Perceptual Clustering with Fuzzy Encoding

Yukinori Suzuki* Member

Akira Kawamura* Non-member

Sato Saga* Non-member

Junji Maeda* Non-member

A human perceives a set of feature points (FPs) as a cluster when he/she finds a mass of collected FPs in a sample space. We have been trying to develop a clustering technique working like clustering in human perception. For such a clustering technique, we introduce the concept of perceptual position in this paper. The perceptual position is based on the assumption that human perception of relative positions among FPs changes depending on the arrangement of FPs around those FPs. To implement a perceptual position, each FP is encoded by a fuzzy set. We then describe a clustering technique using perceptual positions. Computational experiments were carried out to determine the effectiveness of the clustering technique using perceptual position, and the results showed that clustering by the technique using perceptual position is more compatible with clustering by human subjects than is clustering using a conventional fuzzy c-means (FCM) algorithms.

Keywords: Clustering, Perceptual Position, Fuzzy Encoding

1. Introduction

Clustering in pattern recognition and computer vision is essential in knowledge engineering, especially data mining. Clustering techniques enable detection of statistical regularities in a random sequence of input patterns and division of a collection of feature points (FPs) into a number of subgroups. In the individual subgroups, the FPs show a certain degree of closeness or similarity. There are two types of clustering techniques: hard clustering and fuzzy clustering. Hard clustering assigns individual FPs to only one of the clusters for which boundaries are well defined. Fuzzy clustering, on the other hand, can assign individual FPs to multiple clusters with different degrees of typicality. Generally speaking, fuzzy clustering techniques are superior to hard clustering techniques [1]~[5].

Many conventional clustering techniques assume cluster shapes such as a line, a circle, an ellipse, and a rectangle. Clustering techniques are based on such geometrical structures. Therefore, if the distribution of sample data does not conform to one of the geometrical structures, the results of clustering will be greatly different from that based on human intuition. The greater the difference between the shape of the sample data distribution and the shape of the assumed geometrical structure is, the greater is the difference between the clustering results and those based on human intuition. The other problem is noise and/or isolated FPs, which sometimes involve quite different clustering compared to that in the case of no such noise and/or isolated FPs. Thus,

a technique that is robust to noise and/or isolated FPs is needed. Moreover, clustering results depend on the initial clustering to start the technique. Better initial clusters can produce more intuitive clusters.

As we often experience a vision in our daily life, we can perceive clusters in images no matter how many complex clusters there are in the image. We are able to simultaneously find lines, circles, ellipses, free curves, and isolated points in an image. There are no structural assumptions for finding clusters, and our perception of clustering is also robust to noises and/or isolated FPs. Therefore, the human cognitive process for perception is the best model for clustering techniques. The human cognitive process enables clustering to be carried out skillfully in the real world [6]~[9].

In this sense, one of the goals of clustering techniques is the ability to accurately mimic human perception. We have been studying various clustering techniques to achieve this goal. Human vision systems organize FPs into significant perceptual groups that are clusters we perceive. In this manner, a human perceives clusters when he/she finds a mass of FPs in the feature space. The perceptual organization imparts robustness and computational efficiency to the perceptual process for the vision system. In this paper, we introduce the concept of perceptual position using a fuzzy set to approach to the clustering techniques working like clustering human perception. Moreover, a new clustering technique based on perceptual position is proposed. The results of clustering using the proposed technique are compared with the results of clustering performed by human subjects.

The paper is organized as follows. In section 2, we describe the concept of perceptual position and show how

*Muroran Institute of Technology
27-1, Mizumoto-cho, Muroran 050-8585 Japan

it is represented using a fuzzy set. A new clustering technique using perceptual position is presented in section 3. In section 4, the effectiveness of the proposed method is shown by comparing results using this technique with result of clustering by human subjects. Conclusions are presented in section 5.

2. Perceptual Position

An algorithm for clustering that works like clustering in human perception must take in account distances that are perceived by humans which are different from physical distances. The distance perceived by humans is sensational and subjective. We call this distance a perceptual distance in this paper. We assume that humans cluster FPs based on a perceptual distance. Endo and Yamaguchi explained perceptual distance by examples of a painter and a patchworker [6]. They proposed a method to revise a physical distance for realizing a perceptual distance. The revision was carried out using a membership function of a fuzzy set that represents the degree to which two FPs belong to the same mass of the FPs. We propose a more general concept to cluster FPs as they are clustered in human perception in this paper. The concept is a perceptual position. A distance between perceptual positions is a perceptual distance. A perceptual position is based on the following assumption. As shown in Fig. 1, there are two masses of FPs, mass1 and mass2, and one isolated FP. The isolated FP is physically closer to mass1 than to mass2. If humans perceive that the isolated FP is located closer to mass1 than to mass2, it could be perceived that the isolated circle is closer to mass1 than its actual position. On the other hand, we perceive that the isolated FP is more distant from mass2 than it actually is. In this sense, the isolated circle is perceived as if it is drawn towards mass1. This assumption can be explained by Gestalt's principles of perceptual grouping [10]. A perceptual position is introduced to treat this phenomenon computationally.

To model the above assumption computationally, we consider the possibility of the existence of an FP that spreads around in association with other FPs in a local area. The pattern of its spreading is determined by the arrangement of FPs around that FP in the local area. We consider that this spread of the possibility of existence of an FP is the perceptual position. We use a fuzzy set to represent the possibility of the existence of such FP. By the perceptual position, if there is a mass of FPs close to isolated FP, the perceptual position of the mass of FPs and that of the isolated FP overlap each other. This overlap makes the fuzzy distance between the mass of FPs and the isolated FP shorter than the distance when these FPs are represented by crisp sets. This model explains why the distance between the isolated FP and the mass of FPs is perceived as being shorter than the actual distance and also explains why we perceive the isolated FP as being drawn towards the mass of FPs.

All FPs are transformed to a fuzzy set by the following procedures. The perceptual position $s_j (j = 1, 2, ..., N)$ in two dimensional space is a range around a physical
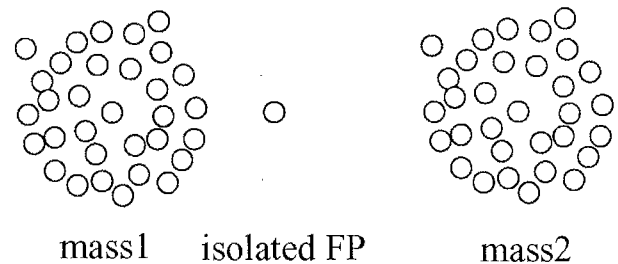

mass1   isolated FP   mass2

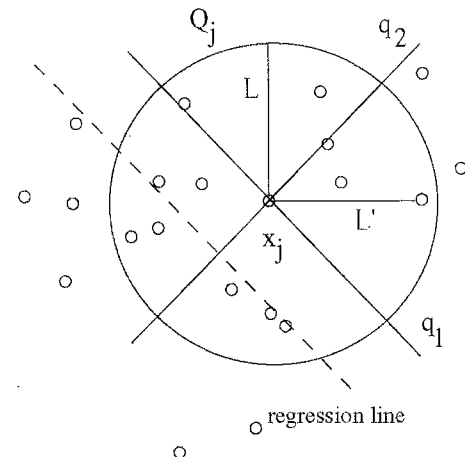Fig. 1. Example of a data set with two masses of FPs and an isolated FP.

Fig. 2. Local rectangular coordinate system for fuzzy encoding.

position x_j, and the shape of it is determined by the arrangement of FPs around x_j in the local area. Here, s_j is represented by a fuzzy set that is characterized by a membership function. It maps the elements of a domain X to the unit interval $[0, 1]$.

$$s_j : X \rightarrow [0, 1]. \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (1)$$

In this sense, a fuzzy set s_j is represented by element $x \in X$, and its grade is membership. We use a domain of a quadrangle in which a membership function of s_j is greater than 0.0. To fix the quadrangle, linear regression is carried out for the FPs within a sphere Q_j centering around x_j with radius L. L' is the distance between the origin and the most-distant FP in Q_j. We then introduce local rectangular coordinates, $q_l (l = 1, 2)$, of which the origin is x_j. The coordinate q_1 is set to be parallel to the regression line and it also passes x_j. Four vertexes of the quadrangle are on respective coordinates as shown in Fig. 2.

Two vertexes, v_l and v_l', on coordinate q_l are located so to have distances from the origin of

$$L_l = \frac{L'}{2(n_c - 1)} n_l, \quad \cdots\cdots\cdots\cdots (2)$$

$$L_l' = \frac{L'}{2(n_c - 1)} n_l', \quad \cdots\cdots\cdots\cdots\cdots\cdots (3)$$

where n_l and n_l' are the number of FPs in Q_j if $q_l \geq 0$ and the number of FPs in Q_j if $q_l < 0$, respectively, n_c
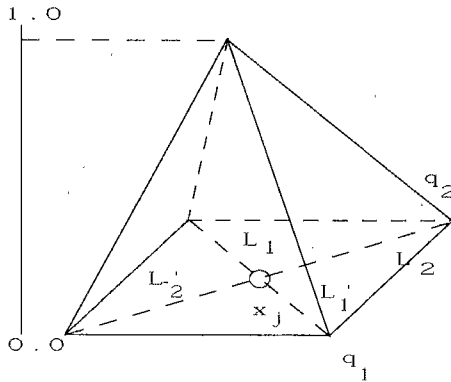
Fig. 3.   Quadrangular pyramid.

is the number of FPs in Q_j. The other two vertexes are fixed in the same manner. The membership function of s_j is defined as a quadrangular pyramid in the domain of the quadrangle. The apex of it is located at x_j and the height is 1.0. The oblique sides of the quadrangular pyramid are obtained to connect the apex and respective vertexes of s_j as illustrated in Fig. 3.

3.   Clustering Technique by Perceptual Positions

Here we present a clustering technique using perceptual positions that are based on FCM algorithm. The FCM algorithm classifies the number of N feature points into C-tuples of prototypes [11]~[14]. Each FP is allowed to belong to multiple prototypes with typicality. The clustering is carried out so as to minimize the objective function given by

$$J(V,U) = \sum_{i=1}^{C} \sum_{j=1}^{N} (u_{ij})^m d_{ij}^2 \quad \cdots\cdots\cdots\cdots\cdots (4)$$

subjected to $\sum_{i=1}^{C} u_{ij} = 1$ for all j.   In (4), $V = (v_1, v_2, ..., v_C)$ is a C-tuple of a prototype, d_{ij}^2 is the distance from x_j to prototype v_i, N is the total number of FPs, C is the number of classes, and $U = [u_{ij}]$ is a $C \times N$ matrix. $U = [u_{ij}]$ has to satisfy the following conditions.

$$u_{ij} \in [0, 1] \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (5)$$

for all i and j,

$$0 < \sum_{j=1}^{N} u_{ij} < N \quad \cdots\cdots\cdots\cdots\cdots (6)$$

for all j, and

$$\sum_{i=1}^{C} u_{ij} = 1. \quad \cdots\cdots\cdots\cdots\cdots\cdots (7)$$

u_{ij} is the grade of membership of x_j in prototype v_i, and $m \in [1, \infty)$ is a weighting exponent that is called a fuzzifier.

In this paper, since x_j is encoded by a fuzzy set that is

symbolized by s_j, the problem is to classify s_j into given C clusters. In this sense, a perceptual position is incorporated into the FCM algorithm. This is performed by repeating the procedure to minimize (4) as follows.

Step (1): Given the number of clusters C into which x_j is classified, the initial partition matrix U^0 is given and counter l is set to zero.

Step (2): The prototype v_i^l is computed by

$$v_i^l = \frac{\sum_{j=1}^{N} (u_{ij})^m s_j}{\sum_{j=1}^{N} (u_{ij})^m}, \quad \cdots\cdots\cdots\cdots\cdots\cdots (8)$$

$(1 \leq i \leq C)$.

Step (3): For $x_j \neq v_i^l$, elements of the partition matrix U^l are renewed to generate U^{l+1} by the equation

$$u_{ij} = \frac{1}{\sum_{k=1}^{C} \left(\frac{\|H(s_j, v_i^l)\|}{\|H(s_j, v_k^l)\|}\right)^{1/(m-1)}}. \quad \cdots\cdots\cdots\cdots (9)$$

$H(s_j, v_i^l)$ is the fuzzy distance between two fuzzy sets, s_j and v_i^l, that is defined as

$$H(s_j, v_i^l) = \frac{\sum_{p=1}^{M} t_p h((s_j)^{t_p}, (v_i^l)^{t_p})}{\sum_{p=1}^{M} t_p}, \quad \cdots\cdots\cdots (10)$$

where $(s_j)^{t_p}$ and $(v_i^l)^{t_p}$ are t_p level sets of s_j and v_i^l, respectively, and $h(\bullet)$ is the Hausdorff distance between two crisp sets [15]. M is the number of levels of s_j. For $x_j = v_i^l$, partition matrix U^l is renewed by

$$\mu_{ij} = \begin{cases} 1 & (j = i) \\ 0 & (j \neq i) \end{cases}. \quad \cdots\cdots\cdots\cdots\cdots\cdots (11)$$

Step (4): If $\|U^{l+1} - U^l\|_G \leq \varepsilon$ is satisfied, the algorithm is terminated and we obtain a final partition matrix. If not, l is replaced by $l + 1$ and we return to step (2). G is a symmetrical positive definite matrix (in this case we use an identity matrix), and ε is a tiny number that is given in advance.

4.   Experiments and Discussion

The human cognitive process for perception carries out clustering skillfully in the real world, and it is considered to be the best model for clustering techniques. In this paper, we have introduced the concept of perceptual position using a fuzzy set to approach to the clustering techniques working like clustering in human perception, and we have described a new clustering technique using the perceptual position.

The results of clustering by the proposed technique are compared here with those of clustering by human subjects. We call clustering by human subjects perceptual clustering. Fig. 4(a) shows a set of sample data that was used for comparison [3]. There are 51 bright
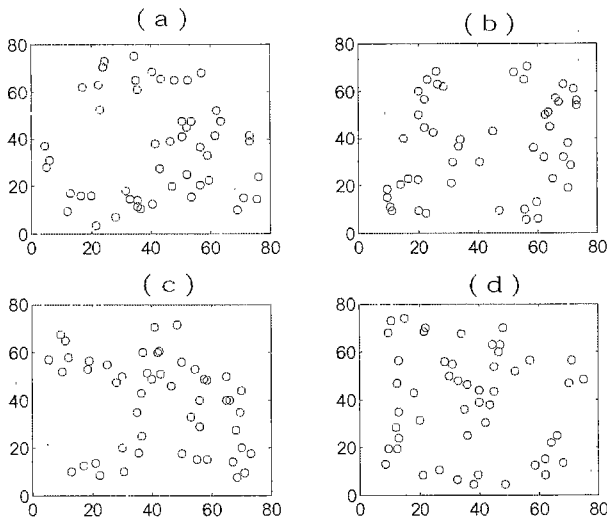
Fig. 4. Four sets of sample data.

Fig. 5. Conceptual figure to explain how to assign the numbers to clusters of the collation that were made by human subjects. The closed circle show FPs and the rectangles are prototypes.

stars near Polaris projected onto the Celestial Equator. These stars are located in a three-dimensional space, and there are nine visual clusters that were established by early astronomers. We made three other sets of sample data by randomly moving the location of each star.

Early astronomers classified the 51 stars into nine clusters in the huge sky. The astronomers saw the whole span of the sky, and then they must have classified stars into individual clusters. In other words they would have seen stars in a local area from a general view. However, as shown in Fig. 4, the subjects were asked to classify the stars in a small range of vision. Then, if we set $C = 9$, it must be too many to classify stars from a general view. The subjects must see only a local area to classify the stars. Therefore, we set $C = 5$ to classify the stars from a general view. In this sense, fifteen human subjects were asked to classify the stars into five clusters for each set of sample data. The results of clustering for one set of sample data performed by the 15 human subjects were integrated to obtain perceptual clustering for that data set.

For the integration, it is necessary to find correspondence between clusters made by one subject and those by other subject. This correspondence was carried out as follows. One typical clustering result was chosen and then cluster numbers were assigned to each cluster. This was used as a reference. Prototype v_i of the reference was computed as

$$v_i = \frac{1}{|\beta_i|} \sum_{x_j \in \beta_i} x_j \quad (i = 1, 2, ..., C), \quad \cdots\cdots\cdots (12)$$

where β_i is a cluster in the result. Prototypes of the other 14 results were then computed using (12). These are collations. One collation was selected, and the prototypes of that collation were compared with the prototypes of the reference. If the location of one prototype of the collation is the closest to the location of one prototype of the reference, we assigned the same prototype number of that prototype of the reference to that prototype of the collation. Thus, the other prototypes of the
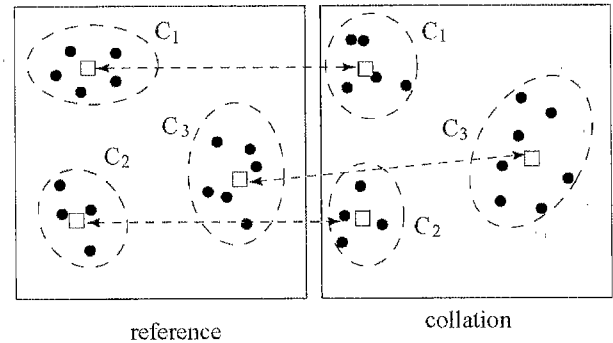
collation were numbered according to the above procedure. In this manner, we assigned numbers to all other prototypes of the collation as shown in Fig. 5.

We then counted how many times individual FPs were classified into individual clusters. Belongingness of each FP to individual clusters was computed to obtain the fuzzy C partition matrix U^p. The elements of U^p are

$$\mu_{ij}^p = \frac{Q}{15}, \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (13)$$

where Q is the number of times x_j is classified into cluster β_i. U^p is a partition matrix of the perceptual clustering. Integrations were also carried out for the other three sets of sample data.

The sets of sample data shown in Fig. 4 were also classified by the technique using a perceptual position, and the C partition matrix U^c was obtained. The difference between U^p and U^c is computed as

$$d_{error} = \sum_{i=1}^{C} \sum_{j=1}^{N} (\mu_{ij}^p - \mu_{ij}^c)^2, \quad \cdots\cdots\cdots\cdots (14)$$

where μ_{ij}^c is an element of U^c. Fig. 6 shows d_{error} for change in L, in which d_{error} at $L = 0$ corresponds to FCM. Since the clustering algorithm depends on initial prototypes, clustering was carried out 5,000 times with changes in initial prototypes. d_{error} is the average value of 5,000 clusterings. Simultaneously, we computed the cluster validity of separation (SE) by

$$SE = \frac{\sum_{j=1}^{N} \sum_{i=1}^{C} \mu_{ij} d_{ij}^2}{|C| \min\{d^2(v_i, o)|v_i, o \in C, v_i \neq o\}},$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (15)$$

where $d(v_i, o)$ is the distance between prototype v_i and prototype o. The smaller the separation is, the better is the clustering.

As shown in Fig. 6, since d_{error} becomes smaller than d_{error} of FCM clustering for change in L other than some d_{error}s, the results of that clustering by the proposed technique is more compatible with clustering by human subjects than is conventional FCM clustering. In Fig.
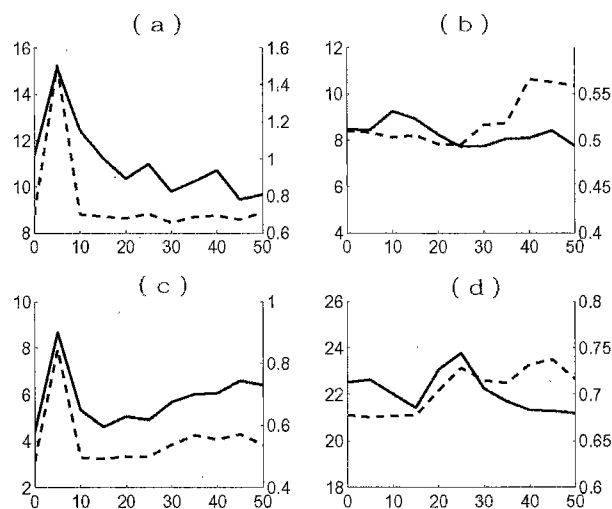
Fig. 6. d_{error} and SE for change in L. The horizontal axis shows L. Left and right vertical axes show d_{error} and SE, respectively. Solid and dotted lines are changes in d_{error} and SE, respectively.

6(a), d_{error} at $L = 10$ is larger than FCM clustering. In this case, SE at $L = 10$ is also larger than FCM clustering. The reason is considered as follows. We introduced the perceptual position in this paper. According to the perceptual position, we consider the possibility of the existence of an FP that spreads around in association with other FPs in a local area. All FPs are transformed into a fuzzy set described in section 2. Consequently, the membership functions of some FPs are overlapped each other. This overlapping may decrease separability of each FP and it is considered to depend on L. Since the overlapping of the membership functions at some L make decrease the separability, d_{error} at that L becomes larger than FCM clustering. In this sense, d_{error} at $L = 10$ is larger than FCM clustering. Therefore, it is essential to choose a suitable L for carrying out clustering like human perception. A suitable L can be determined using SE described below. d_{errors} being larger than FCM clustering in Fig. 6 (b), (c), and (d) are also explained as the same as above mentioned.

For sample data set one, as shown in Fig. 6(a), d_{error} is minimum at $L = 30$ and $L = 45$. SE is also minimum at $L = 30$. For sample data set two, as shown in Fig. 6(b), d_{error} is minimum at $L = 25$ and SE is also minimum at $L = 25$. For sample data set three, as shown in Fig. 6(c), the FCM algorithm, $L = 0$, shows both minimum d_{error} and SE. With the exception of $L = 0$, d_{error} is minimum at $L = 15$ and SE is also minimum at $L = 15$. For sample data set four, as shown in Fig. 6(d), d_{error} is minimum at $L = 15$ and $L = 50$. SE is also minimum neighborhood at $L = 15$. The results of these experiments show that d_{error} changes according to L and that takes minimum value at L, where SE is also minimum. In this sense, we can determine L where d_{error} is minium by a cluster validity of separation.

5. Conclusions

We have introduced the concept of perceptual position

using a fuzzy set to approach the clustering techniques working as human perception. The effectiveness of the perceptual position is demonstrated by the results of experiments in which clustering by the proposed technique was compared with that by human subjects. The results of the experiments show that clustering by the proposed technique is more compatible with clustering by human subjects than is conventional FCM clustering.

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Yukinori Suzuki (Member) received the B.S. degree in electrical engineering from Muroran Institute of Technology in 1980. In 1985 he received Ph. D. degree in biomedical engineering from Hokkaido University. From 1985 to 1989 he was a research associate in the Department of Electronic Engineering in Muroran Institute of Technology. He was a visiting scholar at the Center for Adaptive Systems at Boston University. He is currently an associate professor at the Department of Computer Science and Systems Engineering in Muroran Institute of Technology. He was the program Chair of International Workshop on Soft Computing in Industry, which was held in 1999 in Muroran, Japan. He was a guest editor of Proceedings of the IEEE, 2001 special issue of Industrial Innovations Using Soft Computing. His research interests include computational intelligence and industrial applications.

Akira Kawamura (Non-member) received the B.S. degree in information engineering from Muroran Institute of Technology in 1999. He also received the M.S. degree in information engineering from Muroran Institute of Technology in 2002. His research interests include fuzzy logic and applications to pattern recognition.

Sato Saga (Non-member) received the M.E. and Ph.D. degrees from Hokkaido University, Hokkaido, Japan. From 1987 to 1989, he took on the job of teaching computer science in the Philippines as a member of Japan Overseas Cooperation Volunteers. From 1989 to 1994, he was a researcher at Tecnova Corporation. In 1994, he joined Muroran Institute of Technology, where he currently is an associate professor in the Department of Computer Science and Systems Engineering. His research interests include human interface based on soft computing.

Junji Maeda (Non-member) received the B.S. degree in physics from Hokkaido University in 1972, and the M.E. and Ph.D. degrees in applied physics from Hokkaido University in 1974 and 1985, respectively. He was an Assistant Professor of the Department of Applied Physics at Hokkaido University. In 1988, he joined Muroran Institute of Technology, where he is currently a Professor of the Department of Computer Science and Systems Engineering from 1994. His research interests include image processing and computer vision.