Paper

Some Characterizations of Schur Matrices and Their Application to the Stability of a Polytope of Matrices

Takehiro Mori* Member
Hideki Kokame** Member

Motivated by a general Hurwitz matrix expression involving a triplet of matrices, its Schur counterparts are first derived. This time, they are expressed by a pair of matrices satisfying a certain nonlinear or norm condition. It is then shown that the results can find applications in Schur stability analysis of a polytope of matrices. Using the obtained expessions, two kinds of quadratic Lyapunov functions are proved to work for the polytope:a fixed quadratic function and a parameter-dependent quadratic function. The first kind gives a well known extreme point result on quadratic stability of polytopes of matrices, while the second yields a new result for the stability test of the polytope.

Keywords: Schur stability, Stein inequality, parameter-dependent Lyapunov function, polytope of matrices, quadratic stability

1. Introduction

It is known that any Hurwitz stable matrix can be expressed by a triplet of matrices, two of which are positive definite symmetric and the rest skew-symmetric [(4)]. The expression is shown to effectively give Hurwitz stability conditions for a polytope of matrices by ensuring the existence of two kinds of Lyapunov functions [(8)]. One is a parameter-dependent quadratic Lyapunov function and the other a fixed one which leads to a well known quadratic stability property of matrix polytopes. Polytopic expressions of matrices are now acknowledged as a typical way to model uncertainties involved in state-space representations of control systems and their stability property has attracted attention of those who are interested in robust stability [(1)] [(2)].

The purpose of this paper is to obtain a discrete-time parallel of the above results. We first derive two novel general expressions of Schur matrices. It turns out that the matrices this time can be expressed in terms of two matrices which satisfy certain restrictions. Based on one of these expressions, we explore several characterizations of the Schur property which are centered around the norm condition. We then switch to Schur stability analysis of a polytope of matrices as an application of the above characterizations. As with the continuous-time case, stability conditions are obtained through a fixed quadratic function that guarantees the quadratic stability for the polytope and a parameter-dependent quadratic Lyapunov function ensuring their stability. According to the two Schur stability expressions, we provide respective quadratic stability condition.

The remainder of the paper is plotted as follows. The next section deals with the two expressions which enable to characterize real Schur matrices. The first expression originates from its Hurwitz stability counterpart, while the second one comes from discrete-time Lyapunov equation directly. The third section is devoted to Schur stability conditions of a polytope of matrices derived from using the representations just obtained. Existence conditions of the two sorts of quadratic Lyapunov functions are discussed. The paper concludes with remarks in section 4.

A brief glossary for used symbols is shown in the following. Let X be an $n \times n$ real matrix, $X \in R^{n \times n}$. The eigenvalues and determinant of X are denoted by $\lambda_i(X)$ and $|X|$, respectively. For $X = X'$ with transpose symbol ($'$), $X > 0(\geq 0)$ means positive (semi)definiteness of X, $X < 0(\leq 0)$ negative (semi)definiteness and $X > Y$ with $Y = Y'$ implies $X - Y > 0$. A matrix X is Schur stable, if $|\lambda_i(X)| < 1$ for all $i \in \underline{n} := \{1, \cdots, n\}$ and Hurwitz stable if $\mathrm{Re}\lambda_i(X) < 0$, $\forall i \in \underline{n}$. X is said to be anti-Schur stable, if $|\lambda_i(X)| > 1$, $\forall i \in \underline{n}$. Discrete-time Lyapunov matrix equations(inequalities) are termed in this paper as Stein equations(inequalities) to make a distinction between the continuous-time and discrete-time cases clear. The spectral norm of X is defined by $\|X\| := \max_i\{\lambda_i(XX')\}^{1/2}$. When a matrix X satisfies $X + X' < 0$, such X is denoted as $X \in \mathcal{H}$. Let Π_α be a set of m-tuples of nonnegative numbers defined by:

$$\Pi_\alpha := \{\alpha = (\alpha_1, \cdots, \alpha_m) \mid \sum_{i=1}^{m} \alpha_i = 1,\ \alpha_i \geq 0,\ i \in \underline{m}\}$$

where $\underline{m} := \{1, \cdots, m\}$. For given m matrices, $X_i \in R^{n \times n}$, $i \in \underline{m}$, we define a polytope of the matrices by

* Dept. of Electronics and Information Science, Kyoto Institute of Technology, Matsugasaki, Kyoto 606-8585
** Dept. of Electrical and Electronic Systems, Osaka Prefecture University, Gakuencho, Sakai 599-8531

$$\mathcal{P}(X_i) := \{X_\alpha \mid X_\alpha = \sum_{i=1}^{m} \alpha_i X_i,\ \alpha \in \Pi_\alpha\}.$$

A matrix-valued function $\phi(X)$ with $X \in R^{n\times n}$ and $\phi(X) = \phi(X)'$ is said to be *matrix-convex*, if $\phi(\alpha_1 X_1 + \alpha_2 X_2) \le \alpha_1\phi(X_1) + \alpha_2\phi(X_2)$, $(\alpha_1, \alpha_2) \in \Pi_\alpha$.

2. Some Characterizations of Schur-Matrices

Since the present work is motivated by the Hurwitz matrix characterization [4], we first recall some results about the bilinear trnasformation as a preliminary. By the transformation, Lyapunov equation, Stein equation and their solutions are connected smoothly.

Lemma 1 Consider two matrix equations;

$$A_c'P_c + P_cA_c = -Q_c,\ Q_c = Q_c' \quad \cdots\cdots (1)$$
$$A'PA - P = -Q, \qquad Q = Q'. \quad \cdots\cdots (2)$$

The equation (1) is referred to as Lyapunov equation, whereas (2) Stein equation. The bilinear transformation,

$$A_c = (A - I)(A + I)^{-1} \text{ or } A = (I + A_c)(I - A_c)^{-1} \quad \cdots\cdots (3)$$

transforms (1) to the form of (2) where
i) $Q = 2(I - A_c')Q_c(I - A_c)$, $P = P_c$
ii) $P = \frac{1}{2}(I - A_c')P_c(I - A_c)$, $Q = Q_c$.

In (3), the existence of the inverse of matrices is assumed and this is the case when A and A_c are a Schur matrix and Hurwitz matrix, respectively. We cite the result i) from the reference (4) and ii) from (10), but they can be proven by simple manipulations.

Relevant remarks are given here on the relationships of Lemma 1.

Remark 1 The statement i) indicates that under the transformation Lyapunov and Stein equations share a solution if Q and Q_c are appropriately chosen. Moreover, solutions to Lyapunov inequality, $A_c'P_c + P_cA_c < 0$, and those to Stein inequality, $A'PA - P < 0$, correspond bijectively. On the other hand, ii) shows that equating the right hand sides of (1) and (2) also gives corresponding solutions.

Now, in the reference (4), it is stated that any matrix $A_c \in R^{n\times n}$ is Hurwitz stable , if and only if A_c can be expressed as $A_c = P_c^{-1}(S_c - Q_c)$ where P_c, Q_c and S_c are some n by n real matrices with $P_c = P_c' > 0$, $Q_c = Q_c' > 0$ and $S_c = -S_c'$. This explicit form of Hurwitz matrices can be easily derived by modifying Lyapunov equation [4]. The expression can also be made more compact by setting $R_c = S_c - Q_c$ as

$$A_c = P_c^{-1}R_c,\ R_c \in \mathcal{H}. \quad \cdots\cdots (4)$$

The reference (8) captures the bilinearity of this expression for obtaining Hurwitz stability conditions of a polytope of matrices.

In order to find a discrete-time parallel of the above results, we focus in this section on expressions of Schur matrices in terms of some appropriate matrices. Two expressions will be presented in the following. The first one comes up immediately by way of the bilinear transformation and the above Hurwitz matrix expression.

Theorem 1 A matrix $A \in R^{n\times n}$ is Schur stable, if and only if A has the expression,

$$A = I + 2(P_c - R_c)^{-1}R_c, \quad \cdots\cdots (5)$$

where $P_c = P_c' \in R^{n\times n}$ and $R_c \in R^{n\times n}$ are some matrices satisfying $P_c > 0$ and $R_c \in \mathcal{H}$, respectively.

Proof First of all, we bear in mind the fact that the bilinear transformation (3) is a one-to-one and onto mapping between the set of Hurwitz matrices and that of Schur matrices. This fact almost explains the result, but we follow little more steps to confirm it. Assume Schur stability for A. Notice here that rewriting (3) as

$$A = (I - A_c)^{-1}(I + A_c) = I + 2(A_c^{-1} - I)^{-1} \quad \cdots\cdots (6)$$

and substituting (4) for A_c, we arrive at (5). Since the form of (4) covers every Hurwitz matrix, the resulting (5) also does every Schur matrix due to the above fact. The converse implication, the expression (5) gives Schurness of A, would be apparent. Q.E.D.

Theorem 1 takes a kind of detour route to desired Schur matrix characterizations, since it rests on the transformation and Hurwitz stability characterization. We next show a result, which is direct in the sense that Stein inequality is employed throughout. The following theorem gives therefore a genuine counterpart of the previous Hurwtz matrix expression which comes from Lyapunov equation.

Theorem 2 A matrix $A \in R^{n\times n}$ is Schur stable, if and only if A has the expression $A = MC$ where $M \in R^{n\times n}$ with $|M| \neq 0$ and $C \in R^{n\times n}$ are such that $||CM|| < 1$.

Proof The conclusion follows from the chain of equivalent relations below.

$$\begin{aligned}
& A \text{ is Schur stable} \\
\Longleftrightarrow\ & \exists P = P' > 0,\ A'PA - P < 0 \\
\Longleftrightarrow\ & A'M_1'M_1A - M_1'M_1 < 0, \\
& P = M_1'M_1 \text{ with some } |M_1| \neq 0 \\
\Longleftrightarrow\ & C'C - M_1'M_1 < 0,\ C = M_1A \\
\Longleftrightarrow\ & M'C'CM - I < 0,\ A = MC, \\
& (\text{ by putting } M = M_1^{-1}) \\
\Longleftrightarrow\ & ||CM|| < 1,\ A = MC. \quad \cdots\cdots (7)
\end{aligned}$$

Q.E.D.

Remark 2 The matrices MC and CM share the eigenvalues counting multiplicity and CM is therefore Schur stable as well. This is obvious, if we note they are similar, but it holds in general regardless of their singularity [5]. Obviously, $||CM|| < 1$ implies Schur stability of CM and therefore of $A = MC$. On the contrary, we observe that $||A|| = ||MC|| < 1$ does not generally result from Schur stability of A. However, Theorem 2 exactly designates the class of matrices where the condition $||\cdot|| < 1$ is equivalent to Schur stability: matrices of the form of CM.

Hereafter, we refer the condition $||\cdot|| < 1$ appeared in Theorem 2 for some matrix to the norm condition. Theorem 2 reveals that any Schur matrix can be expressed as a product of a pair of matrices satisfying the norm condition, in the similar manner to the Hurwitz matrix case given in (4). In contrast to Theorem 1, however, Theorem 2 and its proof can yield a variety of ramifications, some of which we list in the following as corollaries. The first one is immediate, if one takes note of the fact that the matrix M can be chosen as $M = P^{-1/2}$ where $P = P' > 0$ is a solution to the Stein inequality, $A'PA - P < 0$.

Corollary 1 A matrix $A \in R^{n\times n}$ is Schur stable, if and only if A is written as $A = P^{-1/2}C$ where the matrices $P = P' > 0, P \in R^{n\times n}$ and $C \in R^{n\times n}$ satisfy $||CP^{-1/2}|| < 1$.

If we further specify C as symmetric, the result gives merely sufficiency, because this restricts the form of A.

Corollary 2 If A has the form of $A = P^{-1/2}W^{1/2}$ with $P = P' > 0$ and $W = W' \geq 0$ satisfying $P > W$, then A is Schur stable.

Proof By the assumptions, we have $A'PA - P = W - P < 0$, showing that the Stein inequality has a solution P. Q.E.D.

Also, as a byproduct of Theorem 2, a parallel result for the anti-Schur property is presented.

Corollary 3 A matrix A is anti-Schur stable, if and only if A is expressed as $A = MC$ with two nonsigular matrices $M \in R^{n\times n}$ and $C \in R^{n\times n}$ satisfying $||M^{-1}C^{-1}|| < 1$.

Proof The anti-Schur property of A is equivalent to the (sign-changed) Stein inequality $A'PA - P > 0$ with $P = P' > 0$. By putting $P = (M')^{-1}M^{-1}$ and $A'PA = C'C$, the proof proceeds in the same way as that of Theorem 2. Q.E.D.

Remark 3 So far as A is nonsingular, A is anti-Schur stable, if and only if A^{-1} is Schur stable. In this case, comparing the above corollary with Theorem 2 leads to the correspondences, $M \leftrightarrow C^{-1}$ and $C \leftrightarrow M^{-1}$. Anti-Schur stability is the mirror concept of Schur stability with respect to the unit circle, appearing in causality discussions. Singularity of A brings to A^{-1} an actually singular situation, eigenvalues at infinity.

3. Schur Stability of a Polytope of Matrices

In this section, the previous results are applied to Schur stability problems of a polytope of matrices, which is a typical representation of uncertainties involved in system models. We are concerned with Schur stability of a polytope of m matrices $\mathcal{P}(A_i)$ which is generated by a given set of Schur stable extreme matrices $\{A_i\}$. The polytope is said to be Schur stable, if so is any of its member matrix. It is well known (1)~(3) that Schur stability of $A_i,\ i \in \underline{m}$ does not automatically imply that of $\mathcal{P}(A_i)$ and we need a stronger condition to ensure the stability of $\mathcal{P}(A_i)$. The results in the previous section are suitable for such stability analysis because the pairs of matrices appearing in the Schur property expressions are all related to the matrix inequalities, which are known to be crucial to treat matrix polytopes.

Schur stability analysis of a matrix polytope can be mainly done by two kinds of quadratic Lyapunov functions: a fixed quadratic Lyapunov function and a parameter-dependent one. In what follows, stability conditions for $\mathcal{P}(A_i)$ will be derived using these two functions.

We start with conditions resulting from a fixed Lyapunov function, which leads to quadratic stability of polytopes. A set of system matrices is said to to be quadratically Schur stable, if there exists a fixed quadratic Lyapunov function proving Schur stability of its every consituent matrix. It is furthermore known that $\mathcal{P}(A_i)$ is quadratically Schur stable, if and only if so is its generating matrix set, $\{A_i\}$. This is due to *matrix convexity* of the left hand side of Stein equation (2) with respect to A (9), that is, $\overline{\phi}(\alpha_1 A_1 + \alpha_2 A_2, P) \leq \alpha_1\overline{\phi}(A_1, P) + \alpha_2\overline{\phi}(A_2, P)$, for any $(\alpha_1, \alpha_2) \in \Pi_\alpha$ where $\overline{\phi}(A, P) := A'PA - P$. It thus follows that quadratic Schur stability of $\{A_i\}$ and that of $\mathcal{P}(A_i)$ are equivalent. This statement holds evidently true as well in case of quadratic Hurwitz stability because of the linearity, or convexity for that matter, of the left hand side of Lyapunov equation (1).

Now, we set out to obtain a quadratic stability condition for $\mathcal{P}(A_i)$ using the expression of Theorem 1.

Theorem 3 Let a set $\{A_i\}$ of Schur stable matrices be given and its members have an expression,

$$A_i = I + 2(P - R_{ci})^{-1}R_{ci},\ i \in \underline{m}, \quad \cdots\cdots\cdots (8)$$

where $P = P' > 0$ and $R_{ci} \in \mathcal{H}$. Then $\mathcal{P}(A_i)$ is quadratically Schur stable. Conversely, quadratic stability of $\mathcal{P}(A_i)$ leads to (8) with some P and R_{ci}.

Proof Let A_i be mapped to a Hurwitz matrix A_{ci} by the bilinear transformation (3). Reminding the proof of Theorem 1, the expression (8) is nothing but to say that A_{ci}'s have the form,

$$A_{ci} = P^{-1}R_{ci},\ i \in \underline{m}, \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots (9)$$

which means the polytope $\mathcal{P}(A_{ci})$ is quadratically Hurwitz stable. Now, Remark 1 asserts that the set of Stein equations, $A_i'\overline{P_i}A_i - \overline{P_i} = -Q_i,\ i \in \underline{m}$ can have a common solution $\overline{P_i} = P$ by suitably choosing $Q_i = Q_i' > 0$. Putting in short, quadratic Hurwitz stability of $\mathcal{P}(A_{ci})$ and quadratic Schur stability of $\mathcal{P}(A_i)$ are equivalent in the sense that they share a common P. Quadratic Schur stability of $\mathcal{P}(A_i)$ thus follows. The argument for the other direction is obvious. Q.E.D.

We can also utilize the other expression of Schur matrices given in Theorem 2 to devise a quadratic Schur stability condition of $\mathcal{P}(A_i)$. We exploit, however, Corollary 1 in place of Theorem 2, because it dispenses with the nonsingularity requirement on M in the theorem. Recall that in Corollary 1 the matrix P amounts to a solution to Stein inequality. The result is shown in the following theorem, which endorses the above-mentioned equivalence between quadratic Schur stability of $\mathcal{P}(A_i)$ and that of $\{A_i\}$ in an alternative way.

Theorem 4 $\mathcal{P}(A_i)$ is quadratically Schur stable, if and only if so is the set $\{A_i\}$. Under this condition, $A_\alpha \in$

$\mathcal{P}(A_i)$ can be expressed as $A_\alpha = P^{-1/2}C_\alpha,\ C_\alpha \in \mathcal{P}(C_i)$ where $P = P' > 0$ is a common solution to the set of Stein inequalities $A_i'PA_i - P < 0,\ i \in \underline{m}$ and C_i are determined by $A_i = P^{-1/2}C_i$.

Proof We prove only sufficiency, since necessity part is immediate. By the assumption that $\{A_i\}$ is quadratically Schur stable and Corollary 1, A_i can be written as $A_i = P^{-1/2}C_i$ where $P = P' > 0$ and $C_i \in R^{n\times n}$ with $||C_iP^{-1/2}|| < 1,\ i \in \underline{m}$. We then have

$$A_\alpha = \sum_{i=1}^{m} \alpha_i A_i = \sum_{i=1}^{m} \alpha_i P^{-1/2}C_i = P^{-1/2}C_\alpha.$$

We can also obtain the norm condition,

$$||C_\alpha P^{-1/2}|| \le \sum_{i=1}^{m} \alpha_i ||C_i P^{-1/2}|| < 1.$$

The second inequality in the above owes to the assumption. Envoking Corollary 1 again readily gives the conclusion that $\mathcal{P}(A_i)$ is quadratically Schur stable with the desired form of A_α. Q.E.D.

Remark 4 The above theorem can be also easily confirmed, if we employ the weighted norm, $||X||_P$ defined by $||X||_P := ||P^{-1/2}XP^{1/2}||$ for the extreme matrices $\{A_i\}$. One of earlier works which pointed out the existence of a common norm satisfying the norm condition for the extreme matrices as a stability condition is the reference (6). Some partial characterizations of a set of Schur matrices having a common solution to the corresponding Stein inequalities are found in the reference (7). As opposed to such characterizations, however, Theorems 3 and 4 give exact conditions for the existence of a common solution. It is remarked that other than the stability analysis of uncertain systems, common Lyapunov function problems are getting focused among those who are interested in analysis of a wide range of systems including fuzzy systems, switching systems and hybrid systems and so forth.

Now, we turn to consider a parameter-dependent Lyapunov function so as to guarantee Schur stability of $\mathcal{P}(A_i)$. The theorem below proves the stability by ensuring the existence of a parameter-dependent solution to the Stein inequality, also assuming the form of expression in Corollary 1 for every extreme matrix.

Theorem 5 Suppose $A_i,\ i \in \underline{m}$ are Schur stable and take the form of

$$A_i = P_i^{-1/2}C,\quad ||CP_i^{-1/2}|| < 1, \quad \cdots\cdots\cdots\cdots (10)$$

where $C \in R^{n\times n}$ and $P_i = P_i' > 0,\ i \in \underline{m}$. Then any $A_\alpha \in \mathcal{P}(A_i)$ is Schur stable and can be expressed as

$$A_\alpha = \bar{P}_\alpha^{\,-1/2}C,\quad \bar{P}_\alpha := P_\alpha^{-2},\ P_\alpha \in \mathcal{P}(P_i^{-1/2}) \quad \cdots\cdots\cdots\cdots (11)$$

The proof is omitted since it follows the same line as that of Theorem 4.

Remark 5 Theorem 5 gives a parameter-dependent quadratic Lyapunov function $\bar{P}_\alpha = (\sum_{i=1}^m \alpha_i P_i^{-1/2})^{-2}$ for a family of discrete-time systems, $z(k+1) = A_\alpha z(k),\ k = 1, 2, \cdots,\ \alpha \in \Pi_\alpha$. In fact, we have $A_\alpha'\bar{P}_\alpha A_\alpha - \bar{P}_\alpha < 0,\ \forall \alpha \in \Pi_\alpha$.

Theorems 4 and 5 thus give a fixed Lyapunov function and a parameter-dependent Lyapunov function for the polytope, respectively. These results run in parallel with their continuous-time counterparts reported in the reference (8), where the bilinearity of the expression of Hurwitz matrices mentioned in the foregoing section is utilized. As with the case of their two continuous-time counterparts, there exists a certain dual relationship between Theorems 4 and 5. To observe this, assume $A_i,\ i \in \underline{m}$ are nonsingular and consider the condition of Theorem 5. Due to Remark 3, the Schur property of $A_i = P_i^{-1/2}C$ is equivalent to the anti-Schur property of $A_i^{-1} = C^{-1}P_i^{1/2}$. This means that in the anti-Schur expression of a matrix $A = MC$ in Corollary 3 M is supposed to be fixed for all $i \in \underline{m}$. In other words, the condition of Theorem 5 is no other than the existence condition of a fixed common solution $U = U' > 0$ to a set of the (sign-changed) Stein inequalities, $A_i'UA_i - U > 0$. On the other hand, as we have seen, the condition of Theorem 4 is just for the existence of a common solution to the Stein inequalities. This interchangeability between fixed matrices and those dependent on i can no longer hold, if any one of A_i's fails to be nonsingular.

At this point, one might wonder why Theorem 1 is not able to join in the scheme to produce a parameter-dependent Lyapunov function in the same way as Theorem 2, by assuming the form of A_i as

$$A_i = I + 2(P_{ci} - R_c)^{-1}R_c,\ i \in \underline{m}. \quad \cdots\cdots\cdots (12)$$

This is explained as follows. The assumption (12) allows that A_{ci}, the continuous-time counterpart of A_i, has the representation $A_{ci} = P_{ci}^{-1}R_c$, which exactly ensures the existence of a parameter-dependent Lyapunov function $(\sum_i^m \alpha_i P_{ci}^{-1})^{-1}$ for the matrix polytope $\mathcal{P}(A_{ci})$ (8). It certainly ascertains Schur stability of the family of matrices which are obtained from $\mathcal{P}(A_{ci})$ through the bilinear transformation. Unfortunately, however, since the transformation can not keep the polytopic form of matrices, $\mathcal{P}(A_i)$ has a subset which is not included in such a family. This implies that the above Lyapunov function does not function for some members of $\mathcal{P}(A_i)$. The situation suggests that we need to contrive another form of parameter-dependent Lyapunov function for $\mathcal{P}(A_i)$ based on the knowledge of P_{ci}. In this case, any Lyapunov function that is *matrix convex* in terms of α would doom to failure, because the matrix-valued function $\overline{\phi}(A, P)$ is not *matrix-convex* with respect to both A and P, although it is convex with respect to only A. This is a major difference between the fixed quadratic Lyapunov function and the parameter-dependent one. In this way, finding a parameter-dependent Lyapunov function via Theorem 1 involves some hurdles to be jumped over.

4. Concluding Remarks

The contribution of this paper is twofold.

The first one is an attempt to obtain a discrete-time counterpart of the known Hurwitz stable matrix expression. To this end, we employed two different approaches: i)combination of the bilinear transformation with the existent Hurwitz matrix representation, ii) direct use of Stein inequality. In either way, Any Schur matrix can be expressed as a function of two matrices that satisfy certain inequalities. In particular, it is shown that in the context of the latter expression a series of characterizations of Schur matrices are possible. They are centered around some norm conditions.

The other contribution is the application of these characterizations to the stability of a polytope of matrices. Two types of Schur stability condition are derived for the polytope based on the obtained characterizations. One of them ensures the existence of a parameter-dependent Lyapunov function, while the other a fixed one which gives the extreme point result on quadratic stability of matrix polytopes. Some of these results have their own continuous-time counterparts and the results run in parallel with them.

In both Hurwitz and Schur cases, a key factor in obtaining the stability results is the parametric expressions of the corresponding matrices. An open question is relationships between a general existence region of eigenvalues of a matrix and a parametric expression of the matrix.

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References

(1) S.P. Bhattacharyya, H. Chapellat, and L.H. Keel: Robust Control, Prentice-Hall, Upper Saddle River, (1995)

(2) B.R. Barmish: New Tools for Robustness of Linear Systems, MacMillan, New York, (1994)

(3) B.R. Barmish and H.I. Kang: "A survey of extreme point results for robustness of control systems", *Automatica*, Vol.**29**, 13 (1993)

(4) S. Barnett and C. Storey: Matrix Methods in Stability Theory, Nelson, London, (1970)

(5) R.A. Horn and C.A. Johnson: Matrix Analysis, Cambridge Univ. Press, Cambridge, 53, (1985)

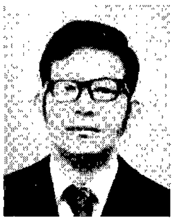
(6) T. Mori and H.Kokame: "The convergence property of interval matrices and interval polynomials", *Int. J. Control*, Vol.**45**, 481 (1987)

(7) Y. Mori, T. Mori, and Y. Kuroe: "A set of discrete-time linear systems which has a common Lyapunov function and its extension", *Proc. of 1998 American Control Conference*, Philadelphia, 2905.

(8) T. Mori and H. Kokame: "A parameter-dependent Lyapunov function for a polytope of matrices", *IEEE Trans. Automat. Contr.*, Vol.**45**, 1516 (2000)

(9) A.W. Marshall and I. Olkin: Inequalities:Theory of Majorization and Its Applications, Academic Press, NY, (1979)

(10) S. Kodama and N. Suda: Matrix Theory for Systems and Control, SICE, Tokyo, (1978)



Takehiro Mori (Member) received B. Eng., M. Eng. and Dr. Eng. degrees in 1968, 1970 and 1977, respectively all from Kyoto University, in Electrical Engineering. From 1974 through 1990, he was a faculty member of the engineering department of the same university, first as a research associate then associate professor. Since 1990, he has been Professor of the Department of Electronics and Information Science, Kyoto Institute of Technology. His research interest includes nonlinear system analysis, robust stability problem and neural network analysis.

Hideki Kokame (Member) received B. Eng., M. Eng. and Dr. Eng. degrees all from Kyoto University in 1968, 1970 and 1978, respectively. From 1973 through 1980, he worked at the same university as a research associate, and from 1980 through 1995 he was a lecturer, associate professor and full professor with Osaka Institute of Technology. Since 1995, he has been a professor with Department of Electrical and Electronic Systems, Osaka Prefecture University. His research interests lie in robust control, control systems design and time-delay system analysis.