Paper

The Effects of Harmonics on Voltage Stability

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Demand for electrical energy has been increasing by the time as related to both technological developments and increase in population. It is necessary to enhance the quality of the electrical energy, especially supplied one. With considering voltage stability of power system, which includes nonlinear loads, during the planning, operating, and controlling of electrical power systems, more reliable and high quality energy is supplied. In this study, the effects of harmonic components on voltage stability, caused by nonlinear elements, have been demonstrated by a synthesis of harmonic power flow analysis and voltage stability one based on Newton-Raphson method. Harmonic modeling for power system and the models obtained from mathematical equations belonging to the steady-state voltage stability have been used together to realize this synthesis. Accordingly, the effects of non-sinusoidal quantities on the voltage stability, neglected so far, are considered and then the analysis is performed under this condition. Thus, the more realistic results were obtained by the harmonic power flow based upon the voltage stability analysis.

keywords: Harmonics, Voltage Stability, Power System and Nonlinear Load.

1. Introduction

1.1 Steady-State voltage stability The planning, operation and control of a power system are governed by stability consideration to a significant extending one. Transient or steady state stability of power system is capable of operating the stable condition without lose of synchronism after a large or small disturbance respectively. Voltage stability, on the other hand, has the ability of the system to provide adequate reactive power support under all operating conditions so as to maintain stable load voltage magnitudes within specified operating limits in the steady-state [1] [2].

The convergence of the Newton-Raphson power flow analysis is suggested as an approach to a stability limit for the steady-state stability one. This stability limit, which is also called critical point, is often designed as the point where the power flow Jacobian is singular [3]. In equation form, this can be expressed as follow,

$$\det[J] = 0 \quad \cdots\cdots (1)$$

Eigenvalues of linearilized dynamics are used as an indicator of the stability limit [4]. The other indicator is Jakobien matrix itself [5]. The analysis can be realized by using the smallest eigenvalues and eigenvectors of a reduced Jakobien matrix as an another method. Each eigenvalue gives a criterion of proximity to voltage instability [6].

1.2 Harmonic components An electrical power system consists of generation, transmission and distribution plants. Besides, there are loads and equipment belongs to individual consumers. These plants make the current and voltage of electrical energy convenient for operation condition of loads in the system by changing waveforms and frequencies, if necessary. The elements in the power systems such as generator, transformer, transmission line, HVDC system, capacitor, converter, inverter, static VAR compensator perform specific functions. The current-voltage characteristics of some of these elements are nonlinear. They are called as nonlinear elements and distort the sinusoidal waveform of current and voltage. These distortions cause the circulating of harmonic components in the system [7]. Harmonics are often used to defining distorted sinewaves associated with currents and voltages of different amplitudes and frequency.

One can compose a distorted periodic waveshape by using different harmonic frequencies with different amplitudes. Conversely, one can also decompose any distorted periodic waveshape into a fundamental wave and a set of harmonics. This decomposition process is called Fourier analysis on which the effects of nonlinear elements in power systems can be analyzed systematically. When steady state harmonics are available, instantaneous voltage and current can be represented by Fourier series as follows [8]:

$$v(t) = \sum_{h=1}^{\infty} v_h(t) = \sum_{h=1}^{\infty} \sqrt{2}V^{(h)} \sin(h\omega_0 t + \theta^{(h)}) \quad \cdots\cdots (2)$$

$$i(t) = \sum_{h=1}^{\infty} i_h(t) = \sum_{h=1}^{\infty} \sqrt{2}I^{(h)} \sin(h\omega_0 t + \delta^{(h)}) \quad \cdots\cdots (3)$$

where the dc terms are usually ignored for simplicity, V_h and I_h are rms values for h^{th} order of harmonic voltage and current, respectively.

Nonlinear elements cause harmonics in transmission and distribution system and affect the quality of energy [9]. The distortions are defined as individual harmonic distortion (HD) and total harmonic distortion (THD) for voltage and current, respectively;

$$HD_V = \frac{V^{(h)}}{V_1} \quad \text{and} \quad HD_I = \frac{I^{(h)}}{I_1} \quad \ldots\ldots\ldots\ldots\ldots\ldots(4)$$

$$THD_V = \frac{\sqrt{\sum_{h=2}^{\infty} V^{(h)^2}}}{V_1} \quad \text{and}\ THD_I = \frac{\sqrt{\sum_{h=2}^{\infty} I^{(h)^2}}}{I_1} \quad \ldots\ldots(5)$$

In order to observe the effects of nonlinear elements on voltage stability, the method based on harmonic power flow is needed instead of conventional power flow method [10]. The harmonic power flow algorithm based on Newton-Raphson method is examined as to be related to the analysis.

2. Harmonic power flow algorithm

Network voltages and currents can be expressed by Fourier series for the harmonic power flow analysis, which was developed by Xia and Heydt [11] [12]. Voltages and nonlinear element parameters form bus variable vector (X). The vector is given in eq. (6),

$$[X] = \Big[\overbrace{\delta_2^{(1)}, |\dot{V}_2^{(1)}|, \delta_3^{(1)}, |\dot{V}_3^{(1)}|}^{Fundamental}, \ldots\ldots, \overbrace{\delta_n^{(1)}, |\dot{V}_n^{(1)}|, \delta_1^{(1)}, |\dot{V}_1^{(5)}|, \ldots, \delta_n^{(5)}, |\dot{V}_n^{(5)}|}^{5^{th}\ Harmonic}, \ldots\ldots, \overbrace{\delta_1^{(L)}, |\dot{V}_1^{(L)}|, \ldots\ldots, \delta_n^{(L)}, |V_n^{(L)}|}^{L^{th}\ Harmonic}, \overbrace{\alpha_m, \beta_m, \ldots\ldots, \alpha_n, \beta_n}^{Nonlinear\ Device\ Variables}\Big]^T = \big[[V^{(1)}][V^{(5)}]\ldots\ldots,[V^{(L)}][\Phi]\big] \quad ..(6)$$

where supercripts and subcripts denote harmonic orders and bus numbers, respectively, L is the maximum harmonic order considered and Φ is the nonlinear device variable vector [13]. The mismatch of real and reactive powers for the linear buses (while $k \in \{1, 2, \ldots, (m-1)\}$) are defined, as (m is the first nonlinear load number),

$$\left.\begin{aligned} \Delta P_k &= (P_k)_{SP} + F_{r,k}^{(1)} \\ \Delta Q_k &= (Q_k)_{SP} + F_{i,k}^{(1)} \end{aligned}\right\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(7)$$

where $(P_k)_{SP}$ and $(Q_k)_{SP}$ are real and reactive powers at bus k, and $F_{r,k}^{(1)}$ and $F_{i,k}^{(1)}$ are the line fundamental real and reactive power respectively. The mismatch of real and reactive power can be calculated for the nonlinear buses as following.

$$\left.\begin{aligned} \Delta P_k^{nonlin} &= (P_k)_{SP} + \sum_{h=1}^{L} F_{r,k}^{(h)} \\ \Delta Q_k^{nonlin} &= (Q_k)_{SP} + \sum_{h=1}^{L} F_{i,k}^{(h)} \end{aligned}\right\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(8)$$

where $k \in \{m, m+1, \ldots, n\}$ and n is the total number of the buses in the system. $F_{r,k}^{(h)}$ and $F_{i,k}^{(h)}$ can be calculated from the eq. (9) (for h=1,5,7,...,L).

$$\left.\begin{aligned} F_{r,k}^{(h)} &= V_k^{(h)} \sum_{j=1}^{n} Y_{jk}^{(h)}.V_j^{(h)}.\cos(\delta_k^{(h)} - \theta_{kj}^{(h)} - \delta_j^{(h)}) \\ F_{i,k}^{(h)} &= V_k^{(h)} \sum_{j=1}^{n} Y_{jk}^{(h)}.V_j^{(h)}.\sin(\delta_k^{(h)} - \theta_{kj}^{(h)} - \delta_j^{(h)}) \end{aligned}\right\} .(9)$$

The harmonic phasor voltage for k[th] bus is $\dot{V}_k^{(h)} = V_k^{(h)} \angle \delta_k^{(h)}$ and (k, j) element of bus admittance matrix calculated for h[th] harmonic frequency is shown as a pharos like $\dot{Y}_{kj}^{(h)} = Y_{kj}^{(h)} \angle \theta_{kj}^{(h)}$. The mismatch vector for the harmonic power flow is defined as [14]:

$$[\Delta M] = \big[[\Delta W], [\Delta I^{(5)}], [\Delta I^{(7)}], \ldots, [\Delta I^{(L)}], [\Delta I^{(1)}]\big]^T \quad \ldots\ldots\ldots(10)$$

where ΔW is the mismatch of power vector and $\Delta I^{(h)}$ is the mismatch current vector for the h[th] harmonic. The mismatch power is given in eq. (11)

$$[\Delta W] = [\Delta P_2, \Delta Q_2, \ldots, \Delta P_{m-1}, \Delta Q_{m-1}, \Delta P_m^{nonlin}, \Delta Q_m^{nonlin}, \ldots, \Delta P_n^{nonlin}, \Delta Q_n^{nonlin}] \ .(11)$$

The mismatch current vector for the fundamental component (h=1) and the harmonic component (h=5, 7, ..., L), which are the elements of the mismatch vector is given in the following equations respectively,

$$\left.\begin{aligned} [\Delta I^{(1)}] &= \big[(I_{r,m}^{(1)} + g_{r,m}^{(1)}), (I_{i,m}^{(1)} + g_{i,m}^{(1)}), (I_{r,m+1}^{(1)} + g_{r,m+1}^{(1)})(I_{i,m+1}^{(1)} + g_{i,m+1}^{(1)}) \ldots, (I_{r,n}^{(1)} + g_{r,n}^{(1)}), (I_{i,n}^{(1)} + g_{i,n}^{(1)})\big]^T \\ [\Delta I^{(h)}] &= \big[I_{r,1}^{(h)}, I_{i,1}^{(h)}, \ldots, I_{r,m-1}^{(h)}, I_{i,m-1}^{(h)}, (I_{r,m}^{(h)} + g_{r,m}^{(h)}), (I_{i,m}^{(h)} + g_{i,m}^{(h)})(I_{r,m+1}^{(h)} + g_{r,m+1}^{(h)}), (I_{i,m+1}^{(h)} + g_{i,m+1}^{(h)}) \ldots, (I_{r,n}^{(h)} + g_{r,n}^{(h)}), (I_{i,n}^{(h)} + g_{i,n}^{(h)})\big]^T \end{aligned}\right\} .(12)$$

where $g_{r,k}^{(h)}$ and $g_{i,k}^{(h)}$ are real and imaginary parts of h[th] nonlinear load current injected to bus-k. $I_{r,k}^{(h)}$ and $I_{i,k}^{(h)}$ are real and imaginary parts of h[th] total line currents of bus-k. In these equations, $I_{r,k}^{(h)}$ and $I_{i,k}^{(h)}$ are also taken to be zero for the harmonic components at linear buses (k=1,2, ..., m-1).

The Newton-Raphson method is implemented to obtain the correction vector by using Jacobian matrix. When the mismatch goes to zero for every term of the mismatch vector, the solution can be obtained from [15]. Thus, by using the Newton-Raphson method, we can get the solution with desired tolerance (in this study the tolerance quantity for the mismatch is 0.0001 p.u.). Correction vector is given in eq. (13) as,

$$[\Delta X] = [J]^{-1}.[\Delta M] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(13)$$

3. Steady-State voltage stability analysis with harmonic power flow

The values obtained from linear power flow analysis are used in steady-state voltage stability analysis performed with conventional methods. It is required to develop a new algorithm to analyze steady-state voltage stability in case the system having nonlinear loads. A synthesis of harmonic power flow algorithm and voltage stability algorithm is done and a new algorithm is developed to realize this analysis [10]. When the Jacobian of a Newton-Raphson power flow becomes singular, the steady- state voltage stability limit (critical point) of the system can be determined easily and rapidly.

With using datum obtained from harmonic power flow, the critical values are calculated for fundamental and harmonic components separately. According to this method, critical transmission angle, $\delta_{critical}$ critical load voltage, $V_{critical}$ and critical load power, $P_{critical}$ for the fundamental, [2] and the harmonic component are given by,

$$\delta_{critical}^{(h)} = \frac{1}{2}\tan^{-1}\left(\frac{K_1}{K_2}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(14)$$

where

$$K_1 = a_1^{(h)}(b_2^{(h)} - b_1^{(h)}\tan\varphi^{(h)}) + a_2^{(h)}(b_1^{(h)} + b_2^{(h)}\tan\varphi^{(h)}) \ .(15)$$

$$K_2 = a_1^{(h)}(b_1^{(h)} + b_2^{(h)}\tan\varphi^{(h)}) + a_2^{(h)}(-b_2^{(h)} + b_1^{(h)}\tan\varphi^{(h)}) \ \ldots.(16)$$

$$V_{critical}^{(h)} = \frac{V_s^{(h)}}{2.K_4} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(17)$$

and

$$P^{(h)}_{critical} = \frac{(V_s^{(h)})^2 . \left[2K_3K_4 - (a_1^{(h)} b_1^{(h)} + a_2^{(h)} b_2^{(h)}) \right]}{4.(K_4)^2 . \left[(b_1^{(h)})^2 + (b_2^{(h)})^2 \right]} \quad \ldots\ldots(18)$$

where

$$K_3 = b_1^{(h)} \cos\delta_{kritik}^{(h)} + b_2^{(h)} \sin\delta_{kritik}^{(h)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(19)$$

$$K_4 = a_1^{(h)} \cos\delta_{kritik}^{(h)} + a_2^{(h)} \sin\delta_{kritik}^{(h)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(20)$$

where h=1, 5, 7, . . ., L.

In this study, the voltage stability analysis including nonlinear loads was done by the following solution algorithm:

Step 1: The harmonic power flow analysis for the available operation conditions of the sample system is applied for computing the load angles, voltages and powers of all buses.

Step 2: Powers at all buses except for the slack bus and the considered bus for voltage stability are transformed into shunt admittance by using bus voltages obtained from harmonic load flow for fundamental and harmonic component with the following equation individually,

$$R = \frac{V^2}{P}, X = \frac{V^2}{Q} \Rightarrow Y = (\frac{1}{R}) + (\frac{1}{jX}) \quad \ldots\ldots\ldots\ldots\ldots\ldots\text{(Step2-a)}$$

Also, the bus including nonlinear load must be transformed into shunt admittance for fundamental and harmonic component. Thus, the operation can be performed for the fundamental component of nonlinear load as above. Since the nonlinear load is modeled as a current source, which injects current to the network, a shunt admittance, Y_h which has the great value must be paid attention for the condition and considered deeply, while the nonlinear load is added to bus admittance matrix. Thus, Y_h is taken into the consideration relating to its direction in the transformation of harmonic components of nonlinear load to the shunt admittance. This is performed with the following equation,

$$Y = (\frac{1}{R}) + (\frac{1}{jX}) + (-jY_h) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(Step2-b)}$$

Step 3: These shunt admittances are added into bus admittance matrix by taking into the consideration by their directions (These operations are not performed for the slack bus and the considered bus for the voltage stability).

Step 4: After obtaining the new bus admittance matrix, the matrix is reordered. In this case, the elements of the considered bus for the voltage stability are in the first row and first column, and the elements of slack bus are in the second row and second column.

Step 5: The elements at the other buses are reduced into the slack bus and the considered bus with matrix algebra method. The final reduced matrix (2×2) is obtained by making new orders at the bus admittance matrix as follow,

$$Y_{bus} = \begin{bmatrix} K & L \\ L^T & M \end{bmatrix} \Rightarrow Y_{bus}(2x2) = K - L.M^{-1}.L^T \quad ..\text{(Step4-a)}$$

Step 6: In a reduced 2-bus system, it is used super-position theorem to find $V^{(h)}_{critical}$ ve $\delta^{(h)}_{critical}$ values which they put the system to instable condition. ($V^{(1)}_{critical}$, $\delta^{(1)}_{critical}$) and ($V^{(5)}_{critical}$, $\delta^{(5)}_{critical}$) values which make J_r, Jacobian matrix singular in appendix part were calculated for h=1 and h= 5 separately. After that, $P^{(1)}_{critical}$ ve $P^{(5)}_{critical}$ values can be calculated by the help of ($V^{(1)}_{critical}$, $\delta^{(1)}_{critical}$) and ($V^{(5)}_{critical}$, $\delta^{(5)}_{critical}$) easily.

4. Numerical Application

In the study, two different load conditions are considered in order to analysis the effects of harmonics on the voltage stability. In the first case, all loads in the example power system 1 (Fig. 1) are linear loads (sinusoidal condition). In the second case, the example power system 2 (Fig. 2) includes linear and nonlinear loads (non-sinusoidal condition). The approximation or system can easily be adjusted for a n-bus system.

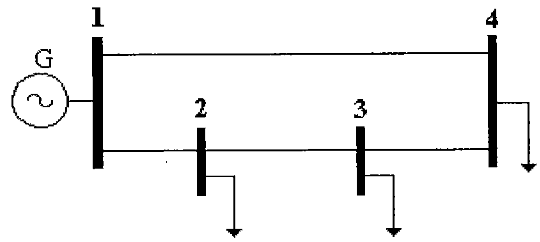

Fig. 1 Single line diagram of the example system 1 including linear loads

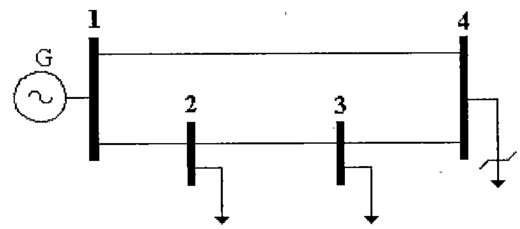

Fig. 2 Single line diagram of the example system 2 including linear and nonlinear loads

The p.u. values of the lines are used in the sample systems, in which voltage stability analysis is performed for 66 kV and 10 MVA base values. The characteristic values of the lines and the load data of the systems are given Table 1 and Table 2, respectively.

Table 1. 4-Bus system impedance and line charging data for the system 1 and 2

From Bus	To Bus	Z, Line Impedance (p.u.)	B, Line Charging (p.u.)
1	2	0.01 + j0.01	j2.1125e-4
2	3	0.02 + j0.08	j8.4500e-4
3	4	0.01 + j0.02	j4.2250e-4
1	4	0.01 + j0.02	j4.2250e-4

Table 2. 4-Bus system load data for the system 1 and 2

Bus No	P_{load} (p.u.)	Q_{load} (p.u.)
1	0.00	0.00
2	0.80	0.80
3	0.90	0.60
4	0.25	0.10

All transmission lines are modeled using a lumped PI model, and bus-1 is the slack bus in the example system. The conventional voltage stability analysis is realized for the example system 1. Developed voltage stability analysis based on the harmonic power flow is realized for the example system 2. Thus, the effects of harmonics on steady-state voltage stability are examined.

The variations of the critical values for various nonlinear load conditions are studied and it is determined the affected buses from the point of voltage stability due to harmonics. The real and imaginary nonlinear load currents for various load conditions are given as [15],

$$g_{r,4}^{(5)} = k[\,(V_4^{(1)})^3\cos(3\delta_4^{(1)}) + (V_4^{(5)})^3\cos(\delta_4^{(5)})\,] \quad (21)$$

$$g_{i,4}^{(5)} = k[\,(V_4^{(1)})^3\sin(3\delta_4^{(1)}) + (V_4^{(5)})^3\sin(\delta_4^{(5)})\,] \quad (22)$$

where k is the coefficient of the nonlinear load current. The voltage stability of bus-2, bus-3 and bus-4 were examined using the various coefficient k for 5th harmonic component. The individual harmonic voltage distortion (HD_V) values of the buses were obtained from the analysis, and the results are given in Table 3.

Table 3. Individual harmonic distortion ratios for system 2

k, coefficient of nonlinear load current	**HD_V (%)**		
	Bus 4	**Bus 3**	**Bus 2**
0.3	2.4584	2.0483	0.2401
0.6	4.9043	4.0859	0.4787
0.9	7.3255	6.1026	0.7147
1.2	9.7110	8.0888	0.9467
1.5	12.0505	10.0358	1.1737

The critical power values obtained from the conventional voltage stability analysis are shown in Table 4. The critical power values obtained from the voltage stability analysis based on harmonic power flow are given in Table 5.

Table 4. The critical power values obtained from voltage stability analysis for the system 1

Bus No	**$P_{critical}$ (Sinusoidal condition)**
2	12.8928
3	06.5291
4	**13.2336**

Table 5. The critical power values obtained from the voltage stability analysis for the system 2

k	**Bus No**	**$P_{critical}$ (Nonsinusoidal condition)**
	2	12.8922
0.3	3	06.5267
	4	**12.9997**
	2	12.8905
0.6	3	06.5195
	4	**12.3367**
	2	12.8876
0.9	3	06.5078
	4	**11.3537**
	2	12.8837
1.2	3	06.4917
	4	**10.2019**
	2	12.8787
1.5	3	06.4716
	4	**09.0297**

As to be seen from the Table 4 and Table 5, load buses have great importance in steady-state voltage stability analysis. Since non-linear load (harmonic source) is on 4th numbered bus, the biggest effect also is on this bus. 2nd and 3rd buses only include linear loads. The variations of critical voltage in the frequency domain at bus-4, bus-3 and bus-2 are illustrated in Fig. 3-Fig. 7.

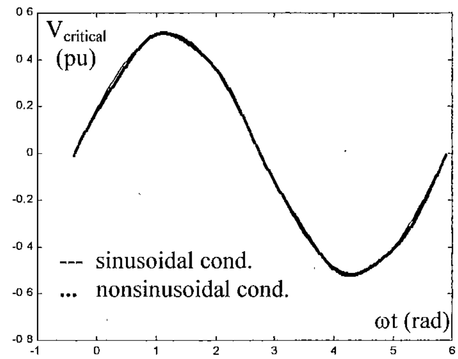

Fig. 3 The variation of critical voltage at the bus 4 (k = 0.3)

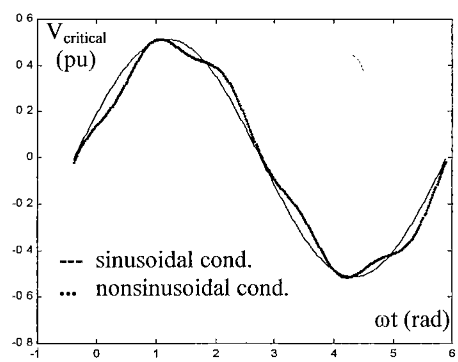

Fig. 4 The variation of critical voltage at the bus 4 (k = 0.9)

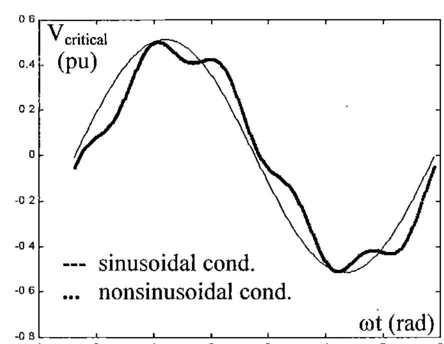

Fig 5 The variation of critical voltage at the bus 4 (k = 1.5)

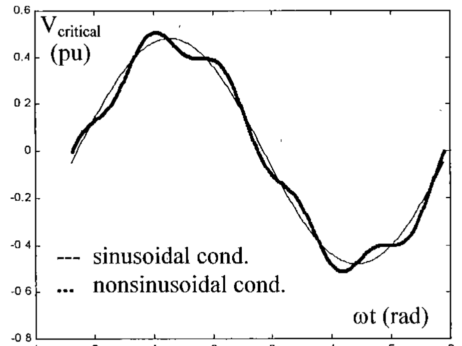

Fig 6. The variation of critical voltage at the bus 3 (k = 1.5)

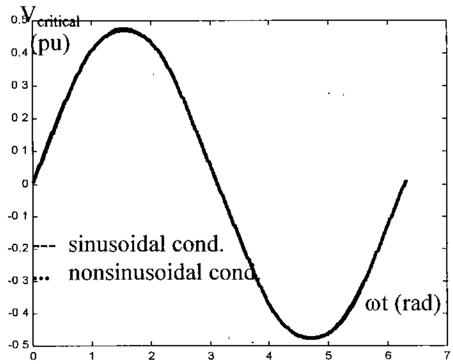

Fig 7. The variation of critical voltage at the bus 2 (k = 1.5)

5. Conclusions

In this paper, the effects of the harmonics on the voltage stability of power systems have been investigated and the following conclusions are determined:

1- Harmonics have negative effects on the voltage stability. The bus on which has the nonlinear load is the most affected bus in the system. Besides, it is determined that the higher harmonic distortion of the negative effect on steady-state voltage stability.

2- The obtained critical values determine maximum power transfer limited by the voltage stability. As to be seen from the paper, the maximum power transfer limit decreases with the increment of the value of the nonlinear load current.

3- By comparing with the results of two analysis (sinusoidal and non-sinusoidal conditions), there is an increment the difference condition between the critical values obtained from the steady-state voltage stability analysis.

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Appendix:
Obtaining of the critical values

Obtaining of the critical values from P-V graphics is not always possible, because of the complex structure of the power system. Thus, when the Jakobian of a Newton-Raphson power flow becomes singular, the steady- state voltage stability limit (critical point) of the system can be determined directly and fast.

The $\underline{I}_r$ current is written by using the equation $\underline{V}_s=\underline{A}.\underline{V}_r+\underline{B}.\underline{I}_r$ as functions of the sending-end and receiving-end voltages, $\underline{V}_s = V_s\angle 0°$ and $\underline{V}_r = V_r\angle -\delta°$ respectively as follows:

$$I_r = \frac{V_s-(a_1+ja_2).V_r.(\cos\delta - j\sin\delta)}{b_1+jb_2} \quad \text{......................(a1)}$$

where $\underline{A}=a_1+ja_2$ and $\underline{B}=b_1+jb_2$ are the generalized circuit constant. Using the above equation in the complex power equation,

$$\underline{S} = \frac{V_sV_r(b_1\cos\delta+b_2\sin\delta)-(a_1b_1+a_2b_2)V_r^2}{b_1^2+b_2^2} + j\frac{V_sV_r(b_2\cos\delta-b_1\sin\delta)-(a_1b_2-a_2b_1)V_r^2}{b_1^2+b_2^2} \quad \text{.(a2)}$$

The real and reactive power P_r and Q_r are determined by separating the real and imaginer part of the eq. (2). Then, the critical values are determined by considering the singularity of the Jakobian matrix in Newton-Raphson power flow for the 2-bus system and defined as two functions [2]:

$$f_1(V_s,V_r,\delta)=P_r-\frac{V_sV_r(b_1\cos\delta+b_2\sin\delta)-(a_1b_1+a_2b_2)V_r^2}{b_1^2+b_2^2} \quad \text{.(a3)}$$

$$f_2(V_s,V_r,\delta)=Q_r-\frac{V_sV_r(b_2\cos\delta-b_1\sin\delta)-(a_1b_2-a_2b_1)V_r^2}{b_1^2+b_2^2} \quad \text{..(a4)}$$

These equations are reordered to form the Jakobian matrix in martix form and it may be written as follow,

$$\begin{bmatrix}\Delta P_r\\ \Delta Q_r\end{bmatrix}=\begin{bmatrix}\frac{\partial f_1}{\partial\delta} & \frac{\partial f_1}{\partial V_r}\\ \frac{\partial f_2}{\partial\delta} & \frac{\partial f_2}{\partial V_r}\end{bmatrix}\begin{bmatrix}\Delta\delta\\ \Delta V_r\end{bmatrix}=\left[J_r\right]\begin{bmatrix}\Delta\delta\\ \Delta V_r\end{bmatrix} \quad \text{....................(a5)}$$

In the equation, the determinant of the matrix must be equal to zero for the singularity of the Jakobian matrix, that is

$$\frac{\partial f_1}{\partial\delta}.\frac{\partial f_2}{\partial V_r}-\frac{\partial f_2}{\partial\delta}.\frac{\partial f_1}{\partial V_r}=0 \quad \text{.......................................(a6)}$$

and if we put these equations in eq. (a6) then,

$$V_s=2V_r.(a_1\cos\delta+a_2\sin\delta) \quad \text{................................(a7)}$$

we get the equation between sending-end and receiving-end voltages at the critical point. In order to determine the critical transmission angle, the expression dependence of V_r and δ is used instead of V_s in power expressions. The real and reactive power P_r and Q_r are related throught the following equation,

$$Q_r=P_r.\tan\varphi \quad \text{..(a8)}$$

Using eq. (a8), the critical transmission angles expressed as,

$$\tan(2\delta)=\frac{a_1(b_2-b_1\tan\varphi)+a_2(b_1+b_2\tan\varphi)}{a_1(b_1+b_2\tan\varphi)+a_2(-b_2+b_1\tan\varphi)} \quad \text{......................(a9)}$$

Then by using the following equations in eq. (a9),

$$K_1=a_1(b_2-b_1\tan\varphi)+a_2(b_1+b_2\tan\varphi) \quad \text{......................(a10)}$$

$$K_2=a_1(b_1+b_2\tan\varphi)+a_2(-b_2+b_1\tan\varphi) \quad \text{......................(a11)}$$

Finally the critical transmission angle is obtained as follow,

$$\delta_{critical}=\frac{1}{2}\tan^{-1}\left(\frac{K_1}{K_2}\right) \quad \text{.......................................(a12)}$$

If we put the eq. (a12) in eq. (a7), we obtain $V_{rcritical}$,

$$V_{rcritical}=\frac{V_s}{2.(a_1\cos\delta_{critical}+a_2\sin\delta_{critical})} \quad \text{...........(a13)}$$

The critical value at the receiving-end is obtained from the eq. (a2), (a7) and (a12)

$$P_{rcritical}=\frac{V_s^2(2K_3K_4-(a_1b_1+a_2b_2))}{(b_1^2+b_2^2).4K_4^2} \quad \text{.................(a14)}$$

where

$$K_3=b_1\cos\delta_{critical}+b_2\sin\delta_{critical} \quad \text{......................(a15)}$$

$$K_4=a_1\cos\delta_{critical}+a_2\sin\delta_{critical} \quad \text{......................(a16)}$$

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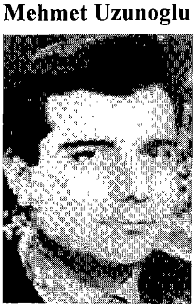

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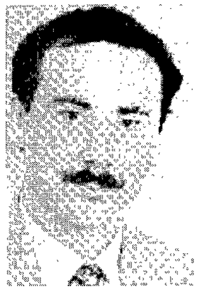
Celal Kocatepe



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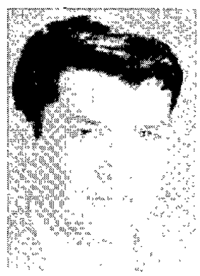

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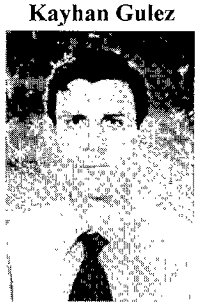

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