Paper

Decentralized Adaptive Robust Stabilization for a Class of Uncertain Large Scale Interconnected Time–Delay Systems

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The problem of decentralized stabilization is considered for a class of large scale linear time–varying systems with delayed state perturbations in the interconnections. In the paper, the upper bounds of the perturbations are assumed to be unknown, and a class of adaptation laws is introduced to estimate such unknown bounds. By employing the updated values of these unknown bounds, a class of decentralized memoryless state feedback controllers is also proposed. Based on Lyapunov stability theory and Lyapunov–Krasovskii functional, it is shown that by employing the proposed decentralized controllers, the solutions of the resulting adaptive closed–loop large scale time–delay system can be guaranteed to be uniformly bounded, and their states can converge uniformly asymptotically to zero. Finally, a numerical example is given to demonstrate the validity of the results.

Keywords: Large scale time–delay systems, decentralized control, adaptive robust control, Lyapunov–Krasovskii functional, uniform boundedness.

1. Introduction

It is well known that the problem of decentralized control of large scale interconnected systems has received considerable attention, and many approaches have been developed to synthesize some types of decentralized local state (or output) feedback controllers (see, e.g. Refs. (1)∼(3) and the references therein).

On the other hand, in many practical control problems, there are a number of time–delay systems, and the existence of delay is frequently a source of instability (see, e.g. Refs. (4), (5) and the references therein). Therefore, the problem of decentralized stabilization for large scale interconnected time–delay systems has also received considerable attention, and some results have also been obtained. In Refs. (6) and (7), for example, the decentralized stabilization problem of linear time–invariant large scale systems with time–delay is considered, and some sufficient conditions on decentralized local state feedback control are derived. In Ref. (8), based on the assumption that each isolated subsystem is strictly feedback positive real, a class of decentralized stabilizing output feedback controllers is proposed for large scale systems with time–varying delays in the interconnections. In Ref. (9), the problem of the decentralized stabilization of large scale nonlinear and linear systems including time–varying delays in the interconnections is considered, and a class of decentralized stabilizing state feedback controllers is presented.

It is worth pointing out that a salient feature of those schemes is that the decentralized state (or output) feedback controllers explicitly depend on the upper bounds of the interconnections. Therefore, for the decentralized controller design problem, one has to assume that such upper bounds are known. However, in a number of practical control problems, when the delayed state perturbations are included in the interconnections, such upper bounds may be unknown, or be partially known. In some cases, it may also be difficult to evaluate the upper bounds of the uncertainties. Thus, one must develop some new controller design methods to relax this assumption. For composite systems with delayed state perturbations, some updating laws to such unknown (or partially known) bounds have been introduced to construct some types of adaptive robust feedback controllers (see, e.g. Refs. (10), (11)). However, few efforts are made to consider the problem of decentralized feedback control for large scale time–delay interconnected systems with the unknown upper bounds of uncertainties because of its complexity. It seems that for such uncertain large scale time–delay interconnected systems, the similar results have not been reported yet in the control literature.

In this paper, we consider the problem of decentralized stabilization for a class of large scale linear time–varying systems with delayed state perturbations in the interconnections. Here, the upper bounds of the perturbations are assumed to be unknown, and control inputs are represented by the nonlinear functions satisfying the condition of the so–called series nonlinearity. For such a class of uncertain large scale interconnected time–delay systems, we want to develop some decentralized stabilizing memoryless state feedback controllers. For this purpose, we first propose some adaptation laws to estimate the unknown bounds of the perturbations in the interconnections. Then, by employing the updated values of these unknown bounds we construct decentralized memoryless state feedback controllers. Moreover, on the basis of the Lyapunov stability theory and Lyapunov–Krasovskii functional, we prove that by employing the proposed decentralized controllers, the solutions of the resulting adaptive closed-loop large scale time-delay system can be guaranteed to be uniformly bounded, and their states can converge uniformly asymptotically to zero.

The paper is organized as follows. In Section 2, the decentralized control problem to be tackled is stated and some standard assumptions are introduced. In Section 3, we propose a class of continuous decentralized adaptive robust controllers for the considered large scale systems. In Section 4, a numerical example is given to illustrate the use of our results. The paper is concluded in Section 5 with a brief discussion of the results.

2. Problem Formulation and Assumptions

We consider an uncertain large scale time-delay system S composed of N interconnected subsystems S_i, $i = 1, 2, \cdots, N$, described by the following differential–difference equations:

$$\frac{dx_i(t)}{dt} = A_i(t)x_i(t) + B_i(t)u_i(t) \quad \cdots\cdots\cdots\cdots (1)$$

where $t \in R^+$ is the time, $x_i(t) \in R^{n_i}$ is the current value of the state, and $u_i(t) \in R^{m_i}$ is the input vector. Each dynamical subsystem is interconnected as

$$u_i(t) = \sum_{j=1}^{N} A_{ij}(\zeta_i, t)x_j(t-h_{ij}) \quad \cdots\cdots\cdots\cdots (2)$$

In (1) and (2), for each $i \in \{1, 2, \ldots, N\}$, $A_i(t)$, $B_i(t)$ are continuous matrices of appropriate dimensions, and the matrices $A_{ij}(\cdot)$ accounts for the interconnection between the subsystems S_i and S_j, which is assumed to be continuous in all their arguments. Moreover, the uncertainty $\zeta_i \in \Psi_i \subset R^{l_i}$ is Lebesgue measurable and take values in a known compact bounding set Ω_i, and the time delays h_{ij}, $i, j = 1, 2, \ldots, N$, are assumed to be any positive constants which is not required to be known for the system designer. In addition, $x(\cdot) \in R^n$ denotes the vector $\left[\ x_1^\top(\cdot)\ \ x_2^\top(\cdot)\ \ \cdots\ \ x_N^\top(\cdot)\ \right]^\top$ where $n = n_1 + n_2 + \cdots + n_N$.

The initial condition for each subsystem with time delays is given by

$$x_i(t) = \chi_i(t), \qquad t \in [t_0 - h_i,\ t_0] \quad \cdots\cdots\cdots\cdots (3)$$

where $\chi_i(t)$ is a continuous function on $[t_0 - h_i,\ t_0]$, and h_i is defined as follows.

$$h_i := \max\left\{\ h_{ij},\ j = 1, 2, \ldots, N\ \right\}$$

For this class of input-interconnected large scale systems with delayed state perturbations in the interconnections, we introduce a decentralized local state feedback controller $\bar{u}_i(t)$ given by

$$\bar{u}_i(t) = p_i(x_i,\ t), \qquad i = 1, 2, \ldots, N \quad \cdots\cdots\cdots (4)$$

for each subsystem which modifies (2) to

$$u_i(t) = \Phi_i\left(\bar{u}_i(t)\right) + \sum_{j=1}^{N} A_{ij}(\zeta_i, t)x_j(t-h_{ij}) \quad \cdots (5)$$

where $p_i(\cdot)\ :\ R^{n_i} \times R^+ \rightarrow R^{m_i}$ is a continuous function which will be proposed later, and the control input nonlinearity is represented by the continuous function $\Phi_i\left(\bar{u}_i\right)$.

Now, the main objective of this paper is to synthesize the decentralized memoryless local state feedback controller $\bar{u}_i(t)$ given in (4) such that some types of stability of the large scale time-delay dynamical system, described by (1) and (5), can be guaranteed in the presence of delayed state perturbations in the interconnections.

Before giving our synthesis approach, we first introduce for the large scale time-delay system the following standard assumptions.

Assumption 2.1. All pairs $\{A_i(\cdot),\ B_i(\cdot)\}$, $i = 1, 2, \ldots, N$, are uniformly completely controllable.

Assumption 2.2. The series nonlinearity $\Phi_i(\cdot)\ :\ R^{m_i} \rightarrow R^{m_i}$ is any continuous function satisfying the following inequality:

$$\gamma_i^0 \bar{u}_i^\top \bar{u}_i \le \bar{u}_i^\top \Phi_i\left(\bar{u}_i\right) \le \gamma_i^* \bar{u}_i^\top \bar{u}_i,\ \forall \bar{u}_i \in R^{m_i} \quad \cdots (6)$$

where γ_i^0 and γ_i^* are two positive constants.

Remark 2.1. It is well known that *Assumption 2.1* is standard and denotes the internally stabilizability of each nominal isolated subsystem, i.e., the subsystem in the absence of the uncertain interconnections. *Assumption 2.2* introduces a condition on the so-called series nonlinearity which can well capture the inexact behaviour of an actuator. In general, γ_i^* is referred to as the gain margin and γ_i^0 as the gain reduction tolerance. It is well known (12) (13) that the optimal state feedback control law, derived from an optimal linear quadratic problem, can tolerate an infinite increase in gain and 50% gain reduction.

For convenience, we now introduce the following notations which represent the bounds of the uncertainties.

$$\rho_{ij}(t) := \max_{\zeta_i} \left\| A_{ij}(\zeta_i, t) \right\|, \quad i,j = 1,2,\ldots,N$$

where $\|\cdot\|$ is the spectral norm of a matrix "$\cdot$". Moreover, $\rho_{ij}(t)$ is assumed to be continuous and bounded for any $t \in R^+$. However, it should be pointed out that the values of $\rho_{ij}(t)$ is unknown.

Remark 2.2. It is well known that the decentralized stabilizing memoryless state feedback controllers proposed in the control literature for the large scale time–delay interconnected system described by (1) and (5) are based on the fact that the bounds of the uncertainties in the interconnections are known. That is, $\rho_{ij}(t)$, $i,j = 1,2,\ldots,N$, are assumed to be the known continuous and bounded functions, and the proposed decentralized control laws include such bounds $\rho_{ij}(t)$, $i,j = 1,2,\ldots,N$. However, in a number of practical control problems, such bounds may be unknown, or it is difficult to evaluate them. Therefore, some updating laws to such unknown bounds must be introduced to construct adaptive robust controllers (see, e.g. Refs. (10) and (11) for composite systems). In this paper, we propose a class of decentralized adaptive robust memoryless state feedback controllers for such uncertain large scale interconnected time–delay systems.

3. Decentralized Adaptive Stabilization

Since the bounds $\rho_{ij}(t)$, $i,j = 1,2,\ldots,N$, have been assumed to be continuous and bounded, we can suppose that there exist some positive constants ρ^*_{ij}, $i,j = 1,2,\ldots,N$, defined by

$$\rho^*_{ij} := \max\left\{ \rho_{ij}(t) \; : \; t \in R^+ \right\} \quad \cdots\cdots\cdots\cdots (7)$$

Here, it is worth pointing out that such positive constants ρ^*_{ij}, $i,j = 1,2,\ldots,N$, are unknown. Therefore, such unknown bounds can not be directly employed to construct decentralized memoryless local state feedback controllers.

Here, without loss of generality, we also introduce the following definition:

$$\psi^*_i := \left[\; (\rho^*_{i1})^2 \quad (\rho^*_{i2})^2 \quad \cdots \quad (\rho^*_{iN})^2 \; \right]^\top$$

where for any $i \in \{1,2,\ldots,N\}$, $\psi^*_i \in R^N$ is still obviously unknown constant vector.

It follows from *Assumption 2.1* that for any symmetric positive definite matrix $Q_i \in R^{n_i \times n_i}$, and any positive constant γ^0_i, the matrix Riccati equation of the form

$$\frac{dP_i(t)}{dt} + A^\top_i(t)P_i(t) + P_i(t)A_i(t) - \gamma^0_i P_i(t)B_i(t)B^\top_i(t)P_i(t) = -Q_i \quad \cdots (8)$$

has a solution which satisfies

$$\alpha^*_{i1} I_{n_i} \le P_i(t) \le \alpha^*_{i2} I_{n_i} \quad \cdots\cdots\cdots\cdots\cdots\cdots (9)$$

for all $t \in R^+$, where α^*_{i1} and α^*_{i2} are positive numbers (see, e.g. Ref. (14)), and $P(t_0)$ is any given positive definite matrix.

Thus, for the large scale time–delay system described by (1) and (5) we propose the following decentralized adaptive robust memoryless state feedback controllers:

$$\bar{u}_i(t) = p_i(x_i(t), t) = -\frac{1}{2} k_i(t) B^\top_i(t) P_i(t) x_i(t) \quad \cdots\cdots (10a)$$

where $i \in \{1,2,\ldots,N\}$, and the control gain function $k_i(t)$ is given by

$$k_i(t) = 1 + \left(\gamma^0_i\right)^{-1} \eta^\top_i \hat{\psi}_i(t) \quad \cdots\cdots\cdots\cdots (10b)$$

and where for any $i \in \{1,2,\ldots,N\}$, $P_i(t) \in R^{n_i \times n_i}$ is the solution of the Riccati equation described by (8), and $\eta_i \in R^N$ is a constant vector defined by

$$\eta_i := \left[\; \alpha^{-1}_{i1} \quad \alpha^{-1}_{i2} \quad \cdots \quad \alpha^{-1}_{iN} \; \right]^\top \quad \cdots\cdots (10c)$$

where α_{ij}, $i,j = 1,2,\ldots,N$, are positive constants which are chosen such that

$$Q_i - \alpha_i N I_{n_i} > 0 \quad \cdots\cdots\cdots\cdots\cdots\cdots (10d)$$

where for each $i \in \{1,2,\ldots,N\}$, the constant α_i is defined as follows.

$$\alpha_i := \max\left\{ \alpha_{ji}, \; j = 1,2,\ldots,N \right\}$$

In particular, for any $i \in \{1,2,\ldots,N\}$, $\hat{\psi}_i(t)$ in (10b) is the estimate of the unknown ψ^*_i which is updated by the following adaptive law:

$$\frac{d\hat{\psi}_i(t)}{dt} = \frac{1}{2}\Gamma_i \eta_i \left\| B^\top_i(t) P_i(t) x_i(t) \right\|^2 \quad \cdots\cdots (11)$$

where $\Gamma_i \in R^{N \times N}$ is any positive definite matrix, and $\hat{\psi}_i(t_0)$ is finite. Moreover, $\hat{\psi}(t) \in R^{N^2}$ denotes

$$\hat{\psi}(t) := \left[\; \hat{\psi}^\top_1(t) \quad \hat{\psi}^\top_2(t) \quad \cdots \quad \hat{\psi}^\top_N(t) \; \right]^\top$$

Remark 3.1. In this paper, though $\hat{\psi}_i(t)$ is called as the estimate of the unknown ψ^*_i, we do not mean that $\hat{\psi}_i(t)$ should converge to ψ^*_i as $t \to \infty$. In the paper, the main purpose is to synthesize some decentralized state feedback controllers such that the considered systems are stable.

For each subsystem, applying the decentralized memoryless state feedback controller given in (10) to (1) and (5) yields the uncertain closed–loop time–delay subsystem $\hat{S}_i$, $i \in \{1,2,\ldots,N\}$, of the form:

$$\frac{dx_i(t)}{dt} = A_i(t)x_i(t) + B_i(t)\Big[\Phi_i(\bar{u}_i(t)) + \sum_{j=1}^{N} A_{ij}(\zeta_i, t)x_j(t-h_{ij})\Big] \cdots (12)$$

where the decentralized control law $\bar{u}_i(t)$ is the one as given in (10).

On the other hand, letting

$$\tilde{\psi}_i(t) = \hat{\psi}_i(t) - \psi_i^*, \quad i \in \{1, 2, \ldots, N\}$$

we can rewrite (11) as the following error system

$$\frac{d\tilde{\psi}_i(t)}{dt} = \frac{1}{2}\Gamma_i\eta_i \left\| B_i^\top(t)P_i(t)x_i(t)\right\|^2 \cdots\cdots\cdots (13)$$

Here, $\tilde{\psi}(t) \in R^{N^2}$ denotes

$$\tilde{\psi}(t) := [\ \tilde{\psi}_1^\top(t) \quad \tilde{\psi}_2^\top(t) \quad \cdots \quad \tilde{\psi}_N^\top(t)\ ]^\top$$

In the following, by $(x, \tilde{\psi})(t)$ we denote a solution of the closed-loop large scale time-delay system and the error system. Then, the following theorem can be obtained which shows the globally boundedness of the solutions of the adaptive closed-loop large scale time-delay system described by (12) and (13).

Theorem 3.1. Consider the adaptive closed-loop large scale time-delay system described by (12) and (13), which satisfies *Assumption 2.1* and *Assumption 2.2*. Then, the solutions $(x, \tilde{\psi})\ (t; t_0, x(t_0), \tilde{\psi}(t_0))$ of the closed-loop large scale time-delay system described by (12) and the error system described by (13) are globally bounded and

(i) $\lim_{t\to\infty} x(t; t_0, x(t_0), \tilde{\psi}(t_0)) = 0 \cdots\cdots\cdots$ (14a)

(ii) $\lim_{t\to\infty} \frac{d\tilde{\psi}(t)}{dt} = 0 \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$ (14b)

Proof: For the adaptive closed-loop large scale time-delay system described by (12) and (13), we first define a Lyapunov-Krasovskii functional candidate as follows.

$$V(x, \tilde{\psi}) = \sum_{i=1}^{N}\left\{x_i^\top(t)P_i(t)x_i(t) + \tilde{\psi}_i^\top(t)\Gamma_i^{-1}\tilde{\psi}_i(t)\right\} + \sum_{i=1}^{N}\sum_{j=1}^{N}\alpha_{ij}\int_{t-h_{ij}}^{t} x_j^\top(\tau)x_j(\tau)d\tau \cdots (15)$$

where for each $i \in \{1, 2, \ldots, N\}$, $P_i(t)$ is the solution of differential Riccati equation (8), Γ_i^{-1} is any positive definite matrix, and α_{ij} is positive constant.

Let $(x(t), \tilde{\psi}(t))$ be the solutions of the closed-loop large scale time-delay system described by (12) and the error system described by (13) for $t \geq t_0$. Then by taking the derivative of $V(\cdot)$ along the trajectories of (12) and (13) we can obtain that

$$\begin{aligned} dV(x,\tilde{\psi})\ /\ dt &= \sum_{i=1}^{N}\Big\{x_i^\top\Big[\frac{dP_i(t)}{dt} + A_i^\top(t)P_i(t) + P_i(t)A_i(t)\Big]x_i \\ &\quad + 2x_i^\top(t)P_i(t)B_i(t)\Phi_i(\bar{u}_i(t)) \\ &\quad + 2x_i^\top(t)P_i(t)B_i\sum_{j=1}^{N}A_{ij}(\zeta_i,t)x_j(t-h_{ij}) \\ &\quad + 2\tilde{\psi}_i^\top(t)\Gamma_i^{-1}\frac{d\tilde{\psi}_i(t)}{dt}\Big\} \\ &\quad + \sum_{i=1}^{N}\sum_{j=1}^{N}\alpha_{ij}\Big[x_j^\top(t)x_j(t) - x_j^\top(t-h_{ij})x_j(t-h_{ij})\Big] \cdots (16) \end{aligned}$$

From (6) and (10) we can obtain that

$$\begin{aligned} &2x_i^\top(t)P_i(t)B_i(t)\Phi_i(\bar{u}_i(t)) \\ &\quad \leq -\gamma_i^0\, k_i(t)x_i^\top(t)P_i(t)B_i(t)B_i^\top(t)P_i(t)x_i(t) \cdots\cdots (17) \end{aligned}$$

Thus, substituting (17) into (16) yields

$$\begin{aligned} dV(x,\tilde{\psi})\ /\ dt &= \sum_{i=1}^{N}\Big\{x_i^\top(t)\Big[\frac{dP_i(t)}{dt} + A_i^\top(t)P_i(t) + P_i(t)A_i(t) \\ &\qquad - \gamma_i^0 P_i(t)B_i(t)B_i^\top(t)P_i(t)\Big]x_i(t) \\ &\quad - \eta_i^\top\hat{\psi}_i(t)x_i^\top(t)P_i(t)B_i(t)B_i^\top(t)P_i(t)x_i(t) \\ &\quad + 2x_i^\top(t)P_i(t)B_i\sum_{j=1}^{N}A_{ij}(\zeta_i,t)x_j(t-h_{ij}) \\ &\quad + \sum_{j=1}^{N}\alpha_{ij}\Big[x_j^\top(t)x_j(t) - x_j^\top(t-h_{ij})x_j(t-h_{ij})\Big] \\ &\quad + 2\tilde{\psi}_i^\top(t)\Gamma_i^{-1}\frac{d\tilde{\psi}_i(t)}{dt}\Big\} \\ &= \sum_{i=1}^{N}\Big\{-x_i^\top(t)Q_ix_i(t) \\ &\quad - \eta_i^\top\hat{\psi}_i(t)\left\|B_i^\top(t)P_i(t)x_i(t)\right\|^2 \\ &\quad + 2x_i^\top(t)P_i(t)B_i\sum_{j=1}^{N}A_{ij}(\zeta_i,t)x_j(t-h_{ij}) \\ &\quad + \sum_{j=1}^{N}\alpha_{ij}\Big[\|x_j(t)\|^2 - \|x_j(t-h_{ij})\|^2\Big] \\ &\quad + 2\tilde{\psi}_i^\top(t)\Gamma_i^{-1}\frac{d\tilde{\psi}_i(t)}{dt}\Big\} \end{aligned}$$

$$\le \sum_{i=1}^{N} \Bigg\{ -x_i^\top(t) Q_i x_i(t) + \sum_{j=1}^{N} \alpha_{ij} \|x_j(t)\|^2$$
$$-\eta_i^\top \hat{\psi}_i(t) \left\|B_i^\top(t) P_i(t) x_i(t)\right\|^2$$
$$-\sum_{j=1}^{N} \alpha_{ij} \Big[\|x_j(t-h_{ij})\|^2$$
$$-2\alpha_{ij}^{-1} \rho_{ij}^* \left\|B_i^\top(t) P_i(t) x_i(t)\right\| \|x_j(t-h_{ij})\| \Big]$$
$$+2\tilde{\psi}_i^\top(t) \Gamma_i^{-1} \frac{d\tilde{\psi}_i(t)}{dt} \Bigg\} \quad \cdots\cdots\cdots (18)$$

It can easily be verified that

$$\sum_{i=1}^{N}\sum_{j=1}^{N} \alpha_{ij} \|x_j(t)\|^2 \le \sum_{i=1}^{N}\sum_{j=1}^{N} \alpha_j \|x_j(t)\|^2$$
$$= N \sum_{i=1}^{N} \alpha_i x_i^\top(t) x_i(t) \quad \cdots\cdots\cdots (19)$$

Therefore, from (19) we can rewrite (18) as

$$dV(x,\tilde{\psi}) \,/\, dt$$
$$\le \sum_{i=1}^{N} \Bigg\{ -x_i^\top(t) \Big[Q_i - \alpha_i N I_{n_i}\Big] x_i(t)$$
$$-\eta_i^\top \hat{\psi}_i(t) \left\|B_i^\top(t) P_i(t) x_i(t)\right\|^2$$
$$-\sum_{j=1}^{N} \alpha_{ij} \Big[\|x_j(t-h_{ij})\|$$
$$-\alpha_{ij}^{-1} \rho_{ij}^* \left\|B_i^\top(t) P_i(t) x_i(t)\right\| \Big]^2$$
$$+\sum_{j=1}^{N} \alpha_{ij}^{-1} \left(\rho_{ij}^*\right)^2 \left\|B_i^\top(t) P_i(t) x_i(t)\right\|^2$$
$$+2\tilde{\psi}_i^\top(t) \Gamma_i^{-1} \frac{d\tilde{\psi}_i(t)}{dt} \Bigg\}$$
$$\le \sum_{i=1}^{N} \Bigg\{ -x_i^\top(t) \Big[Q_i - \alpha_i N I_{n_i}\Big] x_i(t)$$
$$-\eta_i^\top \hat{\psi}_i(t) \left\|B_i^\top(t) P_i(t) x_i(t)\right\|^2$$
$$+\sum_{j=1}^{N} \alpha_{ij}^{-1} \left(\rho_{ij}^*\right)^2 \left\|B_i^\top(t) P_i(t) x_i(t)\right\|^2$$
$$+2\tilde{\psi}_i^\top(t) \Gamma_i^{-1} \frac{d\tilde{\psi}_i(t)}{dt} \Bigg\} \quad \cdots\cdots\cdots (20)$$

Notice that the facts that for any $i \in \{1, 2, \ldots, N\}$, α_i has been chosen such that

$$\tilde{Q}_i := Q_i - \alpha_i N I_{n_i} > 0$$

and

$$\hat{\psi}_i(t) = \tilde{\psi}_i(t) + \psi_i^*$$

where

$$\psi_i^* := \left[\ (\rho_{i1}^*)^2 \quad (\rho_{i2}^*)^2 \quad \cdots \quad (\rho_{iN}^*)^2 \ \right]^\top$$

it follows from (13) and (20) that

$$dV(x,\tilde{\psi}) \,/\, dt$$
$$\le \sum_{i=1}^{N} \Bigg\{ -x_i^\top(t) \tilde{Q}_i x_i(t)$$
$$-\eta_i^\top \hat{\psi}_i(t) \left\|B_i^\top(t) P_i(t) x_i(t)\right\|^2$$
$$+\eta_i^\top \psi_i^* \left\|B_i^\top(t) P_i(t) x_i(t)\right\|^2$$
$$+\tilde{\psi}_i^\top(t) \eta_i \left\|B_i^\top(t) P_i(t) x_i(t)\right\|^2 \Bigg\}$$
$$= -\sum_{i=1}^{N} x_i^\top(t) \tilde{Q}_i x_i(t) \quad \cdots\cdots\cdots (21)$$

Thus, we obtain the following inequality, i.e. that for all $(t, x, \tilde{\psi}) \in R \times R^n \times R^{N^2}$,

$$\frac{dV(x,\tilde{\psi})}{dt} \le -\sum_{i=1}^{N} \lambda_{\min}\left(\tilde{Q}_i\right) \|x_i(t)\|^2 \quad \cdots\cdots (22)$$

Moreover, letting

$$\tilde{x}(t) := \left[\ x^\top(t) \quad \tilde{\psi}^\top(t) \ \right]^\top$$

and

$$\mu_{\min} := \min\Big\{ \lambda_{\min}(\tilde{Q}_i), \ i = 1, 2, \ldots, N \Big\}$$

we can obtain from (22) that for any $t \ge t_0$,

$$\frac{dV(\tilde{x}(t))}{dt} \le -\mu_{\min} \|x(t)\|^2 \quad \cdots\cdots\cdots (23)$$

On the other hand, in the light of the definition, given in (15), of Lyapunov–Krasovskii functional, we can known there always exist two positive constants $\delta_{\min}$ and $\delta_{\max}$ such that for any $t \ge t_0$,

$$\tilde{\gamma}_1(\|\tilde{x}(t)\|) \le V(\tilde{x}(t)) \le \tilde{\gamma}_2(\|\tilde{x}(t)\|) \quad \cdots\cdots (24)$$

where

$$\tilde{\gamma}_1(\|\tilde{x}(t)\|) := \delta_{\min} \|\tilde{x}(t)\|^2$$
$$\tilde{\gamma}_2(\|\tilde{x}(t)\|) := \delta_{\max} \|\tilde{x}(t)\|^2 + \sum_{i=1}^{N}\sum_{j=1}^{N} \alpha_{ij} h_{ij} \sup_{\tau \in [t-h_{ij},\, t]} \|x_j(\tau)\|^2$$

Now, from (23) and (24), we want to show that the solutions $\tilde{x}(t)$ of the adaptive closed–loop large scale time–delay system, described by (12) and (13), are uniformly bounded, and that the state $x(t)$ converges uniformly asymptotically to zero.

By the continuity of the adaptive closed–loop large scale time–delay system described by (11) and (12), it is obvious that any solution $(x, \tilde{\psi})\ (t;\, t_0, x(t_0), \tilde{\psi}(t_0))$ of the system is continuous.

It follows from (23) and (24) that for any $t \geq t_0$,

$$0 \leq \tilde{\gamma}_1(\|\tilde{x}(t)\|) \leq V(\tilde{x}(t)) = V(\tilde{x}(t_0)) + \int_{t_0}^{t} \dot{V}(\tilde{x}(\tau))d\tau \leq \tilde{\gamma}_2(\|\tilde{x}(t_0)\|) - \int_{t_0}^{t} \tilde{\gamma}_3(\|x(\tau)\|)d\tau \quad \cdots\cdots (25)$$

where the scalar function $\tilde{\gamma}_3(\|x(t)\|)$ is defined as

$$\tilde{\gamma}_3(\|x(t)\|) := \mu_{\min} \|x(t)\|^2 \quad \cdots\cdots (26)$$

Therefore, from (25) we can obtain the following two results. First, taking the limit as t approaches infinity on both sides of inequality (25), we can obtain that

$$0 \leq \tilde{\gamma}_2(\|\tilde{x}(t_0)\|) - \lim_{t\to\infty} \int_{t_0}^{t} \tilde{\gamma}_3(\|x(\tau)\|)d\tau \quad \cdots (27)$$

It follows from (27) that

$$\lim_{t\to\infty} \int_{t_0}^{t} \tilde{\gamma}_3(\|x(\tau)\|)d\tau \leq \tilde{\gamma}_2(\|\tilde{x}(t_0)\|) \quad \cdots\cdots (28)$$

On the other hand, from (25) we also have

$$0 \leq \tilde{\gamma}_1(\|\tilde{x}(t)\|) \leq \tilde{\gamma}_2(\|\tilde{x}(t_0)\|) \quad \cdots\cdots (29)$$

which implies that $\tilde{x}(t)$ is uniformly bounded. Since $\tilde{x}(t)$ has been shown to be continuous, it follows that $\tilde{x}(t)$ is uniformly continuous, which implies that $x(t)$ is uniformly continuous. Therefore, it follows from the definition that $\tilde{\gamma}_3(\|x(t)\|)$ is also uniformly continuous. Applying the Barbalat lemma [15] to inequality (28) yields that

$$\lim_{t\to\infty} \tilde{\gamma}_3(\|x(t)\|) = 0 \quad \cdots\cdots (30)$$

Furthermore, since $\tilde{\gamma}_3(\cdot)$ is a positive definite scalar function, it is obvious from (30) that we can have

$$\lim_{t\to\infty} \|x(t)\| = 0$$

which implies that (14a) is satisfied. From (13) and (14a) we can also easily obtain (14b). ■

Remark 3.2. It is obvious that the solutions of the resulting adaptive closed–loop large scale interconnected time–delay systems are continuous. Moreover, the decentralized local state feedback control laws proposed in (10) are memoryless, and the adaptive schemes given in (11) are independent of the time delays. Therefore, in the light of the proof given above, we can know that the time–delay constants $h_{ij},\ i,j = 1,2,\ldots,r$, are not required to be known for the system designer.

Remark 3.3. In the paper, we have considered large scale systems with multiple constant delays. That is, the delays $h_{ij},\ i,j = 1,2,\ldots,r$, have been assumed to be any positive constants. However, by employing the method presented in this paper, one can easily extend the results of this paper to such a class of uncertain large scale interconnected systems with the time–varying delays $h_{ij}(t),\ i,j = 1,2,\ldots,r$. In fact, if assuming that the delays $h_{ij}(t),\ i,j = 1,2,\ldots,r$, are any continuous and bounded nonnegative functions, and their derivatives are less than one, i.e. $\dot{h}_{ij}(t) < 1$, we can use the same Lyapunov–Krasovskii functional as the one given in (15) for the large scale systems with time–varying delays to obtain similar results.

Remark 3.4. From linear control theory, we know that for time–invariant case, A_i and B_i are two constant matrices and the matrix differential Riccati equation, given in (8), can be replaced by the much simpler algebraic Riccati equation of the form

$$A_i^\top P_i + P_i A_i - \gamma_i^0 B_i B_i^\top P_i = -Q_i \quad \cdots\cdots (31)$$

Generally, it is difficult to find an analytical solution of the Riccati differential matrix equation given in (8). However, one may resort to numerical solutions to (8) by some computer algorithms.

4. An Illustrative Example

Here, a large–scale dynamical system, with delayed state perturbations in the interconnections, is composed of three dynamical subsystems described by

$$\frac{dx_1(t)}{dt} = \begin{bmatrix} 3 & 0 \\ -2 & -1 \end{bmatrix} x_1(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \left(\Phi_1(\bar{u}_1(t)) + \sum_{j=1}^{3} A_{1j}(\zeta_1,t)x_j(t-h_{1j})\right) \quad \cdots\cdots (32a)$$

$$\frac{dx_2(t)}{dt} = \begin{bmatrix} -3 & -1 \\ 1 & 2 \end{bmatrix} x_2(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \left(\Phi_2(\bar{u}_2(t)) + \sum_{j=1}^{3} A_{2j}(\zeta_2,t)x_j(t-h_{2j})\right) \quad \cdots\cdots (32b)$$

$$\frac{dx_3(t)}{dt} = \begin{bmatrix} -3 & 1 \\ 0 & 3 \end{bmatrix} x_3(t) + \begin{bmatrix} 1 \\ 1 \end{bmatrix} \left(\Phi_3(\bar{u}_3(t)) + \sum_{j=1}^{3} A_{3j}(\zeta_3,t)x_j(t-h_{3j})\right) \quad \cdots\cdots (32c)$$

where for $i \in \{1,2,3\}$,

$$\Phi_i(\bar{u}_i) = b_i \bar{u}_i, \quad 5 \leq b_i \leq 8$$

and

$$A_{11}(\cdot) = \begin{bmatrix} -1 & \zeta_1(t) \end{bmatrix}, \quad A_{12}(\cdot) = \begin{bmatrix} 1 & 0 \end{bmatrix}$$
$$A_{13}(\cdot) = \begin{bmatrix} \zeta_1(t) & 2 \end{bmatrix}$$

$$A_{21}(\cdot) = \begin{bmatrix} -\zeta_2(t) & \zeta_2(t) \end{bmatrix}, A_{22}(\cdot) = \begin{bmatrix} 0 & \zeta_2(t) \end{bmatrix}$$
$$A_{23}(\cdot) = \begin{bmatrix} 1 & \zeta_2(t) \end{bmatrix}$$

$$A_{31}(\cdot) = \begin{bmatrix} \zeta_3(t) & 2.0 \end{bmatrix}, A_{32}(\cdot) = \begin{bmatrix} \zeta_3(t) & \zeta_3(t) \end{bmatrix}$$
$$A_{33}(\cdot) = \begin{bmatrix} \zeta_3(t) & 0 \end{bmatrix}$$

Here, we choose Q_i as a unit matrix I_i, for each $i \in \{1, 2, 3\}$, and from (6) we can also have $\gamma_1^0 = \gamma_2^0 = \gamma_3^0 = 5$. Then, solving the Riccati equations yields

$$P_1 = \begin{bmatrix} 1.433 & -0.167 \\ -0.167 & 0.430 \end{bmatrix} \quad \cdots\cdots\cdots\cdots (33a)$$

$$P_2 = \begin{bmatrix} 0.196 & 0.131 \\ 0.131 & 0.954 \end{bmatrix} \quad \cdots\cdots\cdots\cdots (33b)$$

$$P_3 = \begin{bmatrix} 0.166 & -0.143 \\ -0.143 & 1.564 \end{bmatrix} \quad \cdots\cdots\cdots\cdots (33c)$$

For the decentralized local state feedback controller given in (10) and the adaptation laws described by (11), we select the following parameters:

$$\alpha_{ij} = 0.25, \quad \Gamma_i = \mathrm{diag}\left\{3,\ 3,\ 3\right\}, \quad i, j = 1, 2, 3$$

Thus, for large scale system (32) with delayed state perturbations in the interconnections, from (10) with (11), we can obtain the decentralized adaptive robust memoryless state feedback controllers, by which the solutions of the resulting adaptive closed–loop large scale time–delay system can be guaranteed to be uniformly bounded, and the states are uniformly asymptotically stable.

For simulation, we give uncertain time–varying parameters $\zeta_i(t),\ i = 1, 2, 3$, time delays $h_{ij},\ i, j = 1, 2, 3$, and initial conditions as follows.

$$\zeta_1 = 1 + 0.5\sin(2t), \quad \zeta_2 = 0.5\sin(3t)$$
$$\zeta_3 = 1 + 0.5\cos(2t)$$

$$h_{i1} = 1.0, \quad h_{i2} = 2.0, \quad h_{i3} = 3.0, \quad i = 1, 2, 3$$

$$x_1(t) = x_2(t) = x_3(t)$$
$$= \begin{bmatrix} 3.0\cos(t) & -3.0\cos(t) \end{bmatrix}^\top, \ t \in [-3,\ 0]$$

$$\hat{\psi}_1(0) = \hat{\psi}_2(0) = \hat{\psi}_3(0) = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^\top$$

$$b_i = \mathrm{random}\left[5,\ 8\right], \quad i = 1, 2, 3$$

where $b_i = \mathrm{random}\left[5,\ 8\right]$ means that b_i can take any value from the interval $\left[5,\ 8\right]$ for simulation.

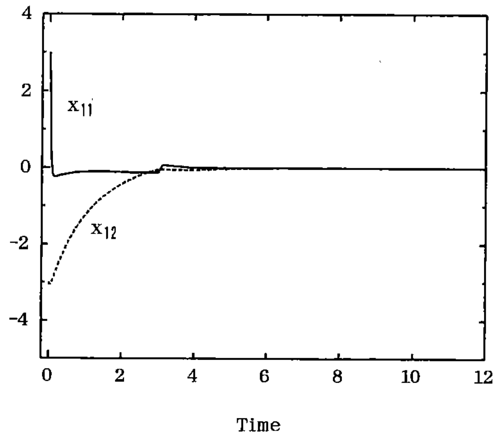

Fig. 1. Response of state variable $x_1(t)$.

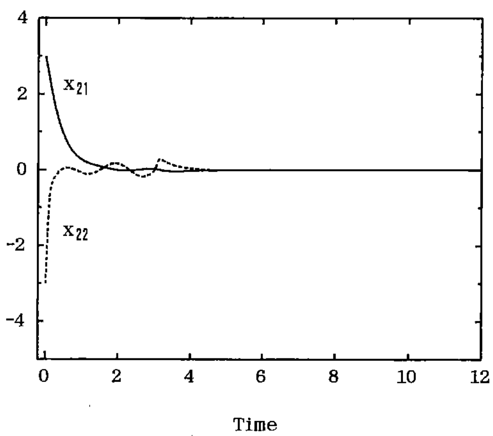

Fig. 2. Response of state variable $x_2(t)$.

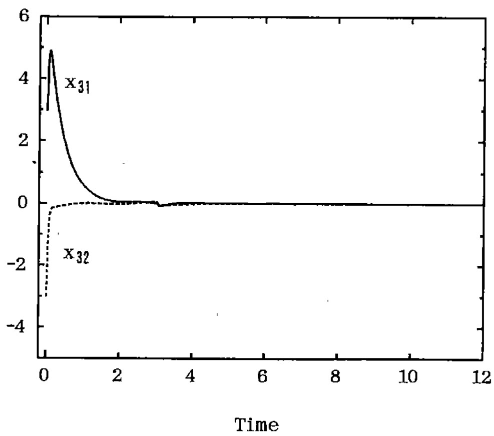

Fig. 3. Response of state variable $x_3(t)$.

With the chosen parameter settings, the results of a simulation are shown in *Figs.1* to *3* for this numerical example.

It can be observed from *Figs.1–3* that the decentralized adaptive robust memoryless state feedback controllers stabilize indeed large scale interconnected system (32) with delayed state perturbations in the interconnections, and the states of the resulting adaptive closed–loop large scale time–delay system converge uniformly asymptotically to zero.

5. Concluding Remarks

The problem of decentralized stabilization has been discussed for a class of large scale linear time–varying systems with delayed state perturbations in the interconnections. Here, the upper bounds of the uncertainties in the interconnections are assumed to be unknown, and control inputs have been represented by the nonlinear functions satisfying the condition of the so–called series nonlinearity. We have proposed the adaptation laws to estimate the unknown upper bounds of the perturbations in the interconnections. Furthermore, by making use of the updated values of these unknown bounds we have constructed a class of decentralized memoryless state feedback controllers. On the basis of the Lyapunov stability theory and Lyapunov–Krasovskii functional, we have shown that by employing the proposed decentralized controllers, the solutions of the resulting adaptive closed–loop large scale time–delay system can be guaranteed to be uniformly bounded, and their states can converge uniformly asymptotically to zero.

Finally, a numerical example is given to demonstrate the synthesis procedure for the proposed decentralized adaptive robust controllers. It is shown from the example and the results of its simulation that the results obtained in the paper are effective and feasible. Therefore, our results may be expected to have some applications to practical decentralized control problems of large scale dynamical systems with delayed state perturbations in the interconnections.

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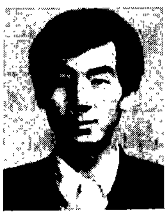

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