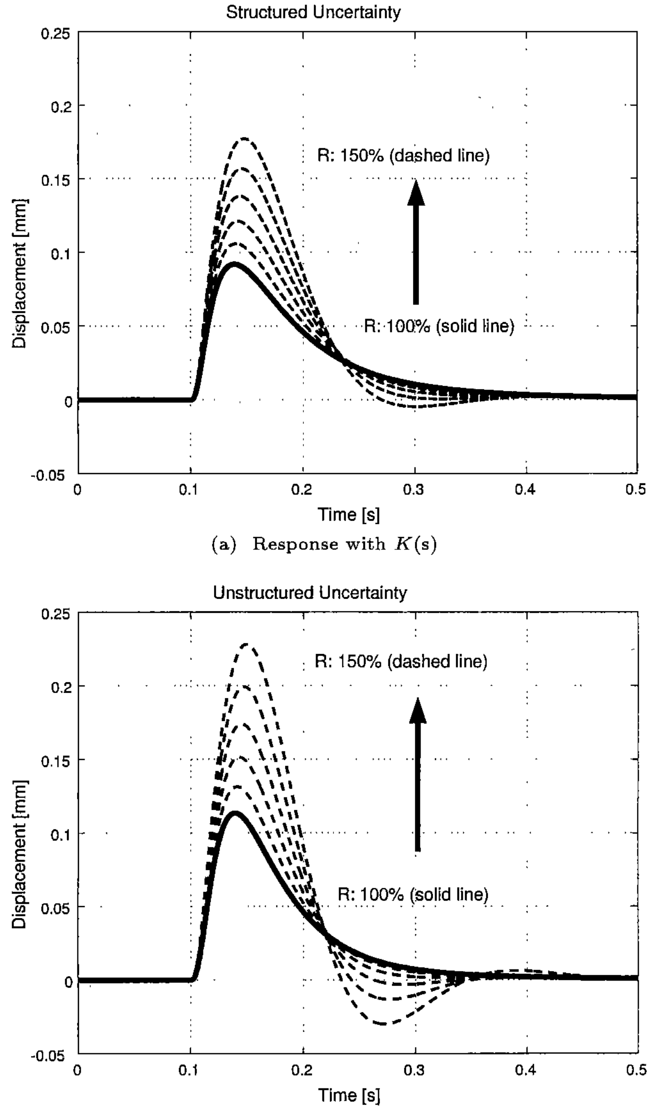
In order to evaluate the proposed set of plant models $\tilde{G}(s)$ and the robust performance analysis result, we carried several numerical simulations with $K(s)$ and $K_2(s)$. All simulation results are time response of the rotor position against step-type disturbances, and shown in Fig.9. To evaluate that the system keeps stability and



(a) Frequency Response of $\sigma(K(s))$ and $\sigma(K_2(s))$

(b) $\mu_\Delta[F_l(P, K)]$ and $\mu_\Delta[F_l(P_2, K)]$

Fig. 8. Robust Performance Analysis



(a) Response with $K(s)$

(b) Response with $K_2(s)$

Fig. 9. Simulation Results

performance against the 15% parametric perturbations, we employed the following simulation models to validate the robust performance of the controller $K(s)$ and $K_2(s)$.

- **Nominal Model** (nominal parameters): solid line
- **Perturbed Model 1** (R: ×110%) : dashed line
- **Perturbed Model 2** (R: ×120%) : dashed line
- **Perturbed Model 3** (R: ×130%) : dashed line
- **Perturbed Model 4** (R: ×140%) : dashed line
- **Perturbed Model 5** (R: ×150%) : dashed line

Time responses of the position g_{l1} of the rotor against a step-type disturbance 90.9[N] are shown in Fig.9(a), where 90.9[N] is a steady state force of the rotor, and the used controller is $K(s)$. According as the perturbation of R increases time responses of g_{l1} get into deterioration.

By the way, time responses of the position g_{l1} of the rotor with the controller $K_2(s)$ are shown in Fig.9(b), where same 90.9[N] step-type disturbance was added. According as the perturbation of R also increases time responses of g_{l1} deteriorate awfully. From these two figures, we can obtain the following conclusion of this section.

- Time responses with the controller $K(s)$ in Fig.9(a) is better than with $K_2(s)$ in Fig.9(b).
- Until 15% perturbation, time responses in Fig.9(a) show relatively good performance, but for the bigger perturbation, the controller still keep certain performance level.

6. Conclusion

In this paper, we proposed the novel model and uncertainty structure of magnetic bearings via LFT, which contains less conservativeness for robust performance analysis. We structurally described four types of uncertainties by using real/ complex bounded numbers/matrices. Next, we designed a robust controller by μ-analysis and synthesis which achieves robust performance. Finally, we evaluate the proposed model and uncertainty structure concerned to the robust performance analysis by comparing an unstructured uncertainty model.

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References

(1) G. Schweitzer, R. Siegwart, F. Lösch, R. Berksun (Editor): *Proceedings of the Seventh International Symposium on Magnetic Bearings,* ETH Zurich ISMB, 2000.

(2) F. Matsumura and T. Yoshimoto, "System Modeling and Control Design of a Horizontal-Shaft Magnetic-Bearing System," *IEEE Trans. on Magnetics*, vol.22, no.3, pp.196-203, 1986.

(3) M. Fujita, K. Hatake, and F. Matsumura, "Loop Shaping Based Robust Control of a Magnetic Bearing," *IEEE Control Systems Magazine*, pp.57-65, Aug., 1993.

(4) M. Fujita, K. Hatake, F. Matsumura, and K. Uchida, "An Experimental Evaluation and Comparison of H_∞/μ Control for a Magnetic Bearing," *Proc. of 12th IFAC World Congress*, pp.393-398, 1993.

(5) S. Skogestad and I. Postlethwaite, *Multivariable Feedback Control,* John Wiley & Sons, 1996.

(6) K. Nonami and T. Ito, "μ Synthesis of a Flexible Rotor Magnetic Bearing Systems," *IEEE Trans. on Control Systems Technology*, vol.4, no.5, pp.503-512, 1996.

(7) M. Hirata, K. Nonami and T. Ohno, "Robust Control of a Magnetic Bearing System Using Constantly Scaled H_∞ Control," *Proc. of the Sixth Int. Symp. on Magnetic Bearings,* pp.713-722, 1998.

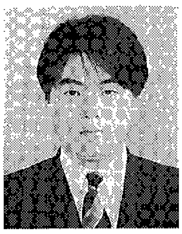
(8) R. L. Fittro and C. R. Knospe, "μ Control of a High Speed Spindle Thrust Magnetic Bearings," *Proc. of the 1999 IEEE International Conference on Control Applications,* pp.570-575, Hawaii, USA, August 22-27, 1999.

(9) F. Lösch, C. Gähler and R. Herzog, "Low Order μ-Synthesis Controller Design for a Large Boiler Feed Pump Equipped with Active Magnetic Bearings," *Proc. of the 1999 IEEE International Conference on Control Applications,* pp.564-569, Hawaii, USA, August 22-27, 1999.

(10) Toru Namerikawa and M. Fujita, "Uncertain Model and μ-Synthesis of a Magnetic Bearing," *Proc. of the 1999 IEEE International Conference on Control Applications,* pp.558-556, Hawaii, USA, August 22-27, 1999.

(11) P.M. Young, "Controller design with real parametric uncertainty," *Int. J. of Control*, vol.65, no.3, pp. 469-509, 1996.

(12) P.M. Young and J.C. Doyle, "A Lower Bound for the Mixed μ Problem," *IEEE Trans. on Automatic Control,* vol.42, no.1, pp. 123-128, 1997.



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