Paper

Short Optical Pulse Switching in Three-core Nonlinear Fiber Couplers

Non-member Zhongyuan Yu (Beijing University of Posts and Telecommunications)
Non-member Xiaoguang Zhang (Beijing University of Posts and Telecommunications)
Non-member Shuichao Yang (Beijing University of Posts and Telecommunications)

The propagation and switching of short optical pulses in a three-core nonlinear fiber coupler have been investigated with a variational method within the framework of the Lagrange equation. The analytical solutions were directly obtained from the coupled nonlinear Schrodinger equations. It is shown that the soliton switching behavior predicted by our analytical method agrees well with results from numerical analysis. In addition, the coupling length and the switching threshold of solitons in a nonlinear coupler were obtained.

Key words: nonlinear fiber coupler, optical switching, coupling length, switching threshold

1. Introduction

There is currently a growing interest in the optical waveguide switched directional couplers, for application in optical fiber telecommunication systems. Several all-optical switching devices have been proposed and analyzed in the literatures [1-4]. The non-linear directional coupler (NLDC) was a promising example, which exhibits all optical switching. Switching of short pulse in three-core and more generally in n-core [5,6] is of special interest because of the possibility of very high switching speeds (as high as the femto-second range). Switching is the process of energy redistribution among the cores for a given input. Since the transmission characteristics of this device are determined by the input power of the signal, by increasing the input power above a critical level the incoming signal can be switched from one core to the other depending on the coupler length.

The triple-core nonlinear couplers have two type of different structures, one is triangular symmetrical structure with three cores in an equilateral-triangle arrangement; another is transverse structure with three cores in a parallel equidistant arrangement.

In this paper, using the technique developed by Anderson and coworkers [7-9], we derived the three linearly coupled nonlinear Schrödinger equations (NLSEs), which govern the propagation of pulses in a nonlinear triple-core directional coupler (with the cores in a parallel arrangement), by using a variational method. We obtained the analytical solutions by putting trial functions into NLSEs and variation equations. We also obtained the numerical solutions directly from NLSEs using split-step Fourier transform method. It is shown that the soliton switching behavior predicted by our analytical method agrees well with results from numerical analysis. In addition, the coupling length and the switching threshold of soliton in a nonlinear coupler were obtained.

2. The Analysis of the Coupled Equations

In order to analyze short optical pulse switching behavior in the non-linear directional coupler, we use the approach based on the Lagrange equations of motion for a finite number of degrees of freedom. This technique has been successfully used in many dynamical problems for optical solitons. The Lagrangian of the transverse structure coupler system is

$$\begin{aligned} L &= \int_{-\infty}^{+\infty} L' d\tau \\ &= \int_{-\infty}^{+\infty} \Big[\frac{i}{2}\Big(u_1{}^*\frac{\partial u_1}{\partial \xi} - u_1\frac{\partial u_1{}^*}{\partial \xi} + u_2{}^*\frac{\partial u_2}{\partial \xi} - u_2\frac{\partial u_2{}^*}{\partial \xi} - u_3{}^*\frac{\partial u_3}{\partial \xi} - u_3\frac{\partial u_3{}^*}{\partial \xi}\Big) - \\ &\quad -\frac{1}{2}\left|\frac{\partial u_1}{\partial \tau}\right|^2 - \frac{1}{2}\left|\frac{\partial u_2}{\partial \tau}\right|^2 - \frac{1}{2}\left|\frac{\partial u_3}{\partial \tau}\right|^2 + \frac{1}{2}|u_1|^4 + \\ &\quad + \frac{1}{2}|u_2|^4 + \frac{1}{2}|u_3|^4 + \\ &\quad + K(u_2{}^* u_1 + u_2{}^* u_3 + u_2 u_1{}^* + u_2 u_3{}^*)\Big] d\tau \end{aligned} \qquad (1)$$

where u_k are mode field amplitudes in soliton units with core k, $k=1,2,3$. $\xi = z/L_D = z|\beta_2|/T_0^2$ is the normalized length and $\tau = t/T_0$ is time in soliton units, with pulse width T_0 and second order dispersion β_2. Here, $K=z_0/L_C$ is the coupling parameter with a normalization associated to the soliton period $z_0 = \pi L_D/2$ and the coupling length L_C, which is the linear coupling length required to complete transfer of energy from one core to the other.

The coupled NLSEs can been derived from the variation equations corresponding to the variation principle

$$\delta \int L\Big(u_k, u_k{}^*, \frac{\partial u_k}{\partial \xi}, \frac{\partial u_k{}^*}{\partial \xi}, \frac{\partial u_k}{\partial \tau}, \frac{\partial u_k{}^*}{\partial \tau}\Big)\partial\xi\partial\tau = 0 \qquad (2)$$

Varying Eq. (1) with respect to $u_k{}^*$ leads to the variation equation

$$\frac{\delta L}{\delta u_k{}^*} = \frac{\partial}{\partial \xi}\frac{\partial L}{\partial\big(\frac{\partial u_k{}^*}{\partial \xi}\big)} + \frac{\partial}{\partial \tau}\frac{\partial L}{\partial\big(\frac{\partial u_k{}^*}{\partial \tau}\big)} - \frac{\partial L}{\partial u_k{}^*} = 0 \qquad (3)$$

For $k=1,2,3$, we obtain

$$\begin{aligned} & i\frac{\partial u_1}{\partial \xi} + \frac{1}{2}\frac{\partial^2 u_1}{\partial \tau^2} + |u_1|^2 u_1 + K u_2 = 0 \\ & i\frac{\partial u_2}{\partial \xi} + \frac{1}{2}\frac{\partial^2 u_2}{\partial \tau^2} + |u_2|^2 u_2 + K(u_1 + u_3) = 0 \\ & i\frac{\partial u_3}{\partial \xi} + \frac{1}{2}\frac{\partial^2 u_3}{\partial \tau^2} + |u_3|^2 u_3 + K u_2 = 0 \end{aligned} \qquad (4)$$

There are linearly coupled NLSEs, which govern the propagation of pulses in the nonlinear triple-core directional coupler with parallel equidistant arrangement. In the equations, the group velocity dispersion and the nonlinearity of Kerr effect are considered, while the high order dispersion and fiber loss are neglected.

We assume the pulse profiles $u_k(\xi, \tau)$ take the forms

$$\begin{aligned} u_1(\xi,\tau) &= a\sqrt{r}\,\mathrm{sech}(r\tau)\cos^2[\theta(\xi)] \\ &\quad \exp(i\phi + i\psi(\xi) + iq\tau^2) \\ u_2(\xi,\tau) &= \frac{a\sqrt{r}}{\sqrt{2}}\mathrm{sech}(r\tau)\sin[2\theta(\xi)] \\ &\quad \exp(i\phi + iq\tau^2) \\ u_3(\xi,\tau) &= a\sqrt{r}\,\mathrm{sech}(r\tau)\sin^2[\theta(\xi)] \\ &\quad \exp(i\phi - i\psi(\xi) + iq\tau^2) \end{aligned} \tag{5}$$

where $\theta(\xi)$ is the coupling angle which determines the power coupling among the three cores, $\psi(\xi)$ is the relative phase and q is the chirp parameter. For these trial functions a and r are constants of motion. The parameters ϕ and q have no influence on the other parameters.

Substituting Eqs. (5) into Eq. (1) and varying the resulting Lagrangian with respect to each of the parameters a, r, θ, ψ and q. The reduced Lagrangian takes the form

$$\begin{aligned} L &= -2a^2\cos[2\theta(\xi)]\frac{\partial\psi}{\partial\xi} - \frac{2}{3}a^4 r\sin^2[2\theta(\xi)] + \\ &+ \frac{1}{4}a^4 r\sin^4[2\theta(\xi)] + \\ &+ \frac{4}{\sqrt{2}}Ka^2\sin[2\theta(\xi)]\cos[\psi(\xi)] - \\ &- \frac{1}{3}a^2r^2 + \frac{2}{3}a^4 r + q^2 B \end{aligned} \tag{6}$$

where

$$B = \int_{-\infty}^{\infty} - 2a^2 r\tau^2\mathrm{sech}^2(r\tau)d\tau \tag{7}$$

From Eq.(6), we obtain the Hamiltonian for the evolution of the dynamical system

$$\begin{aligned} H &= \frac{\partial\psi}{\partial\xi}\frac{\partial L}{\partial(\frac{\partial\psi}{\partial\xi})} + \frac{\partial\theta}{\partial\xi}\frac{\partial L}{\partial(\frac{\partial\theta}{\partial\xi})} - L \\ &= -2a^2\cos[2\theta(\xi)]\frac{\partial\psi}{\partial\xi} - L \\ &= \sin[2\theta(\xi)]\{\frac{2}{3}a^4 r\sin[2\theta(\xi)] - \\ &- \frac{1}{4}a^4 r\sin^3[2\theta(\xi)] - \frac{4}{\sqrt{2}}Ka^2\cos[\psi(\xi)]\} + \\ &+ \frac{1}{3}a^2r^2 - \frac{2}{3}a^4 r - q^2 B \end{aligned} \tag{8}$$

In general the problem of soliton switching in a fiber coupler can be considered to be a particular case of the trajectories of the Hamiltonian dynamic system. The equation H=*constant* determines the form of a dynamical trajectory on the plane (θ, ψ). As shown in Eqs. (5), the power transfer from pulse u_1 in core 1 into pulses u_2 and u_3 in cores 2 and 3 is described by a trajectory involving the evolution of θ.

Since we are mainly interested in studying the coupling and the switching of solitons in a nonlinear coupler, we consider that a soliton is initially launched into core 1 only:

$$\begin{aligned} u_1(0,\tau) &= A_1\mathrm{sech}(A_1\tau) \\ u_2(0,\tau) &= 0 \\ u_3(0,\tau) &= 0 \end{aligned} \tag{9}$$

The initial condition is then $\theta=0$ in Eq. (5). In this case the dynamical evolution starts at $H=0$ and will proceed along a trajectory with $H=0$. According to Eq. (8), there are two possible equations satisfying this requirement

$$\begin{aligned} &\frac{1}{3}a^2r^2 - \frac{2}{3}a^4 r - q^2 B = 0 \\ &\sin[2\theta(\xi)]\{\frac{2}{3}a^4 r\sin[2\theta(\xi)] - \frac{1}{4}a^4 r\sin^3[2\theta(\xi)] - \\ &- \frac{4}{\sqrt{2}}Ka^2\cos[\psi(\xi)]\} = 0 \end{aligned} \tag{10}$$

The first equation shows that the chirp parameter is

$$q^2 = \frac{1}{3B}a^2 r(r - 2a^2) \geq 0 \tag{11}$$

The relation can be obtained

$$a\sqrt{r}\frac{1}{r} \geq \frac{1}{\sqrt{2}} \tag{12}$$

where $A = a\sqrt{r}$ is the amplitude, and $\tau_0=1/r$ is the of half pulsewidth at $1/e$ point, respectively. We have $A\tau_0 \geq \frac{1}{\sqrt{2}}$ now. When $A\tau_0 = \frac{1}{\sqrt{2}}$, this is adiabatic propagation case in which $q=0$ is required. When $A\tau_0 > \frac{1}{\sqrt{2}}$, it is non-adiabatic propagation case in which $q\neq 0$. In fact, the short pulses propagation and the switching behavior in a nonlinear coupler is greatly influenced by the initial chirp.

If we take the variation of the Lagrangian (6) with respect to $\psi(\xi)$ according to

$$\frac{\partial L}{\partial\psi} - \frac{\partial}{\partial\xi}\left(\frac{\partial L}{\partial(\frac{\partial\psi}{\partial\xi})}\right) = 0 \tag{13}$$

which implies that

$$\begin{aligned} &\sin[2\theta(\xi)] = 0 \\ &\frac{\partial\theta}{\partial\xi} = -\frac{K}{\sqrt{2}}\sin[\psi(\xi)] \end{aligned} \tag{14}$$

According to Eqs.(8) and (10) we have two solutions with $H=0$. The first one is just the former expression of Eq. (14). And the second one is

$$\cos[\psi(\xi)] = \frac{a^2 r}{6K'}\sin[2\theta(\xi)](1 - \frac{3}{8}\sin^2[2\theta(\xi)]) \tag{15}$$

where $K' = K/\sqrt{2}$. The second solution is special interest to our study. Substituting Eq.(15) into (14), we get

$$\frac{d\theta}{\sqrt{1 - (\frac{a^2 r}{6K'})^2\sin^2[2\theta(\xi)] + \frac{3}{8}(\frac{a^2 r}{6K'})^2\sin^4[2\theta(\xi)]}} = -K' d\xi \tag{16}$$

3. Results and discussions

We can define the transmission T_k as follows:

$$T_k = \frac{\int_{-T}^{T}|u_k(\xi_L,\tau)|^2 d\tau}{\int_{-T}^{T}|u_1(0,\tau)|^2 d\tau} \tag{17}$$

Using Eqs. (5) directly, one can express the power transmission coefficients as

$$\begin{aligned} T_1 &= \cos^4[\theta(\xi_L)] \\ T_2 &= \frac{1}{2}\sin^2[2\theta(\xi_L)] \\ T_3 &= \sin^4[\theta(\xi_L)] \end{aligned} \tag{18}$$

In the following, we will analyze the characteristics of short pulses in the triple core directional coupler.

First, we used so-called semi-numerical method to calculate the coupling length by solving Eq. (16) obtained with variation method. The Eq.(16) is readily solved by a fourth Runge-Kutta method. We found that: when $\xi_L = 2.237/K$, $\theta(\xi_L) = \pi/2$. So ξ_L is considered as the coupling length of the triple-core directional coupler.

Second, we also used entirely numerical method to simulate switching characteristics of three-core directional fiber coupler by solving NLSEs using split-step Fourier transform method. Fig.1 shows the periodic transference of pulses among three channels of the coupler with $K=1$ and with functions in Eqs. (9) as input signals.

Fig.2(a) and (b)show the numerical results of transmission coefficients as the function of distance with $K=1$, and $K=0.5$, respectively. The spatial period of T_2 is a half of T_1. This shows a good agreement with the analytical results from Eqs. (18). Especially, from Fig.2, we found the coupling length $\xi_L=2.2/K$, which nearly the result of that by semi-numerical method. It is different from the case of continuous wave in twin-core coupler, in which $\xi_L= \pi/2K$.

Fig.3 shows the apparent switching property in the three-core coupler. The threshold power is about 3.8. When input power is greater than threshold, the energy would remain in core 1, whose case is also show in Fig.4.

In section 2, we have obtained an analytical result from Eqs.(12) that: when chirp parameter $q \neq 0$, the non-adiabatic propagation of pulses will take place, which would distort the switching property. The numerical results in Fig.5 confirm this judgment.

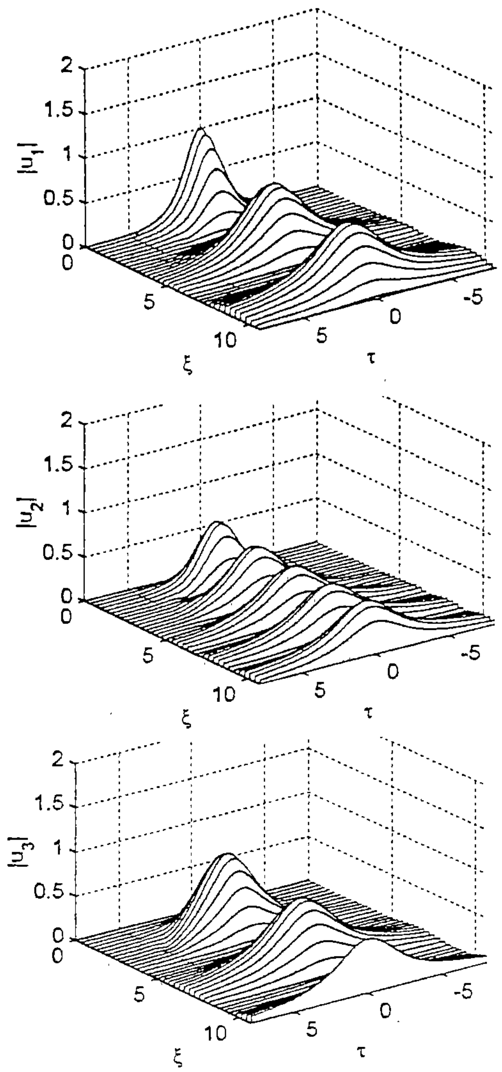

Fig.1. The evolution of pulse transmitted in three-core directional fiber coupler, with parameters $K=1, a=1$ and $r=1$.

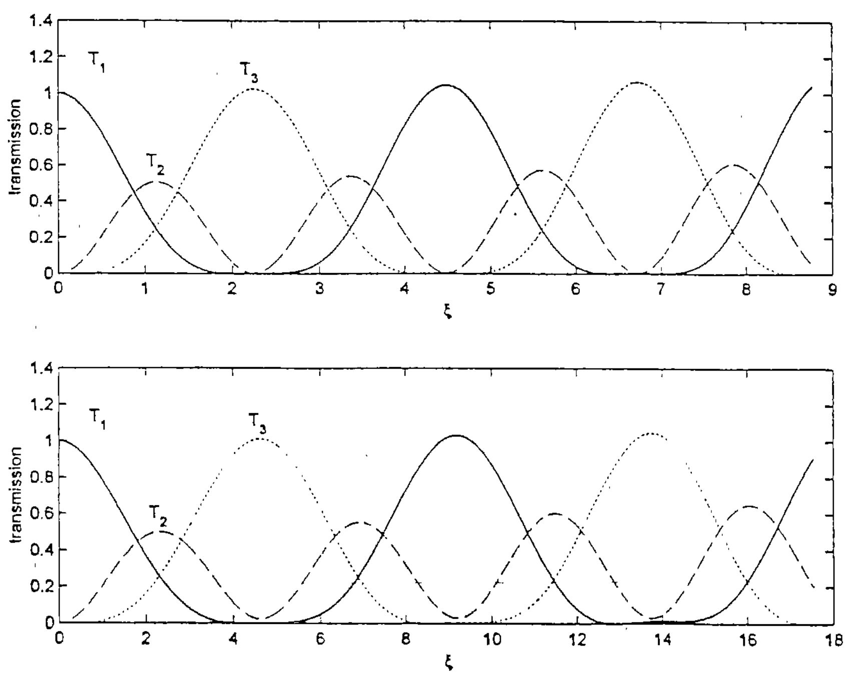

Fig.2. The transmission coefficients versus distance in three channels. The spatial periods of T_1 and T_2 are $2.2/K$ (a) $K=1$; (b) $K=0.5$

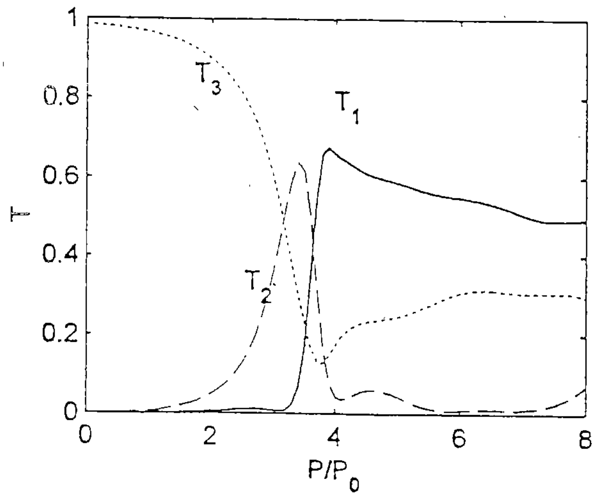

Fig.3. The transmission coefficients versus input power

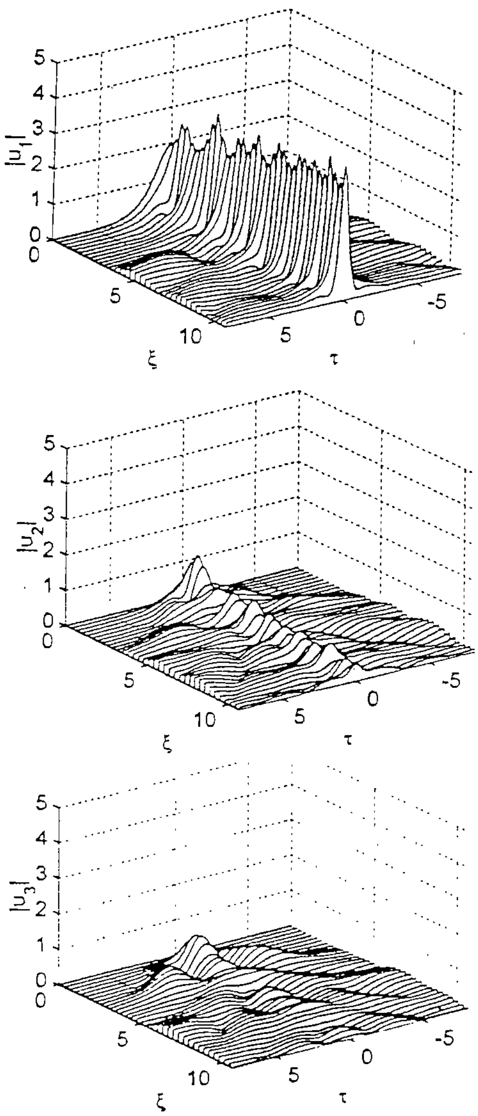

Fig.4. The transmission case when input power greater than threshold with parameters $K=1$, $a=\sqrt{3.8}$ and $r=1$

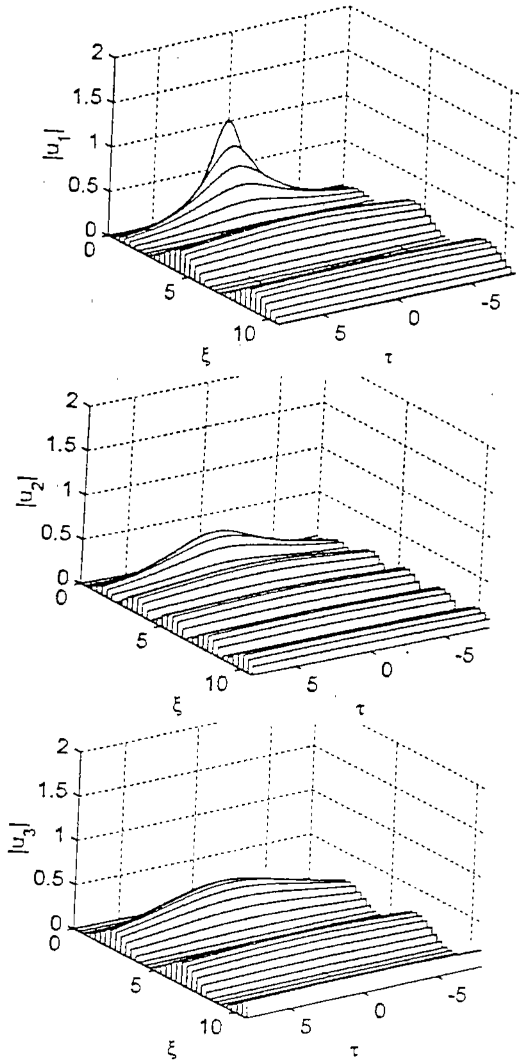

Fig.5. The transmission case when the input pulses have initial chirp. $K=1$, $a=1$, $r=1$ and $q=-1$

(Manuscript received January 22,2001, revised July 16,2001)

This work is partly supported by the National Natural Science Foundation of China (60077026), Foundation for University Key Teacher by the Ministry of Education, and BUPT Science Foundation

References

(1) T. P. Valkering, J. van Honschoten, H. J. W. M. Hoekstra: Opt Commu. 159, 215-220(1999)
(2) T. P. Valkering, P. T. de Boer, H. J W M Hoekstra: Physica D, 123, 223-234(1998)
(3) P L. Chu, B A. Malomed, G D. Peng: J pt. Soc Am. B, 10, 1379-1385(1993)
(4) G P Agrawal: Nonlinear Fiber Optics, Second Edition, Academic Press, New York, 1995
(5) C Mapalagama, R. T. Deck: J. Opt. Soc. Am B, 9, 2258-2264(1992)
(6) N N Akhmediev, A. V Buryak J Opt. Soc Am B, 11, 804-809(1994)
(7) D Anderson, M. Lisak, T. Reichel: J. Opt Soc. Am B, 5, 207(1988)
(8) Ajit Kumar, A. Kumar: Opt. Quantum Electro 30, 39-47(1998)
(9) P. M. Ramos, C. R. Paiva: IEEE J Quantum Electron. 35, 983-989(1999)

zhongyuan Yu (non-member) received the M. S. degree and Ph. D. degree in theoretical physics from Lanzhou University in Lanzhou, China, in 1982 and 1997 She is presently a Professor in Physics at Department of Physics, School of Science, Beijing University of Posts and Telecommunications in Beijing, China. Now her research interests include optical fiber communication and nonlinear optics. She is one of Chinese Physics Society members.

Xiaoguang Zhang (non-member) received the B. S. degree and M.S. degree in Physics from Peking University in Beijing, China, in 1985 and 1988. He is currently a Professor in Physics at Department of Physics, School of Science, Beijing University of Posts and Telecommunications in Beijing, China. His research interests include nonlinear optics, optical electronics and optical solitoncommunications. Prof. Zhang received the Science and Technology Progress Award from China Ministry of Information Industry in 1999 for his work in the project entitled "The Experimental and Theoretical Research on Optical Soliton Transmissions". He is a member of Chinese Physical Society, Chinese Optical Society and a senior member of Communications Institute of China

Shuichao Yang (non-member) received the bachelor'degree in Management Information System from Beijing University of Posts and Telecommunications, , in 1999 . He is currently a post graduate in Physics Electronics at Department of Physics, School of Science. Beijing University of Posts and Telecommunications in Beijing, China. His research interests include nonlinear optics, optical communications, and optical soliton communications.