Scattering of Electromagnetic Waves by Columnar Dielectric Gratings with Elliptically Layered Media

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In this paper, the scattering problems by columnar dielectric gratings with elliptically layered media are analyzed using the combination of improved Fourier series expansion method and the multilayer method. Numerical results are given for the transmitted scattered characteristics for the case of incident angle and frequency by varying the grating shape whose profile is the elliptic cylinders, and whose interior distribution of permittivity is an elliptically layered medium for both TM and TE waves. The influences of the incident angle and frequency of the transmitted power are compared between inhomogeneous case and homogeneous case.

Keywords: Columnar Dielectric Grating, Inhomogeneous Media, Scattering

1. Introduction

Dielectric gratings have found applications in various areas such as integrated optics and acousto-optics, optical filters, and holography. Recently, the refractive index can easily be controlled to make the periodic structures such as fiber grating and photonic crystal waveguide by the development of manufacturing technology of optical devices. Thus, the scattering and guiding problems of the inhomogeneous gratings have been considerable interest, and many analytical and numerical methods which are applicable to the dielectric gratings having an arbitrarily periodic structures have been proposed. In the multilayer method[3,4], as the inhomogeneous region is divided into an assembly of stratified thin layers with modulated index, the order of the matrix depends on the number of layers. However, in our approach, the order of characteristic matrix equation depends on the modal truncation number, but does not depend on the number of layers. Therefore the range of applicability to periodic structures is much wider[9] than that of other method, and our method can be applied easily to the guiding problems, such as planar slanted gratings[10].

In this paper, the scattering of electromagnetic waves by dielectric gratings with elliptically layered media[9] are analyzed using the combination of improved Fourier series expansion method[6] and the multilayer method[11].

Numerical results are given for the transmitted scattered characteristics for the case of incident angle and frequency by dielectric gratings whose shape of grating is an elliptic cylinder, and whose interior distribution of permittivity is an elliptically layered medium for both TM and TE waves. The influences of the incident angle and frequency of the transmitted power are compared between inhomogeneous case and homogeneous case.

2. Method of Analysis

We consider the columnar dielectric grating with elliptically layered media as shown in Fig.1(a). The grating is uniform in the y-direction and the permittivity \( \varepsilon(x,z) \) is an arbitrary periodic function of \( z \) with period \( p \), Fig.1(a) shows the configuration whose shape of the grating is an elliptic cylinder with the cross section of \( a \times d/2 \),

\[
\left( \frac{x+d}{2}/(d/2) \right)^2 + (z/a)^2 = 1
\]

and whose interior distribution of permittivity \( \varepsilon(x,z) \) is elliptically layered medium

\[
\varepsilon(x,z) \begin{cases} 
\varepsilon_2 & b = 1 - \varepsilon_1/\varepsilon_2 \quad : \text{inside of elliptic cylinder} \\
\varepsilon_1 & \quad : \text{outside of elliptic cylinder} 
\end{cases}
\]

The permeability is assumed to be \( \mu_0 \). The time dependence is \( \exp(-i\omega t) \) and suppressed throughout. In the formulation, the TM (the magnetic field has only the y-component) case is discussed. For the TE (the electric field has only the y-component) case, only numerical results are presented.

When the plane wave is assumed to be incident from \( x > 0 \) at the angle \( \theta_0 \), the scattering fields in the regions \( S_1(x \geq 0) \) and \( S_2(x \leq -d) \) are expressed[9] as

\[
S_{1}(x \geq 0) : H_{y}^{(1)} = e^{i\theta_{0}(\sin \theta_{0} - \sin \theta_{y})} \sum_{n=-N}^{N} t_{n}^{(1)} e^{ik_{y}(x+2\pi n/p)}
\]

(3)

\[
S_{2}(x \leq -d) : H_{y}^{(3)} = e^{ik_{0}z} \sum_{n=-N}^{N} t_{n}^{(3)} e^{-ik_{y}(x-zd-2\pi n/p)}
\]

(4)

\[
F_{x}^{(1)} = (-i\omega \varepsilon_{0})^{-1} \left( \partial H_{y}^{(1)} / \partial x \right)
\]

(5)


\[
 k_j^{(i)} = \sqrt{k^2 - (k_j \sin \theta_j + 2 \pi n_j / p)^2} ; k_1 = 2 \pi / \lambda , \ j = 1, 3 ,
\]

where \( \lambda \) is the wavelength in free space and \( k_j^{(i)} \) and \( t_n^{(3)} \) are unknown coefficients to be determined from boundary conditions. In inhomogeneous grating region \( S_2 \) \((-d < x < 0)\), the permittivity profile \( \varepsilon(x,z) \) of Eq.(2) is generally not separable with respect to the \( x \) and \( z \) coordinates. Therefore, it is difficult to obtain the analytical solution, such as Mathieu function, in the inside of columnar dielectric gratings. However in our method, the region \( S_2 \) is divided into thin layers \( d_s = d / M \), as shown in Fig.1(b), so that the dielectric distribution of thin layer is approximated to step profile with respect to \( z \).

\[
\varepsilon^{(i)}(z) = \varepsilon_0 (1 - 0.5) d_s = 1 - M . \tag{6}
\]

It is important to notice in Eq.(6) that the electromagnetic fields in inhomogeneous grating \( S_2 \) can be obtained by the eigenvalue equation as follows.

In each layer \((l = 1 \sim M)\), the magnetic field can be written as \( H^{(i)} = H^{(i)}(z) e^{il z} \), where \( h^{(i)} \) is the propagation constant in the \( x \)-direction, and \( H^{(i)}(z) \) must satisfy the following wave equation\([6]\):

\[
\begin{align*}
 \frac{d^2 H^{(i)}(z)}{dz^2} + \frac{1}{\varepsilon^{(i)}(z)} \frac{d}{dz} \left( \frac{d H^{(i)}(z)}{dz} \right) & = 0,
\end{align*}
\]

\[
\tag{7}
\]

Taking into account the Floquet's theorem, \( H^{(i)}(z) \) can be approximated by the finite Fourier series as:

\[
H^{(i)}(z) = e^{\jmath \beta x \sin \theta_j} \sum_{l=-L}^{L} \psi_{\varepsilon}^{(i)}(z) e^{il \pi \rho} . \tag{8}
\]

If \( \varepsilon^{(i)}(z) \) is expressed as \( \varepsilon^{(i)}(z) = \varepsilon^{(i)}(0) / g^{(i)}(z) \), substituting Eq.(8) into Eq.(7) and multiplying both sides by \( g^{(i)}(z) f^{(i)}(z) = e^{-il \pi \rho} \), and rearranging after integrating with respect to \( z \) in the interval \( 0 < z < p \), we get the following eigenvalue equation in regard to \( \eta^{(i)} \)[6]:

\[
\begin{align*}
 \Lambda_s U^{(i)} & = \left[ \eta^{(i)} \right] \Lambda_s U^{(i)} , \tag{9}
\end{align*}
\]

\[
\begin{align*}
 \Lambda_1 & = \left[ \eta^{(i)} \right] , \ \Lambda_2 & = \left[ \phi^{(i)} \right] , \ i = 1 \sim M ,
\end{align*}
\]

where

\[
\begin{align*}
 \xi^{(i)} & = \left[ \xi_{\varepsilon}^{(i)}, \ldots, \xi_{\eta}^{(i)}, \ldots, \xi_{\phi}^{(i)} \right]^T , \ T : \text{transpose} ,
\end{align*}
\]

\[
\begin{align*}
 \eta^{(i)} & = k_2^{(i)} z_2^{(i)} - \gamma^{(i)} , \ \gamma^{(i)} & = 2\pi (n - m) \eta^{(i)} / p - q^{(i)} ,
\end{align*}
\]

\[
\begin{align*}
 f^{(i)} & = \frac{1}{p} \int_0^p f^{(i)}(z) g^{(i)}(z) e^{2\pi (n - m) i / p} dz ,
\end{align*}
\]

\[
\begin{align*}
 q^{(i)} & = \frac{2 \pi}{p} \int_0^p \frac{f^{(i)}(z) d\left\{ \frac{g^{(i)}(z)}{dz} \right\} e^{2\pi (n - m) i / p} dz} ,
\end{align*}
\]

\[
\begin{align*}
 s^{(i)} & = \frac{1}{p} \int_0^p \left\{ \frac{f^{(i)}(z)}{g^{(i)}(z)} \right\}^2 e^{2\pi (n - m) i / p} dz ,
\end{align*}
\]

\[
\begin{align*}
 y^{(i)} & = (k_j \sin \theta_j + 2 \pi n_j / p) , \ m, n = (-N, \ldots, 0, \ldots, N) .
\end{align*}
\]

It is also important to notice in Eq.(9) that Fourier coefficients \( \eta_{\varepsilon}^{(i)}, \eta_{\eta}^{(i)} \) and \( \eta_{\phi}^{(i)} \) can be obtained without numerical integration. For example, \( e^{il \pi / p} = (e^{il \pi / p} + e^{-il \pi / p}) / 2 \), the analysis is made easy to put \( f^{(i)}(z) = 0 \) and \( g^{(i)}(z) = (e^{il \pi / p} + e^{-il \pi / p}) / 2 \). Therefore the range of applicability to periodic structures is much wider than that of other methods.[13]

However for the case of Eq.(2), \( f^{(i)}(z) \) or \( g^{(i)}(z) \) contain discontinuity such as the step function, so that \( (n - m) \eta^{(i)} \) does not converge, because \( \eta^{(i)} \) is \( O(1 / |n - m|) \), \( |n - m| \to \infty \) \( |\eta^{(i)}| \) is less than \( K/|n - m| \), where \( K \) is independent of \( |n - m| \), therefore the solution of Eq.(9) does not converge to the correct value [10]. To solve this difficulty in our method, the function containing the discontinuity is approximated by Fourier series of \( N_r \) terms

\[
\begin{align*}
f^{(i)}(z) \text{ or } g^{(i)}(z) = \sum_{n=-N}^{N} T\left\{ e^{il \pi \rho} \right\} e^{2\pi (n - m) i / p} . \tag{10}
\end{align*}
\]

\( N_r \) in Eq. (10) is related to the modal truncation number \( N \) by \( N = \sigma N_r \) (\( \sigma \gg 1 \)). For the step function case, we have obtained that \( \sigma = 1.5 \) is sufficient to get the proper solution when \( N \) and \( N_r \) are increased[9,10].

Substituting Eq.(10) into \( \eta_{\varepsilon}^{(i)}, \eta_{\eta}^{(i)} \) and \( \eta_{\phi}^{(i)} \) of Eq.(9), the electromagnetic fields using the solution of Eq.(9) in each layer are expressed as

\[
\begin{align*}
 S_x & = (d < x < 0) : \quad H^{(i)} = \sum_{l=-L}^{L} \left[ A_{\psi}^{(i)} e^{il \pi \rho} + B_{\psi}^{(i)} e^{il \pi \rho} \right] f^{(i)}(z) ,
\end{align*}
\]

\[
\begin{align*}
d_s = d / M , \ i = 1 \sim M ,
\end{align*}
\]

\[
\begin{align*}
 E^{(i)} = \left\{ -\jmath \varepsilon^{(i)}(z) \right\}_z 
\end{align*}
\]

\[
\begin{align*}
 f^{(i)}(z) = e^{\jmath \beta x \sin \theta_j} \sum_{n=-N}^{N} \psi_{\varepsilon}^{(i)} e^{2\pi (n - m) i / p} , \tag{12}
\end{align*}
\]
where $A^{(4)}_n$, $B^{(4)}_n$ are unknown coefficients to be determined by boundary conditions.

From the boundary conditions at $x = 0$, $x = -l \cdot d_\lambda$ ($l = 1 - M - 1$), and $x = -d$, we get the following homogeneous matrix equation in regard to $A^{(M)}$ by matrix algebra\cite{11}:

$$\mathbf{W} \cdot A^{(M)} = \mathbf{F},$$

where the elements of matrix $\mathbf{W}$ and $\mathbf{F}$ are obtained by reference\cite{13,14}, whose matrix order has been reduced to the modal truncation number $(2N + 1)$, but is independent of the numbers of layers rather than that of other methods\cite{13,14,15}.

The mode power transmission coefficients $|T^{(TM)}_n|^2$ is given by

$$|T^{(TM)}_n|^2 = \mathcal{R} \{ k^{(0)}_n \} |k^{(0)}_n|^2 \left( \varepsilon_\lambda / \varepsilon_\theta \right),$$

where superscript (TM) indicates TM wave case.

### 3. Numerical Analysis

We consider an elliptically layered medium for Eq.(2) in the grating region. The shapes of gratings are the elliptic cylinder in Eq.(1). In this case, we put $g(z) = 1$ and $f^{(0)}(z) = \varepsilon^{(0)}(z)$.

The values of parameters chosen are $\varepsilon_\lambda = \varepsilon_\theta = \varepsilon_0$, $d/p = 2/3$ and $\varepsilon_\lambda / \varepsilon_\theta = 3 [\varepsilon_0(0,z)/\varepsilon_0 = 1 \sim 3]$, because the aim of this paper is to provide inhomogeneous and homogeneous medium.

First, we consider the inhomogeneous case ($b = 0$ in Eq.(2)). Figures 2(a) and 2(b) show the convergence of the [0]th-mode
power transmission coefficients $|T_{n}^{(TM)}|^2$ and $|T_{n}^{(TE)}|^2$ versus $p/\lambda$ for the case of $\theta = 30^\circ$ with elliptical layered media. (a) TM wave, (b) TE wave.

Fig.6 $|T_{n}^{(TM)}|^2$ and $|T_{n}^{(TE)}|^2$ versus a normalized frequency $p/\lambda$ magnified view for $0.56 \leq p/\lambda \leq 0.64$ in Fig.5. (a) TM wave, (b) TE wave.

Fig.7 Mode power transmission coefficients $|T_{-1}^{(TM)}|^2$ and $|T_{-1}^{(TE)}|^2$ versus a normalized frequency $p/\lambda$ for the case of $-1$th-mode with elliptical layered media. (a) TM wave, (b) TE wave.

In general resonance occur at two particular angles, first is Wood's anomaly at

$$\theta_{n}^{TM} = \sin^{-1}(n/(p/\lambda + 1)), \quad n = \pm 1, \pm 2, \ldots \tag{15}$$

The second is the strong resonance due to the coupling with the $n$-th-mode at

$$\theta_{n}^{TM} = \sin^{-1}(\pm(p/\lambda + 1)/n), \quad n = \pm 1, \pm 2, \ldots \tag{16}$$

where $\beta$ is propagation constant in the free modes. For the case of Figure 4, $\theta_{n}^{TM}$ appears at $\theta_{n}^{TM} \approx 0^\circ$.

From Figure 3 and 4, we note the following features:

1. The minimum points ($\theta_{n}^{TM} = 9.23^\circ$, $12.59^\circ$ and $16.96^\circ$) of coupling resonance curve for TM wave moves toward larger
Fig. 8 Mode power transmission coefficients $|T_{0}^{(TM)}|$ and $|T_{0}^{(TE)}|$ versus incident angle $\theta_0$ for the case of [0]th-mode with elliptical layered media. (a) TM wave, (b) TE wave.

Fig. 9 Mode power transmission coefficients $|T_{0}^{(TM)}|$ and $|T_{0}^{(TE)}|$ versus a normalized frequency $p/\lambda$ for the case of [0]th-mode with elliptical layered media. (a) TM wave, (b) TE wave.

$\theta_0 \geq (\beta p/2\pi) > 1$ as $2a/d$ increases as well as TE wave for $\theta_0 = 10.44^\circ$, $15.70^\circ$ and $22.3^\circ$. This is attributed to the effect of grating shape, so that when the equivalent permittivity connection with the propagation constant $\beta$ in the grating region is larger at $2a/d$. increases.

For the $|T_{0}^{(TM)}|$, the discrepancies are large than that of $|T_{0}^{(TE)}|$ around $\theta_0 = 30^\circ$ because of Bragg angle at $p/\lambda = 1$. On the other hand, for the TE wave, $|T_{0}^{(TE)}|$ has also a symmetric shape around $\theta_0 = 30^\circ$ as well as $|T_{0}^{(TM)}|$. Therefore, the $\theta_0$ dependence at coupling resonance is more significant for the TE wave than that of TM wave.

It is interest the peak of $|T_{0}^{(TM)}|$ at $2a/d = 1.0$ moves toward at $\theta_0 = 90^\circ$ only for TE wave.

Figures 5(a) and 5(b) show $|T_{0}^{(TM)}|$ and $|T_{0}^{(TE)}|$ for various values of normalized frequency $(p/\lambda)$ at $\theta_0 = 30^\circ$ with the same parameters as in Fig. 3. Figures 6 give the magnified view for the range of $0.56 < p/\lambda < 0.64$ in Fig. 5 as the same scale both TM and TE wave. Figure 7(a) and 7(b) show the $-1$th-mode power transmission coefficients $|T_{-1}^{(TM)}|$ and $|T_{-1}^{(TE)}|$. Comparing the TM wave with the TE wave, from in Figure 5, 6 and 7, we note the following features for the effect of grating shape:

1. The characteristic tendencies are approximately same at $p/\lambda < 0.7$, but for about $p/\lambda > 0.7$, the effect of the grating shape is more significant for TE case.

2. The minimum points $(p/\lambda)^2 = 0.609$, 0.621 and 0.634 of coupling resonance curve for TM wave moves toward larger $p/\lambda$ for $\theta_0$, as $2a/d$ decreases as well as TE wave for $(p/\lambda)^2 = 0.570$, 0.590 and 0.611. This is attributed to the effect of grating shape, so that when the equivalent permittivity connection with the propagation constant $\beta$ is smaller $(p/\lambda)$ for $\theta_0 < 1$ as $2a/d$ increases. The coupling resonance curve for TM wave is sharper than that of TE wave. This is attributed to the effect of an attenuation constant in the free mode.[13]

3. $|T_{-1}^{(TM)}|$ and $|T_{-1}^{(TE)}|$ have a population as $2a/d$ decreases. But for $p/\lambda > 1$, the effect of the grating shape is more significant at $2a/d = 1.0$ than that of $2a/d = 1.0$.

Next, we consider the homogeneous case ($b = 0$ in Eq. (2)) comparison with the above inhomogeneous case at $2a/d = 1.0$.

Figures 8 and 9 show the $|T_{0}^{(TM)}|$ and $|T_{0}^{(TE)}|$ when $\varepsilon_z/\varepsilon_0$ is 2.5 and $\varepsilon_0/\varepsilon_z = 0.5$ and $1.3$ respectively under the same condition Fig. 5 and Fig. 6 for both TM and TE wave. From in Figure 8 and 9, comparing the inhomogeneous case with homogeneous case, we note that the following features:

1. The TM wave in Fig. 8(a), the characteristic tendencies for the inhomogeneous case are approximately the same at $\varepsilon_z/\varepsilon_0 = 2.5$. The minimum points $p/\lambda^2$ of coupling resonance curve are $7.33^\circ$ $(\varepsilon_0/\varepsilon_z = 2)$, $11.61^\circ$ $(\varepsilon_0/\varepsilon_z = 2.5)$ and $12.59^\circ$ $(\varepsilon_0/\varepsilon_z = 1.3)$ as the equivalent permittivity in
creases. On the other hand, for the TE wave Fig.8(b), the characteristic tendencies for the inhomogeneous case are approximately the same around $\theta_0 = 30^\circ$ at $\varepsilon_2/\varepsilon_0 = 2.0$. The minimum points $\theta_0^{c}$ of coupling resonance curve are $8.58^\circ (\varepsilon_2/\varepsilon_0 = 2)$, $13.63^\circ (\varepsilon_2/\varepsilon_0 = 2.5)$ and $15.62^\circ (\varepsilon_2/\varepsilon_0 = 1.5)$. It is also interest for TE wave the peak of $|T_{01}|^2$ only appears at near the $\theta_0 = 90^\circ$ in inhomogeneous case. This is attributed to the effect of inhomogeneous media. However it will be investigated more detailed numerical results for the distribution of power flow density, and for the case of guiding problem in the next time.

(2) For the TM wave in Fig.9(a), the characteristic tendencie for the inhomogeneous case are approximately the same about $p/\lambda = 1.15$ at homogeneous case $\varepsilon_2/\varepsilon_0 = 2.5$. The minimum points $(p/\lambda)_c$ of coupling resonance curve are $0.637 (\varepsilon_2/\varepsilon_0 = 2)$, $0.621 [\varepsilon(0, z)/\varepsilon_0 = 1-3]$ and $0.620 (\varepsilon_2/\varepsilon_0 = 2.5)$. For the TE wave in Fig.9(b), the characteristic tendencies in inhomogeneous case are approximately the same at $\varepsilon_2/\varepsilon_0 = 2.0$ about 0.75 $p/\lambda = 1.1$. The minimum points $(p/\lambda)_c$ of coupling resonance curve are $0.624 (\varepsilon_2/\varepsilon_0 = 2)$, $0.597 (\varepsilon_2/\varepsilon_0 = 2.5)$ and $0.590 [\varepsilon(0, z)/\varepsilon_0 = 1-3]$. The effects of the inhomogeneous media are more significant on the grating shape than those with homogeneous media.

4. Conclusions

In this paper, we have analyzed the scattering of electromagnetic waves by columnar dielectric gratings with elliptically layered media using improved Fourier series expansion method and multilayer method. Numerical results are given for the transmitted scattered characteristics for the case of incident angle and frequency for both TM and TE waves between inhomogeneous case and homogeneous case. It is shown that the influences on the grating shape are more significant for the inhomogeneous case than homogeneous case. Finally, this work was partially supported by a Nihon University Provisions Research Grants G00-079 in 2000. The authors also would like to thank Mr. Ryoji Terada at graduate student of Nihon University (now he is Fujitsu Ltd.) for help with making graphics in this work.

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