Paper

A Study of Subsidiary Techniques Based on Combined Iterative Method for a Nonlinear Transients Analysis

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This paper presents some extensions for Combined Iterative Method (CIM) based on Modified Predictor-Corrector Iteration (MPCI) and Newton Raphson Iteration (NRI) to extend Electromagnetic Transient Simulation for including nonlinear elements in an electrical network solution. It is important to adopt an iterative method optimally in CIM to improve convergency, because a solution obit depends on a multi-dimensional plane between initial and real solutions. An effective construction of MPCI and Jacobian matrices based on a multi-dimensional solution with Dommel's method, and an effective expression of MPCI and NRI on EMTP-Type simulators have been also proposed to realize a high accuracy, high stability and saving a calculation time simultaneously for any kinds of circuits. Some examples are demonstrated using the proposed algorithm, and the results are compared with those by a basic nodal conductance approach (NCA), EMTDC/PSCAD solutions and measured results. The comparison proves the validity of the proposed methods, which is applicable to EMTP-Type simulators to solve a nonlinear circuit.

Keywords: Combined Iterative Method, Optimum Adoption, Optimum handling, Unified Expression, EMTP-type Simulator.

1. INTRODUCTION

A nodal-conductance approach (NCA) with Dommel's method [1] is a basis of many Electro-Magnetic Transients Programs such as EMTP, ATP [2] and PSCAD/EMTDC [3] for linearized circuits. In this paper, the simulators based on Dommel's method are defined as an EMTP-type simulator.

Previous works in a nonlinear transients analysis were based on Newton-Raphson method in SPICE [4] and non-iterative method [2] in ATP, but each has restrictions of the number and configuration of nonlinear elements, and involves problems such as instability, low efficiency, inaccuracy in convergency. A linear interpolation [5, 6] can represent simple nonlinear devices using piecewise linear approximations with small error caused by the piecewise modeling and the arrangement of nonlinear elements, and results in a stable and an accurate calculation for an arbitrary number of the nonlinear devices. However, it is difficult to express all kinds of nonlinear devices in this way, and the method has comparatively big error in simulating an oscillation circuit. Therefore it is important to complete a more general, accurate and stable solution method and representation of the nonlinear elements.

The combined iterative method (CIM) [7] is a useful scheme for a transient calculation of nonlinear circuits, and very expandable on EMTP-type simulator. CIM consists of Modified Predictor-Corrector Iteration (MPCI) [9] and Newton Raphson Iteration (NRI) algorithms. It is important to adopt an iterative method optimally in CIM to improve convergency, because a solution obit depends on a multi-dimensional plane between initial and real solutions. An effective construction of MPCI and Jacobian matrices based on a multi-dimensional solution with Dommel's method, and an effective expression of MPCI and NRI on an EMTP-Type simulator are also proposed to realize high accuracy, high stability and saving calculation time simultaneously for any kinds of circuits.

Some examples are demonstrated using the proposed algorithm, the calculated results are compared with those by a basic NCA, EMTDC/PSCAD solutions and measured results. The comparison proves the validity of the proposed methods in a simulation of a nonlinear circuit by EMTP-Type simulators.

2. OPTIMUM ORDERING OF NODES

When the circuit includes some nonlinear elements, which are expressed as nonlinear conductances, a nodal-conductance matrix **G** can depend on many factors such as an instantaneous node voltage solution v(t) and branch current i(t). That is to say, the retriangulation of **G** is required whenever the factors change at a time step or an iterative step. In MPCI, the retriangulation of **G** is not required at each time step and each iterative step, because a nonlinear element is expressed as a piecewise linear conductance and a variable current injection. An optimum ordering of nodes in G is illustrated in Fig.1.

When one or more nonlinear elements operate at that time step, the portion of the matrix from the smallest number of nodes involved in the operating nonlinear elements to the end must be retriangulated, but the execution is complied with the scheme of optimum handling introduced at chapter 5.

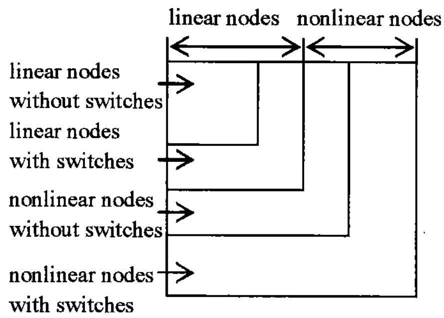


Fig.1 Optimum ordering of nodes in G

3. ITERATIVE EQUATIONS

CIM employs two kinds of iterations efficiently. One of the iterative methods in CIM is MPCI method. The solution $\mathbf{v}^{(0)}(t)$ of the following equation gives the first estimation of the iteration (prediction).

$$\mathbf{G}(t)\mathbf{v}^{(0)}(t) = \mathbf{J}(t) + \mathbf{J}(t, \mathbf{v}(t-\Delta t)) \quad (1)$$

The improved solutions are repeatedly obtained by the following iteration scheme (correction).

$$\mathbf{G}(t)\mathbf{v}^{(k)}(t) = \mathbf{J}(t) + \mathbf{J}(t, \mathbf{v}^{(k-1)}(t)) \quad (2)$$

where $k = 1, 2, \ldots$: the number of iterations.

A typical problem gives N functional relations to be zeroed, which involves variables x_i, $i = 1, 2, \ldots, N$.

$$F_i(x_1, x_2, \ldots, x_N) = 0 \qquad i = 1, 2, \ldots, N. \tag{3}$$

Each of the functions F_i in eq. (3) can be expanded in Taylor series, and the following equation can be derived as a circuit equation for NRI in a matrix notation [7].

$$\mathbf{Jc} \cdot \delta\mathbf{x} = -\mathbf{F} \quad \left(Jc_{ij} = \frac{\partial F_i}{\partial x_j} \right) \tag{4}$$

where $\mathbf{x}$: vector of values x_i, $\mathbf{F}$: vector of functions F_i.

When the example case which includes a nonlinear element between node i and j is illustrated, Jacobian matrix $\mathbf{Jc}$ can be expressed as a following form [7].

$$\frac{\partial \mathbf{F}}{\partial \mathbf{v}} = \begin{bmatrix} G_{11} & \cdots & G_{1i} & G_{1j} \\ \vdots & \ddots & \vdots & \vdots \\ G_{i1} & \cdots & \left(G_{Lii} + \frac{df}{dv}\right) & \left(G_{Lij} - \frac{df}{dv}\right) \\ G_{j1} & \cdots & \left(G_{Lji} - \frac{df}{dv}\right) & \left(G_{Ljj} + \frac{df}{dv}\right) \end{bmatrix} = \mathbf{Jc} \tag{5}$$

where G_L: sum of conductances of linear elements, $i = f(v)$: $v - i$ characteristic of a nonlinear element.

4. UNIFIED EXPRESSION

The feature of NRI procedures makes it difficult to save the calculation time of a nonlinear circuit, because an optimum handling explained in chapter 5 can not be applied. Therefore, it is important that MPCI and NRI equations eqs. (1), (2) and (4) can be expressed as a same form using Dommel's method and CIM basis.

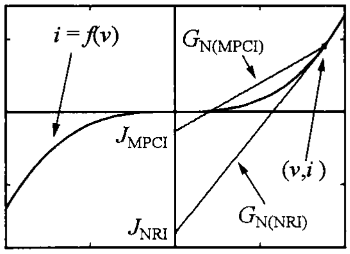
$\mathbf{F} = \mathbf{Gv} - \mathbf{J}$ is substituted for eq. (4), and the following equation can be derived.

$$\mathbf{Jc} \cdot \delta\mathbf{v} + \mathbf{Gv} = \mathbf{J} \tag{6}$$

As well as the derivations of eq. (5) [7], an example circuit, which includes a nonlinear element between node i and j, is illustrated for the verification of an unified expression of iterative equations. When a nonlinear element is composed of a piecewise linear or nonlinear conductance and a nonlinear current injection, the conductance matrix can be expressed as follows.

$$G(t) = \begin{bmatrix} G_{11} & \cdots & G_{1i} & G_{1j} \\ \vdots & \ddots & \vdots & \vdots \\ G_{i1} & \cdots & (G_{Lii} + G_N) & (G_{Lij} - G_N) \\ G_{j1} & \cdots & (G_{Lji} - G_N) & (G_{Ljj} + G_N) \end{bmatrix} \tag{7}$$

Because the conductance G_N of a nonlinear element is explained as in Fig. 2, it means that G_N in eq. (7) is equivalent to df/dv in eq. (5) for NRI. Therefore, in CIM procedures, both MPCI and NRI equations can be constructed in the same eq. (1) and (2), but it should be noticed that the expression of MPCI conductance, of which a nonlinear element is composed, is piecewise linear, not nonlinear as NRI conductance as shown Fig.2.



$i = G_{N(NRI)} v + J_{NRI}$ *for NRI Method*

$i = G_{N(MPCI)} v + J_{MPCI}$ *for MPCI Method*

Fig.2 Difference of MPCI and NRI conductance

5. HANDLING OF MATRIXES

5.1 The advantage of LU decomposition

Matrix $\mathbf{A}$ can be written as a product of two matrices $\mathbf{L}$ and $\mathbf{U}$ (lower and upper triangular matrices respectively). When we solve the linear set by the following decomposition [9],

$$\mathbf{A} \cdot \mathbf{x} = (\mathbf{L} \cdot \mathbf{U}) \cdot \mathbf{x} = \mathbf{L} \cdot (\mathbf{U} \cdot \mathbf{x}) = \mathbf{L} \cdot \mathbf{y} = \mathbf{b} \tag{8}$$

$\mathbf{L} \cdot \mathbf{y} = \mathbf{b}$ in eq. (8) is solved to get the vector $\mathbf{y}$. Then $\mathbf{U} \cdot \mathbf{x} = \mathbf{y}$ is solved to get the real solution vector $\mathbf{x}$.

An advantage of breaking up one linear set into two successive ones is that the solution of a triangular set of equation is quite trivial. Thus, eq. (8) can be solved by forward and backward substitution.

$$y_1 = \frac{b_1}{L_{11}}, \quad y_i = \frac{1}{L_{ii}} \left[b_i - \sum_{j=1}^{i-1} L_{ij} y_j \right] \quad i = 2, 3, \cdots, N \tag{9}$$

$$x_N = \frac{y_N}{U_{NN}}, \; x_i = \frac{1}{U_{ii}} \left[y_i - \sum_{j=i+1}^{N} U_{ij} x_j \right] \quad i = N-1, \cdots, 1 \tag{10}$$

The most important advantage for EMTP-type simulators is that once we have the LU decomposition of $\mathbf{A}$, we can solve with many right-hand sides $\mathbf{b}$ without reconstructions of $\mathbf{A}$.

5.2 Crout's algorithm for LU decomposition

Very efficient procedure is Crout's algorithm [9], which quite trivially solves $\mathbf{A} = \mathbf{L} \cdot \mathbf{U}$ by just arranging the equations in the following order.

a) Set $L_{ii} = 1$, $i = 1, \ldots, N$.

b) For each $j = 1, 2, 3, \ldots, N$: First, for $i = 1, 2, \ldots, j$, use the following equation to solve for U_{ij},

$$U_{ij} = A_{ij} - \sum_{k=1}^{i-1} L_{ik} U_{kj}, \tag{11}$$

Second, for $i = j+1, j+2, \ldots, N$, use eq. (12) to solve for L_{ij},

$$L_{ij} = \frac{1}{U_{jj}} \left(A_{ij} - \sum_{k=1}^{j-1} L_{ik} U_{kj} \right) \tag{12}$$

Note that the both procedures have to be done before going on to the next j.

It is obvious from the above that L and U on the right-hand side of eqs. (11) and (12) are already determined by the time when those are needed. Every A_{ij} is used only once and never again, i.e. the corresponding L_{ij} or U_{ij} can be stored in the location where the A_{ij} was used to occupy. In brief, Crout's method fills in the combined matrix of $\mathbf{L}$ and $\mathbf{U}$ by columns from left to right, and within each column from top to bottom.

5.3 Relations between LU decomposition and Norton-Thevenin equivalent for a nonlinear matrix equation

For an efficient and fast construction of the nonlinear conductance of MPCI and Jacobian matrix of NRI, an optimum handling method using relations between LU decomposition and Norton-Thevenin equivalent for nonlinear matrix equations as in Fig.3 is proposed. Once the equivalent circuit is determined, it is very efficient and fast to simulate a nonlinear circuit, because the part of linear equivalent circuit as in Fig.3 needs not to be changed during the whole time steps. To define this equivalent circuit automatically for all kinds of electromagnetic circuits, a partial LU decomposition and a partial forward and backward substitution based on Crout's algorithm are proposed.

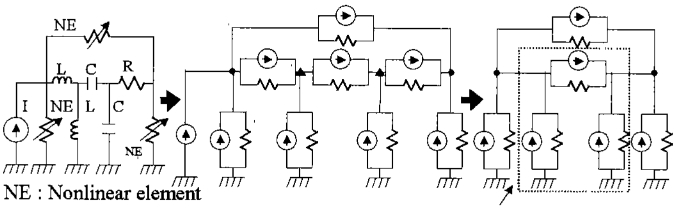


Fig. 3 Equivalent circuit for a nonlinear circuit

Node 1, 2,..., M are linear nodes and node M+1,..., N are nonlinear nodes on a general nonlinear conductance matrix.

a) Set $L_{ii} = 1, i = 1,\ldots,N$.

b) For each $j = 1,2,\ldots,M$: First, for $i = 1,2,\ldots,j$, use eq. (11) to set U_{ij}. Second, for $i = j+1, j+2,\ldots,N$ use eq. (12) to set L_{ij}.

c) For $j = M+1$: For $i = 1,2,\ldots,M$, use eq. (11) for U_{ij}.

From the above steps a), b) and c), the general nonlinear conductance matrix **A** is renewed to the following matrix form.

$$\begin{bmatrix} U_{11} & \cdots & U_{1M} & U_{1M+1} & \cdots & A_{1N} \\ \vdots & \ddots & \vdots & \vdots & & \vdots \\ L_{M1} & \cdots & U_{MM} & U_{MM+1} & \cdots & A_{MN} \\ L_{M+11} & \cdots & L_{M+1M} & A_{M+1M+1} & \cdots & A_{M+1N} \\ \vdots & & \vdots & \vdots & \ddots & \vdots \\ L_{N1} & \cdots & L_{NM} & A_{NM+1} & \cdots & A_{NN} \end{bmatrix} \quad (13)$$

It should be noticed that renewed elements $U_{11}, L_{21}, L_{31},\ldots, L_{N1}, U_{12}, U_{22}, L_{32},\ldots, L_{NM},\ldots, U_{M\text{-}1M+1}, U_{MM+1}$ are constructed from the only linear elements of **A**. Therefore, the renewed elements are constant for the whole simulation time steps and is performed only once before advancing to the time step loop.

d) The linear part of **y** ($y_1, y_2,\ldots, y_M$) in eq. (8) is calculated from the renewed elements $L_{11}, L_{21}, L_{31},\ldots, L_{N1}, L_{22}, L_{32},\ldots, L_{N2},\ldots, L_{NM}$ by use of eq. (9), and also this part is constant and is performed only once before advancing to the time step loop.

e) After the LU decomposition of nonlinear parts of **A**, the nonlinear part of **y** ($y_{M+1}, y_{M+2},\ldots, y_N$) and the nonlinear part of **x** ($x_N, x_{N-1},\ldots, x_{M+1}$) are calculated iteratively by CIM procedure. However, it should be noticed that the decomposition of nonlinear parts of **A** occurs only when the conductance $G_{N(MPCI)}$ or $G_{N(NRI)}$ in Fig.2..

f) After convergence, the nonlinear part of **x** ($x_M, x_{M-1},\ldots, x_1$) is calculated, and we can proceed to next time step.

During iterative terms, the following equations ($\mathbf{L_n}\cdot\mathbf{y_n} = \mathbf{b_n}$ and $\mathbf{U_n}\cdot\mathbf{x_n} = \mathbf{y_n}$) are executed for the nonlinear conductance matrix as a partial substitution method.

$$\begin{bmatrix} L_{M+1M+1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ L_{NM+1} & \cdots & L_{NN} \end{bmatrix} \begin{bmatrix} y_{M+1} \\ \vdots \\ y_N \end{bmatrix} = \begin{bmatrix} b_{M+1} - \sum_{i=1}^{M} L_{M+1i} y_i \\ \vdots \\ b_N - \sum_{i=1}^{M} L_{Ni} y_i \end{bmatrix} \quad (14)$$

$$\begin{bmatrix} U_{M+1M+1} & \cdots & U_{M+1N} \\ \vdots & \ddots & \vdots \\ 0 & \cdots & U_{NN} \end{bmatrix} \begin{bmatrix} x_{M+1} \\ \vdots \\ x_N \end{bmatrix} = \begin{bmatrix} y_{M+1} \\ \vdots \\ y_N \end{bmatrix}$$

If $\mathbf{L_n}\cdot\mathbf{U_n}$ is symmetrical, eq. (14) shows that every kind of nonlinear circuits can be expressed as the equivalent circuit having only (N - M) nonlinear nodes as in Fig.4 which does not include any linear nodes connected with only linear elements. Actually in all the kinds of electrical circuits on an EMTP-type simulator, $\mathbf{L_n}\cdot\mathbf{U_n}$ become symmetrical (see A.1 for details and verification). From eq. (14), an equivalent conductance matrix and an equivalent current injection vector can be $\mathbf{L_n}\cdot\mathbf{U_n}$ and $\mathbf{b_n}$ respectively.

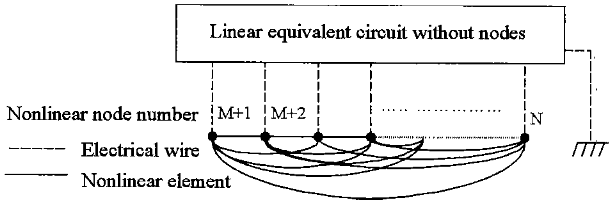


Fig. 4 Relation between LU decomposition and a nonlinear equivalent circuit

A comparison of calculation times required for LU decomposition, backward and forward substitution is given in Table 1 for the normal and partial method, and the comparison is based on the normal method. A difference of the calculation times heavily depends on the ratio of the number of nonlinear nodes and linear nodes. From Table 1, the proposed partial method is very efficient for a nonlinear circuit simulation. This method seems to be more powerful with the optimum sparse matrix method adopted in an EMTP-type simulator.

Table 1 Comparison of the calculation times

	Normal method	Partial method
Fig.6	1.00	0.667
Fig.9	1.00	0.704
Features	slow	fast

6. EFFECTIVE ADOPTION OF ITERATIONS

NRI gives a very efficient means of converging to a root, if a sufficiently good initial value can be guessed. Therefore, it is very important to exploit how the improved solution is calculated from a first estimated solution which is not a sufficiently good initial value. So far, an effective adoption of iterations has not been discussed sufficiently, only methods of an iteration for nonlinear circuit have been researched [10].

CIM consists of two kinds of iterative methods, the most important feature of MPCI is very stable if a sufficiently good initial value cannot be guessed, but the speed of convergency heavily depends on the direction of each nonlinear conductance and a multi-dimensional plane between an initial solution and a real one. On the other hands, the most important feature of NRI is that the speed of convergency is very fast, but the stableness heavily depends on the multi-dimensional plane. Therefore, a standard of a good initial value need to be defined.

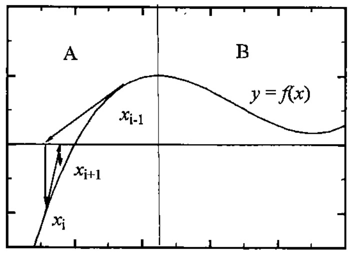


Fig. 5 Process of convergency

When the sign of first and second derivatives of nonlinear functions is not changed on the multi-dimensional plane between an initial and real solutions, the initial value is regarded as sufficiently good (region A in Fig.5), but if the sign is changed during convergency, MPCI method is employed to approach to a real solution (region B in Fig.5). Namely, if the sign of $(x_{i+1} - x_i)\times(x_i - x_{i-1})$ changes their own sign during their iterations successively, MPCI method replaces NRI one from the next iterative term in CIM procedures. It means that a solution dosen't exist close to the solution x_i. If the sign of $(x_{i+1} - x_i)\times(x_i - x_{i-1})$ doesn't change successively, NRI method has been used until convergency.

7. EXAMPLES

7.1 Dynamic arc

(a) Primary and secondary arc and re-ignition voltage

An arc expression used in this example has been proposed in Ref. [11], consists of following equations.

$$\frac{dg}{dt} = \frac{1}{T}(G - g) \quad , G = \frac{|i|}{Vl} \tag{15}$$

$$\left.\begin{array}{l} V = 15 \quad \text{V/cm} \\ T = \dfrac{\alpha I_p}{l} \end{array}\right\} \textit{for primary arc} \tag{16}$$

$$\left.\begin{array}{l} V = 75.0 \times I_s^{-0.4} \quad \text{V/cm} \\ T = \dfrac{\beta I_s^{1.4}}{l(t_r)} \end{array}\right\} \textit{for secondary arc} \tag{17}$$

$$l(t_r)/l_o = \begin{cases} 1.0 & t_r \le t'\text{s} \\ a \times t_r + b & t_r > t'\text{s} \end{cases} \tag{18}$$

where l: arc length, G: static arc conductance, T: arc time constant, i: arc current, V: arc voltage gradient, $[a, b, t']$: constants heavily depend on the velocity around the arc column, α : 2.85×10^{-5}, I_p : peak primary arc current calculated when the fault is assumed as a bolted fault, $\beta = 2.51\times10^{-3}$ for the small value of the arc current, t_r : time from the initiation of secondary arc.

The re-striking phenomenon can be modeled using the re-ignition voltage. Based on available experimental results, an empirical expression for this voltage is determined in the following form :

$$V_r(t_r) = \{c(I_s)\cdot T_e + d\}(t_r - T_e)h(t_r - T_e)\times 10^3 \quad [\text{V/cm}]$$

$$h(t) = \begin{cases} 0 & t < 0 \\ 1 & t \ge 0 \end{cases} \tag{19}$$

where V_r : arc re-ignition voltage, T_e : time from the initiation of secondary arc to the last current zero crossing, $c(I_s) = 1620/(2.15 + I_s)$, $d = 5.0$. To model arc re-striking, the re-ignition voltage is compared with the arc branch voltage. If $v_{arc} \ge V_r$, the arc re-ignition takes place. If $v_{arc} < V_r$ over a period between two consequent current zero crossings, the secondary arc is extinguished.

(b) Arc simulation with CIM

The arc model is expressed as the parallel connection of a constant conductance and a nonlinear current source to avoid reconstructing the conductance matrix G for MPCI. This expression is very efficient to simulate not only this example case, but also complex systems for arc, because an arc characteristic dose not have an abrupt change such as a power electronics model.

The constant conductance of the proposed arc model is evaluated in the following equation:

$$G = \frac{I_p}{Vl}, or \frac{I_s}{Vl} \tag{20}$$

The equation representing a conductance of the arc can be derived from eq. (15) using the trapezoidal rule and MPCI rule of the first iteration.

$$g_{(0)}^{(0)}(t) = g(t - \Delta t)\frac{2T - \Delta t}{2T + \Delta t} + \frac{2\Delta t}{2T + \Delta t}\frac{|i(t - \Delta t)|}{Vl} \tag{21}$$

After the second step, the time varying arc conductance g is renewed in the following equation:

$$g_{(k)}^{(n)}(t) = g(t - \Delta t)\frac{2T - \Delta t}{2T + \Delta t} + \frac{2\Delta t}{2T + \Delta t}\frac{\left|g_{(k)}^{(n-1)}(t)\, v(t)\right|}{Vl} \tag{22}$$

where n : iteration times in arc model, k : iteration times in the main program (combined iteration method). These iterative methods are applied to the secondary arc as well as the primary arc.

(c) Synthetic test circuit as an Example

The example shows the correctness of the arc simulation and some advantages. A synthetic test circuit, shown in Fig. 6 is used

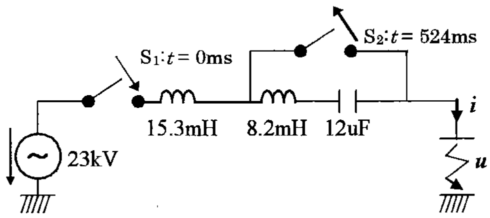
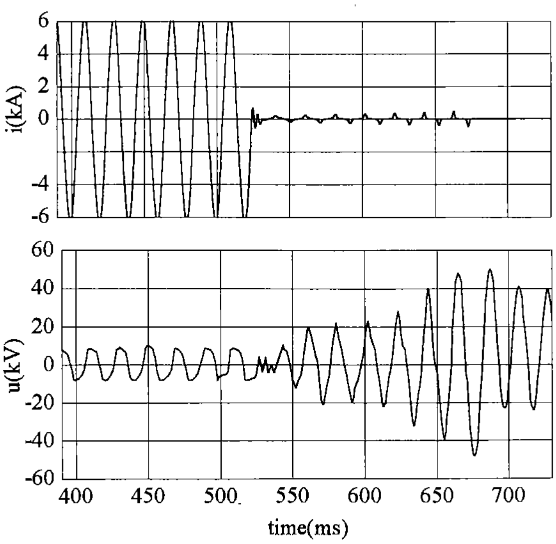
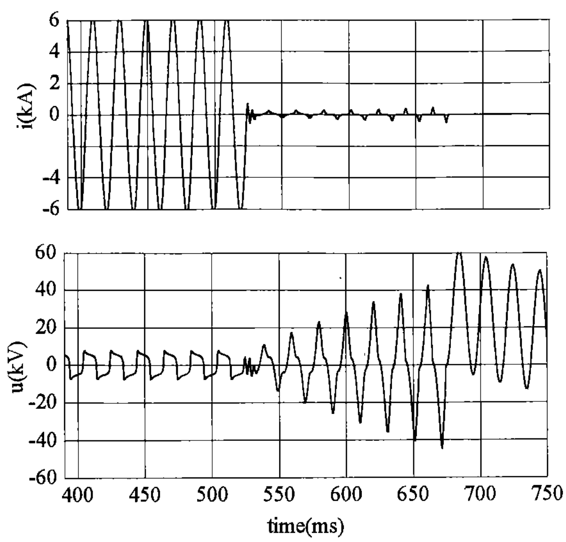


Fig. 6 Synthetic test circuit

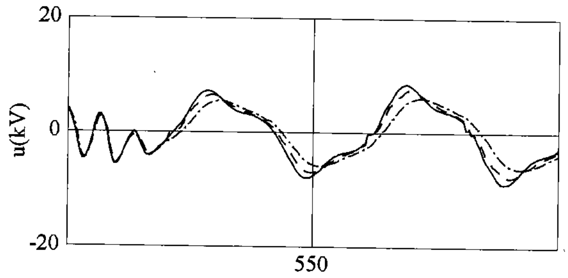


(a) Measured results

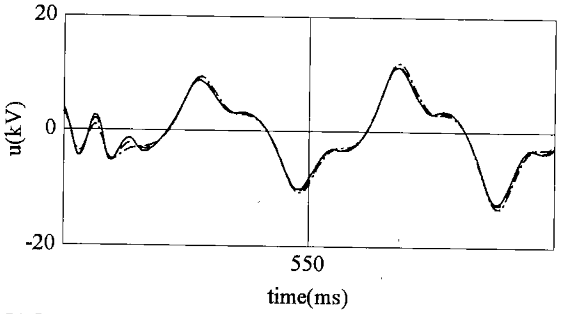


(b) Combined iterative method

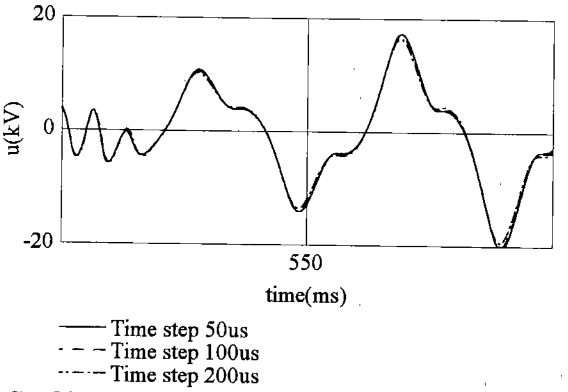
Fig. 7 The fault arc results in synthetic test circuit



(a) EMTDC with constant conductance in arc model



(b) EMTDC with changing conductance in arc model



(c) Combined iterative method with constant conductance

Fig. 8 The synthetic test circuit with different time steps

for field measurements [12]. The arc along 380-kV insulation string was initiated by means of a fuse wire, when switch S_1 in Fig. 6 was closed. Initially switch S_2 was closed, and in this state, high sinusoidal current of 4.3kA rms (primary arc current) flowed through the arc. At t = 524ms the switch S_2 was opened, and an arc current of 90 A flowed. The measured voltage u, currents i are shown in Fig. 7(a). Simulated results with the combined iterative method (Δt = 50μs) are shown in Fig. 7(b). As observed, the simulation results show a close agreement with the measured result. Results of a further investigation are presented in Fig. 8. In an EMTDC simulation, the arc model is represented as a current source and a single conductance, which do not change during a cycle. The other EMTDC simulation uses a current source and a conductance, of which the value changes several times during a cycle in order to minimize the error between the numerical arc value and the using a constant conductance. The third simulation uses the proposed combined iterative method. As it can be seen in Fig. 8, the results obtained with the combine iterative method are almost the same using different time steps even in the case with only single constant conductance shown in eq. (20) in the arc model representation.

7.2 Oscillator circuit using a tunnel diode

An oscillator circuit using a tunnel diode which is approximated in i = $2.83v^3$ - $2.02v^2$ + $0.37v$ around the negative conductance characteristic is adopted as an example case, which shows some advantages and the correctness of CIM in EMTP-type simulator. The oscillator circuit is shown in Fig.9. Simulated results with CIM adopting proposed schemes (Δt = 150, 5ns), and basic NCA results without iterations (Δt = 150, 5ns) are shown in Fig.10. As it can be seen in Fig.10, the CIM results show a close agreement with a calculated result with small Δt, and are almost the same using different time steps.

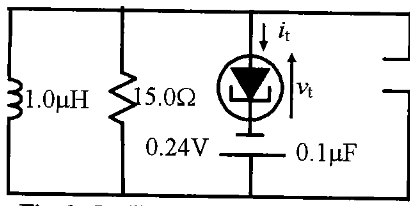
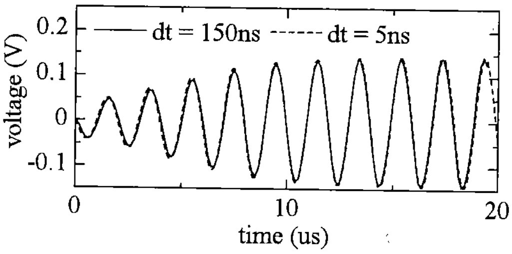
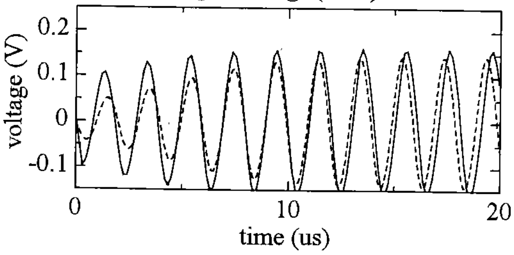


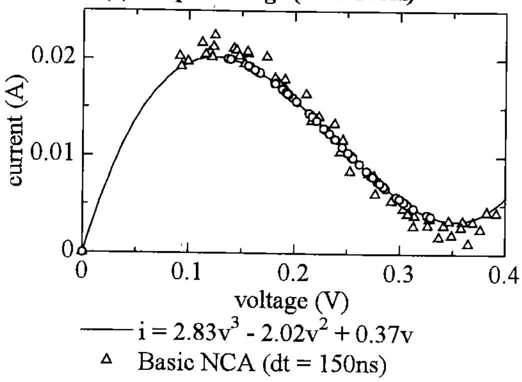
Fig. 9 Oscillator circuit using a tunnel diode



(a) Output voltage (CIM)



(b) Output voltage (basic NCA)



(c) $v_t - i_t$ characteristic

Fig. 10 Calculated results

8. CONCLUSIONS

An extension of an NCA solution method has been proposed to allow an arbitrary number and configuration of nonlinear elements in a subnetwork by use of a combined iteration method with the congenial and subsidiary techniques.

A subsidiary technique of a CIM solution method has been proposed to realize stableness and saving a calculation time on an arbitrary number and configuration of nonlinear elements in a subnetwork. One is an optimum handling method of the nonlinear conductance matrix of CIM. The unified expression of nonlinear matrices has made it possible to save a calculation time using the optimum handling method based on the theory of Crout's algorithm. It has also proven that an effective adoption of iterations contributes to the stableness of CIM simulations.

The calculated results agree well with experimental results (where available), calculated results using a small time step and interpolated solution methods, and theoretical results. The proposed schemes have been shown to be accurate and stable even for a large time step, and can survive simultaneous and abrupt changes due to nonlinear elements.

It is possible that the proposed method can be applied to other EMTP-type programs, because it is an extension of the basic NCA method.

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A.1 Proof of the symmetrical characteristic of the reduced nonlinear conductance matrix

The pre-reduced nonlinear conductance matrix **A**, is completely symmetrical in EMTP-Type simulators.

$$\begin{bmatrix} L_{11} & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & & \vdots \\ L_{M1} & \cdots & L_{MM} & 0 & \cdots & 0 \\ L_{M+11} & \cdots & L_{M+1M} & L_{M+1M+1} & \cdots & 0 \\ \vdots & & \vdots & \vdots & \ddots & \vdots \\ L_{N1} & \cdots & L_{NM} & L_{NM+1} & \cdots & L_{NN} \end{bmatrix} \begin{bmatrix} U_{11} & \cdots & U_{1M} & U_{1M+1} & \cdots & U_{1N} \\ \vdots & \ddots & \vdots & \vdots & & \vdots \\ 0 & \cdots & U_{MM} & U_{MM+1} & \cdots & U_{MN} \\ 0 & \cdots & 0 & U_{M+1M+1} & \cdots & U_{M+1N} \\ \vdots & & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & 0 & \cdots & U_{NN} \end{bmatrix} = \mathbf{A} \qquad \text{(A.1)}$$

If we reduce the first line and column from eq. (A.1), the following equation is derived.

$$\begin{bmatrix} L_{22} & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & & \vdots \\ L_{M2} & \cdots & L_{MM} & 0 & \cdots & 0 \\ L_{M+12} & \cdots & L_{M+1M} & L_{M+1M+1} & \cdots & 0 \\ \vdots & & \vdots & \vdots & \ddots & \vdots \\ L_{N2} & \cdots & L_{NM} & L_{NM+1} & \cdots & L_{NN} \end{bmatrix} \begin{bmatrix} U_{22} & \cdots & U_{2M} & U_{2M+1} & \cdots & U_{2N} \\ \vdots & \ddots & \vdots & \vdots & & \vdots \\ 0 & \cdots & U_{MM} & U_{MM+1} & \cdots & U_{MN} \\ 0 & \cdots & 0 & U_{M+1M+1} & \cdots & U_{M+1N} \\ \vdots & & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & 0 & \cdots & U_{NN} \end{bmatrix} = \mathbf{C} \qquad \text{(A.2)}$$

From the symmetrical elements C_{ij} and C_{ji} in the matrix **C**, C_{ij} subtracted C_{ji} can be calculated as follows to prove that incidental matrix **C** is symmetrical.

$$\begin{aligned} C_{ij} - C_{ji} &= \sum_{k=2}^{i} L_{ik} U_{kj} - \sum_{k=2}^{i} L_{jk} U_{ki} \qquad (i < j) \\ &= A_{ij} - L_{i1} U_{1j} - \left(A_{ji} - L_{j1} U_{1i} \right) \\ &= L_{j1} U_{1i} - L_{i1} U_{1j} \quad \left(From\ A_{ij} - A_{ji} = 0 \right) \qquad \text{(A.3)} \\ &= \frac{L_{i1}}{L_{11}} \left(L_{j1} U_{11} - U_{1j} L_{11} \right) \begin{pmatrix} From\ A_{1j} = L_{11} U_{1j} \\ = A_{j1} = L_{j1} U_{11} \end{pmatrix} \\ &= \frac{L_{i1}}{L_{11}} \left(A_{j1} - A_{1j} \right) = 0 \end{aligned}$$

Therefore, the reduced matrix **C** is symmetrical. Namely, reduced all matrices which are reduced some lines and columns from a symmetrical matrix are symmetrical.

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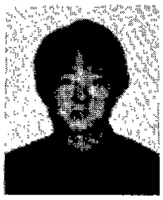

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