Paper

A Method of Robust Model Reference Adaptive Control for Discrete-Time Systems in the Presence of Unmodeled Dynamics

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A method for the design of a robust MRAC(model reference adaptive control) for discrete-time systems in the presence of unmodeled dynamics is proposed. This controller robustly stabilizes the nominal plant in the presence of unmodeled dynamics and achieves the desired model reference adaptive control simultaneously. Furthermore, in this method, we introduce the output-loop compensator for the unmodeled dynamics. Sufficient condition for stabilizing the nominal plant in the presence of unmodeled dynamics is established. Finally, the results of computer simulation are presented to illustrate the effectiveness of the proposed method.

Keywords: MRAC, robust control, unmodeled dynamics, discrete-time system

1. Introduction

The use of robust adaptive control techniques is motivated by the need to automatically adjust the parameters of the controller for the plant having unknown parameters(1)~(8). An adaptive controller is formed by combining an on-line parameter estimator, which provides estimates of unknown parameters at each instant, with a control law that is obtained from the known parameter case. Significant progress has been made on the convergence of adaptive control algorithms. Here it is important to note that the conventional adaptive control strategies cannot be successfully applied to systems in the presence of unmodeled dynamics(9)~(11).

The first task of a control engineer in designing a control system is to obtain a mathematical model that describes the actual plant to be controlled. The actual plant, however, may be too complex and its dynamics may not be completely understood. Developing a mathematical model that describes accurately the physical behavior of the plant over an operating range is a challenging task. Even if a detailed mathematical model of the plant is available, such a model may be of high order leading to a complex controller whose implementation may be costly and whose operation may not be well understood. This makes the modeling task even more challenging because the mathematical model of the plant is required to describe accurately the plant as well as be simple enough from the control design point of view. While a simple model leads to a simpler control design, such a design must possess a sufficient degree of robustness with respect to the unmodeled plant characteristics. To study and improve the robustness properties of control designs, we need a characterization of the types of plant uncertainties that are likely to be encountered in practice. Once the plant uncertainties are characterized in some mathematical form there can be used to analyze the stability and performance properties of controllers designed using simplified plant models but applied to plants with uncertainties.

In this paper we introduce a method to solve the robust MRAC for discrete-time systems in the presence of unmodeled dynamics. Here, we adopt the polynomial approach for designing the controller. This controller robustly stabilizes the nominal plant in the presence of unmodeled dynamics and achieves the MRAC simultaneously. Sufficient condition for robustly stabilizing the nominal model in the presence of unmodeled dynamics is established. This paper proposes a method for solving an important problem related to model reference adaptive control system: the unmodeled dynamic problem. We show how the effect of the unmodeled dynamic can be decoupled from the plant output. If the proposed method is used, it is possible to simply satisfy both MRAC condition and robust stability condition for the unmodeled dynamics.

2. Problem Statement

Consider the following single-input single-output discrete-time system

$$y(t) = \tilde{G}(z)u(t) \quad \cdots\cdots (1)$$

where

$$\tilde{G}(z) = G(z) + \Delta G(z) \quad \cdots\cdots (2)$$

$$G(z) = \frac{z^{-d}B(z)}{A(z)} \quad \cdots\cdots (3)$$

$$A(z) = 1 + \sum_{i=1}^{n} a_i z^{-i}, \quad B(z) = \sum_{j=0}^{m} b_j z^{-j}$$

$G(z)$ is the nominal plant model, $\Delta G(z)$ is unmodeled dynamics, and z^{-1} is the backward shift operator,

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$z^{-1}y(t) = y(t-1)$, d represents the known time delay. Furthermore, $u(t)$ and $y(t)$ are the plant input and output, respectively. The polynomials $A(z)$ and $B(z)$ are relatively coprime polynomials (but having unknown coefficients, and $b_0 \neq 0$), and the plant is a minimum phase system. Furthermore, it is assumed that the unmodeled dynamics $\Delta G(z)$ satisfies the following relation for $z = e^{j\omega}$ in the frequency domain,

$$|\Delta G(z)| \leq |W(z)|, \quad z = e^{j\omega}, \omega \in [0, 2\pi] \quad \cdots\cdots (4)$$

where, $W(z) = W_n(z)/W_d(z)$ is a function prescribing the class of the unmodeled dynamics and is known. Furthermore, $W_n(z)$ and $W_d(z)$ are stable polynomials. Under these circumstances, the objective of the controller design is to obtain the result $\lim_{t\to\infty}(y(t) - y_m(t)) = 0$. The output $y_m(t)$ of the reference model to the command input $r(t)$ is given by

$$y_m(t) = G_m(z)r(t) \quad \cdots\cdots (5)$$

where

$$G_m(z) = \frac{z^{-d}B_m(z)}{A_m(z)} \quad \cdots\cdots (6)$$

$$A_m(z) = 1 + \sum_{i=1}^{\nu} a_{mi}z^{-i}, \quad B_m(z) = \sum_{j=0}^{\mu} b_{mj}z^{-j}$$

and $A_m(z)$ and $B_m(z)$ are stable polynomials. Furthermore, the polynomial $A_m(z)$ and $B_m(z)$ are relatively coprime polynomials, the coefficients a_{mi}, b_{mj} ($i = 1, 2, \cdots, \nu; j = 0, 1, 2, \cdots, \mu$) of the polynomials $A_m(z)$, $B_m(z)$ are specified beforehand such that the model $G_m(z)$ is stable and performs the desirable responses for the arbitrary bounded reference input $r(t)$. The desired response characteristics are set by designing the model $G_m(z)$.

3. Design Method for Known Systems

The problem to be considered in the paper is (1) to design a robust controller which can guarantees robust stability when the unmodeled dynamics exits and (2) to achieve the model matching completely even when the unmodeled dynamics exists.

First, let us consider the case when $\Delta G(z) = 0$. Then, the plant can be rewritten as

$$y(t) = G(z)u(t) = \frac{z^{-d}B(z)}{A(z)}u(t) \quad \cdots\cdots (7)$$

In this case, let us construct a control system such that the model matching can be achieved completely for the nominal system $G(z)$ of the controlled system of eq. (7).

Now, using the pole-zero placement method, it is possible to design a controller such that the closed-loop transfer function of the system from the reference input $r(t)$ to plant output $y(t)$ matches some desired transfer function.

If we choose an asymptotically stable polynomial $T(z)$, then it is known that there exist unique polynomials $R(z)$ and $S(z)$, which satisfy the following Diophantine equation [12],[13]

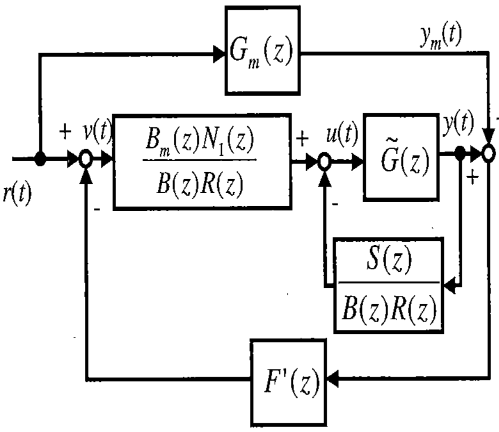


Fig. 1. The block diagram of the proposed system.

$$A(z)N(z) + z^{-d}M(z) = A_m(z)N_1(z) \quad \cdots\cdots (8)$$

where $deg\ N_1(z) = l (l \leq n)$, $deg\ N(z) = l + \nu - n$ and $deg\ M(z) = n - 1$. $N_1(z)$ is the part of the desired closed-loop characteristic polynomial which should not influence the reference tracking.

Let the control input $u(t)$ be given by

$$u(t) = T_1(z)v(t) - T_2(z)y(t) \quad \cdots\cdots (9)$$

where

$$T_1(z) = \frac{B_m(z)N_1(z)}{B(z)R(z)} \quad \cdots\cdots (10)$$

$$T_2(z) = \frac{S(z)}{B(z)R(z)} \quad \cdots\cdots (11)$$

$$v(t) = r(t) - e^0(t) \quad \cdots\cdots (12)$$

$$e^0(t) = F'(z)[y(t) - y_m(t)] \quad \cdots\cdots (13)$$

$$F'(z) = \frac{A_m(z)F_2(z)}{B_m(z)[1 - F_1(z)]} \quad \cdots\cdots (14)$$

$F'(z)$ denotes a compensator. $F_1(z)$, $F_2(z)$, $R(z)$ and $S(z)$ are transfer functions. This model matching controller given in eq. (9) could be thought of as a combination of feedback having the transfer function $T_1(z)$, a feedforward with the transfer function $T_2(z)$ and the output-loop compensator $F'(z)$.

The block diagram of the proposed system is shown in Fig. 1.

Our goal is to design a controller that makes the output of the system be given by $[z^{-d}B_m(z)/A_m(z)]r(t)$. When the control input of eq. (9) is synthesized by utilizing $T_1(z)$ of eq. (10) and $T_2(z)$ of eq. (11), in order to achieve the model matching completely, $R(z)$ and $S(z)$ must satisfy

$$A(z)R(z) + z^{-d}S(z) = A_m(z)N_1(z) \quad \cdots\cdots (15)$$

Here, by using $N(z)$ and $M(z)$ of eq. (8), the polynomials $R(z)$ and $S(z)$ which satisfy the following relations are selected,

$$R(z) = N(z) + z^{-d}K(z) \quad \cdots\cdots (16)$$

$$S(z) = M(z) - K(z)A(z) \quad \cdots\cdots (17)$$

where, $K(z)$ is a rational function introduced for the purpose that the robust stability condition described in Chapter 4 will be satisfied simply and the model matching will not be destroyed. By utilizing the proposed method, the model matching condition and the robust stability condition for additive perturbations can be satisfied simply by properly selecting $K(z)$. When the control is performed by the control input in eq. (9), the model matching can be realized completely by using the above method. This can be confirmed as follows.

Substituting the input $u(t)$ in eq. (9) into eq. (1), and using eqs. (10) $\sim$ (17), the following equation can be obtained.

$$y(t) = \frac{z^{-d}B_m(z)}{A_m(z)}r(t) = G_m(z)r(t) \quad \cdots\cdots\cdots (18)$$

Hence, it is confirmed that when the control is to be performed by utilizing the input $u(t)$ in eq. (9), the transfer function of the system between $r(t)$ and $y(t)$ becomes equal to that of the reference model $G_m(z)$.

When the unmodeled dynamics $\Delta G(z)$ exists, substituting eq. (9) into eq. (1), and using eqs. (10) $\sim$ (14), and eqs. (16), (17), it can be shown that

$$y(t) = \frac{z^{-d}B_m(z)}{A_m(z)}[1 + \Gamma(z)\Delta_1(z)]r(t) \quad \cdots\cdots\cdots (19)$$

where

$$\Gamma(z) = 1 - F_1(z) \quad \cdots\cdots\cdots (20)$$

$$\Delta_1(z) = \frac{z^d A^2(z)\Delta G(z)}{A_m(z)N_1(z)B(z)F(z) + \Omega(z)\Delta G(z)} \quad \cdots\cdots\cdots (21)$$

$$F(z) = 1 - F_1(z) + z^{-d}F_2(z) \quad \cdots\cdots\cdots (22)$$

$$\Omega(z) = A(z)S(z)[1 - F_1(z)] + A_m(z)N_1(z)A(z)F_2(z) \quad \cdots\cdots\cdots (23)$$

Furthermore, the error between plant output and the desired output can be given by

$$e(t) = y(t) - y_m(t) = \frac{z^{-d}B_m(z)}{A_m(z)}[1 - F_1(z)]\Delta_1(z)r(t) \quad \cdots\cdots (24)$$

It is desired to eliminate the effect of unmodeled dynamics $\Delta G(z)$ on output $y(t)$. It is clear that the objective is accomplished, if $F_1(z)$ is chosen as

$$F_1(z) = \rho\frac{\bar{H}(z)}{H(z)} \quad \cdots\cdots\cdots (25)$$

where $H(z)$ is a stable polynomial, and $\bar{H}(z) = z^{-\alpha}H(z^{-1})$, $\alpha = degH(z)$. Furthermore, $\rho \simeq 1$ and $\rho < 1$. Thus $F_1(z)$ is an allpass filter. Here the constant ρ is introduced to avoid the singular case in eq. (14). From eq. (25), we obtain

$$|F_1(z)| = |\rho\frac{\bar{H}(z)}{H(z)}| = \rho \simeq 1$$

$$z = e^{j\omega}, \omega \in [0, 2\pi] \quad \cdots\cdots\cdots (26)$$

By using eqs. (19), (20), (25), (26), we obtain

$$y(t) \simeq \frac{z^{-d}B_m(z)}{A_m(z)}r(t) \quad \cdots\cdots\cdots (27)$$

As can be seen in eq. (27), the transfer function of the system between $r(t)$ and $y(t)$ becomes almost equal to that of the reference model. Therefore, it is possible to realize model matching even in the presence of unmodeled dynamics.

Furthermore, we choose $F_2(z)$, which satisfies the condition that $1 - F_1(z) + z^{-d}F_2(z)$ is stable.

4. Design Method for Parameter Variation Systems

When the coefficients of eq. (1) are unknown, or the plant parameters vary suddenly during operation, the problem of estimation of the unknown parameters of plant arises. Eq. (7) can be written as

$$y(t) = -\sum_{i=1}^{n} a_i z^{-i}y(t) + \sum_{j=0}^{m} b_j z^{-d-j}u(t) = \alpha^T X(t) \quad \cdots\cdots\cdots (28)$$

where T denotes the transpose, and

$$\alpha^T = [a_1, a_2, \cdots, a_n, b_0, b_1, \cdots, b_m] \quad \cdots\cdots\cdots (29)$$

$$X(t) = [-y(t-1), \cdots, -y(t-n), u(t-d), \cdots, u(t-d-m)]^T \quad \cdot\ (30)$$

The vector α represents the unknown parameters of plant to be estimated. This is accomplished by using an identification model described by the equation

$$\hat{y}(t) = \hat{\alpha}^T(t)X(t) \quad \cdots\cdots\cdots (31)$$

where

$$\hat{\alpha}^T(t) = [\hat{a}_1(t), \hat{a}_2(t), \cdots, \hat{a}_n(t), \hat{b}_0(t), \hat{b}_1(t), \cdots, \hat{b}_m(t)]$$

and $\hat{y}(t)$ is an estimate of the output $y(t)$ at time t, and $\hat{\alpha}(t)$ is an adjustable parameter vector. The parameter adjustment law, which ensures that the estimated parameters can converge to their true values, is given by

$$\hat{\alpha}(t) = (1 - \eta|\varepsilon_0(t)|)\hat{\alpha}(t-1) - \Gamma(t-1)X(t)\varepsilon_0(t) \quad \cdots\cdots\cdots (32)$$

$$\Gamma(t) = \frac{1}{\sigma}[\Gamma(t-1) - \frac{\lambda\Gamma(t-1)X(t)X^T(t)\Gamma(t-1)}{\sigma + \lambda X^T(t)\Gamma(t-1)X(t)}] \quad \cdots\cdots\cdots (33)$$

$$\varepsilon_0(t) = \hat{y}(t) - y(t) = \frac{\hat{\alpha}^T(t-1)X(t) - y(t)}{1 + X^T(t)\Gamma(t-1)X(t)} \quad \cdots (34)$$

$$\Gamma(0) = \delta I, \quad \delta > 0 \quad \cdots\cdots\cdots (35)$$

where $\eta > 0, 0 < \sigma \leq 1$ and $0 < \lambda < 2$ [(2)~(4)].

The control input $u(t)$ in the adaptive case is given by

$$u(t) = \frac{B_m(z)N_1(z)}{\hat{B}(z)\hat{R}(z)}v(t) - \frac{\hat{S}(z)}{\hat{B}(z)\hat{R}(z)}y(t) \quad \cdots (36)$$

where $\hat{R}(z)$, $\hat{B}(z)$ and $\hat{S}(z)$ are the estimates of $R(z)$, $B(z)$ and $S(z)$, respectively.

5. Robustness Analysis

Now, we will discuss the robust stability briefly.

5.1 Robustness Analysis for Known Systems

Theorem: If the condition in eq. (37) is satisfied, the closed-loop system using the control input in eq. (9) is a robust stabilizing controller.

$$\left|\frac{\Omega(z)W(z)}{A_m(z)N_1(z)B(z)F(z)}\right| < 1$$
$$z = e^{j\omega}, \omega \in [0, 2\pi] \quad \cdots\cdots\cdots\cdots\cdots (37)$$

Proof: When the control input obtained from eq. (9) is applied to the plant in eq. (1), the closed-loop characteristic equation can be given by

$$A_m(z)N_1(z)B(z)F(z) + \Omega(z)\Delta G(z) = 0 \quad \cdots (38)$$

In the above equation $A_m(z)N_1(z)B(z)F(z)$ is the nominal closed-loop characteristic equation and $\Omega(z)\Delta G(z)$ is an increment due to the unmodeled dynamics of the plant. It is important to note that the closed-loop system obtained by using nominal controller to control the current plant is stable, then the system is robustly stable. Using small gain theorem(14)~(17) and eq. (37), it can be shown that if the nominal closed-loop is asymptotically stable, then the closed-loop will be stable even if the current plant is used in the loop, when

$$\left|\frac{\Omega(z)\Delta G(z)}{A_m(z)N_1(z)B(z)F(z)}\right| < 1$$
$$z = e^{j\omega}, \omega \in [0, 2\pi] \quad \cdots\cdots\cdots\cdots\cdots (39)$$

Let us choose $W(z)$ as

$$|\ \Delta G(z)\ | \le |\ W(z)\ |, \qquad z = e^{j\omega}, \omega \in [0, 2\pi] \quad \cdots (40)$$

then eq. (39) is satisfied if the condition in eq. (37) holds.

Since the nominal closed-loop system $A_m(z)N_1(z) \times B(z)F(z)$ is stable, therefore, if the condition in eq. (37) holds, the closed-loop stability for the current plant can be achieved by using the nominal controller for all unmodeled dynamics.

(Q.E.D.)

Let us now define the following equation.

$$J(z) = \frac{\Omega(z)W(z)}{A_m(z)N_1(z)B(z)F(z)} \quad \cdots\cdots\cdots\cdots\cdots (41)$$

By using eqs. (16), (17), (23) and (41), we obtain

$$J(z) = \frac{\Omega(z)W(z)}{A_m(z)N_1(z)B(z)F(z)}$$
$$= \frac{[A_m(z)N_1(z)F_2(z) + M(z)(1 - F_1(z))]A(z)W(z)}{A_m(z)N_1(z)B(z)F(z)}$$
$$- \frac{A^2(z)(1 - F_1(z))W(z)}{A_m(z)N_1(z)B(z)F(z)}K(z) \quad \cdots\cdots\cdots (42)$$

In the above equation, $A_m(z)$, $N_1(z)$, $F_2(z)$, $M(z)$, $F_1(z)$, $A(z)$, $W(z)$, $B(z)$, $F(z)$ are all known. Then, the problem is equivalent to finding the rational function $K(z)$ such that $J(z)$ given in eq. (42) can satisfy $|\ J(z)\ | < 1$. In this way, the rational function $K(z)$ is independent of the unmodeled dynamics and the plant, and the model matching will not be destroyed. Therefore, the rational function $K(z)$ can easily be implemented by using the methods of Ref. (10) and Ref. (11).

Furthermore, $A(z)$ is a stable polynomial in many cases. When $A(z)$ is a stable polynomial, the transfer function $K(z)$ can easily be obtained as

$$K(z) = \frac{L_1(z)W(z)Q(z)}{L_1(z)A(z)W(z)(1 - F_1(z))}$$
$$- \frac{\rho_1\bar{L}_1(z)A_m(z)N_1(z)B(z)F(z)}{L_1(z)A^2(z)W(z)(1 - F_1(z))} \quad \cdots (43)$$

where $Q(z) = A_m(z)N_1(z)F_2(z) + M(z)(1 - F_1(z))$, $L_1(z)$ is a stable polynamial, and $\bar{L}_1(z) = z^{-\beta}L_1(z^{-1})$, $\beta = degL_1(z)$. Furthermore, $0 < \rho_1 < 1$.

Substituting eq. (43) into eq. (42), we obtain

$$J(z) = \rho_1\frac{\bar{L}_1(z)}{L_1(z)} \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (44)$$

From eq. (44), we obtain

$$|\ J(z)\ | = |\ \rho_1\frac{\bar{L}_1(z)}{L_1(z)}\ | = \rho_1 < 1$$
$$z = e^{j\omega}, \omega \in [0, 2\pi] \quad \cdots\cdots\cdots\cdots\cdots (45)$$

From eqs. (42), (45), we can choose $K(z)$ which satisfies the condition in eq. (37). This implies that the current closed-loop system can be stabilized by the nominal controller.

Furthermore, when $A(z)$ is an unstable polynomial, the transfer function $K(z)$ can be obtained by using H_∞ control theory(10),(11).

5.2 Robustness Analysis for Unknown Systems

The robust stable condition in the adaptive case is given by

$$|\ J'(z)\ | = \left|\frac{\hat{\Omega}(z)W(z)}{A_m(z)N_1(z)\hat{B}(z)F(z)}\right| < 1$$
$$z = e^{j\omega}, \omega \in [0, 2\pi] \quad \cdots\cdots\cdots\cdots\cdots (46)$$

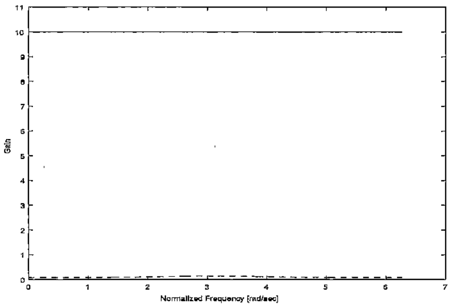
where

$$\hat{\Omega}(z) = \hat{A}(z)\hat{S}(z)[1 - F_1(z)]$$
$$+ A_m(z)N_1(z)\hat{A}(z)F_2(z) \quad \cdots\cdots\cdots (47)$$

The transfer function $K(z)$ in the adaptive case is given by

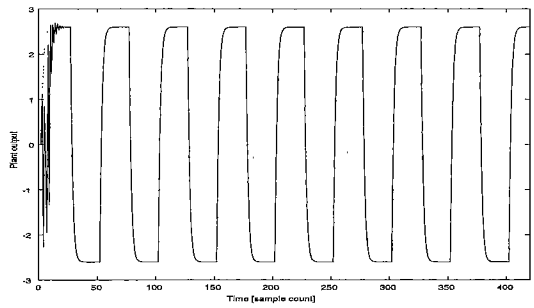
$$K(z) = \frac{L_1(z)W(z)Q(z)}{L_1(z)\hat{A}(z)W(z)(1 - F_1(z))}$$
$$- \frac{\rho_1\bar{L}_1(z)A_m(z)N_1(z)\hat{B}(z)F(z)}{L_1(z)\hat{A}^2(z)W(z)(1 - F_1(z))} \quad \cdots (48)$$

By using eqs. (16), (17), (46) $\sim$ (48), we obtain

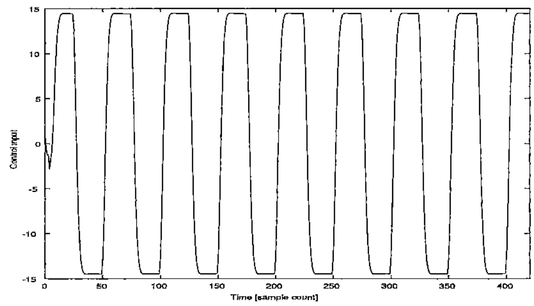


- -: $\mid \Delta G(e^{j\omega}) \mid$, —: $\mid W(e^{j\omega}) \mid$

Fig. 2. Gains of $\mid \Delta G(e^{j\omega}) \mid$ and $\mid W(e^{j\omega}) \mid$.



(a) Plant output $y(t)$ (—) and reference output $y_m(t)$ ($\cdots$)



(b) Input $u(t)$

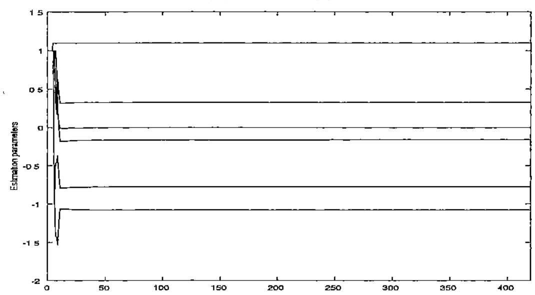


Fig. 3. Results of robust MRAC.

$$\mid J'(z) \mid = \mid \rho_1 \frac{\bar{L}_1(z)}{L_1(z)} \mid = \rho_1 < 1$$

$$z = e^{j\omega}, \omega \in [0, 2\pi] \quad \cdots\cdots\cdots\cdots\cdots \quad (49)$$

Therefore, we can choose $K(z)$ in eq. (48) which satisfies the condition in eq. (46). This implies that the current closed-loop system can be stabilized by the nominal controller in the adaptive case.

6. Simulations

In this section, the results of simulations are presented to give an indication of the performance of the robust MRAC scheme for discrete-time systems in the presence of unmodeled dynamics.

Let us consider the system described by the following equation

$$\tilde{G}(z) = \frac{z^{-3}\tilde{G}_{num}(z)}{\tilde{G}_{den}(z)}$$

$$\tilde{G}_{num}(z) = 1.1 - 1.16z^{-1} + 0.247z^{-2} + 0.1286z^{-3} - 0.04548z^{-4}$$

$$\tilde{G}_{den}(z) = 1 + 0.7z^{-1} - 0.04z^{-2} - 0.127z^{-3} - 0.0297z^{-4} - 0.0018z^{-5}$$

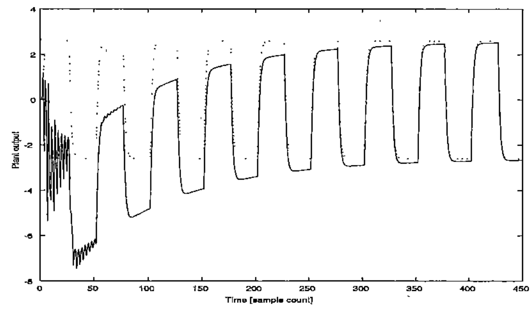
Let the estimation model (nominal plant $G(z)$) be described as

$$G(z) = \frac{z^{-3}(b_0 + b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3})}{1 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3}}$$

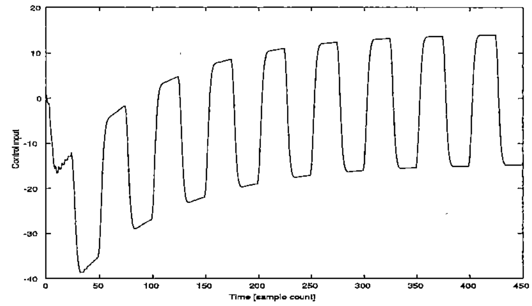
For the unmodeled dynamics $\Delta G(z)$($\Delta G(z) = \tilde{G}(z) - G(z)$) of the above equation, let us consider the case when $W(z)$ is selected as

$$W(z) = 10$$

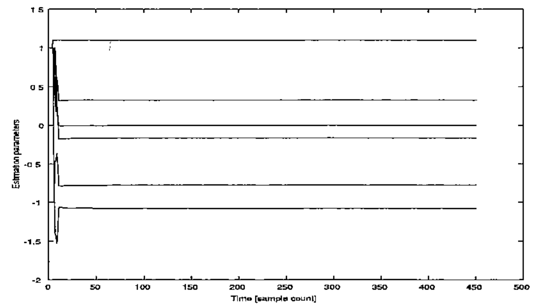
Furthermore, we choose some parameters as follows.



(a) Plant output $y(t)$ (—) and reference output $y_m(t)$ ($\cdots$)

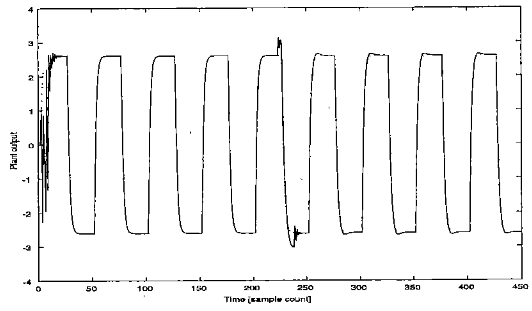


(b) Input $u(t)$

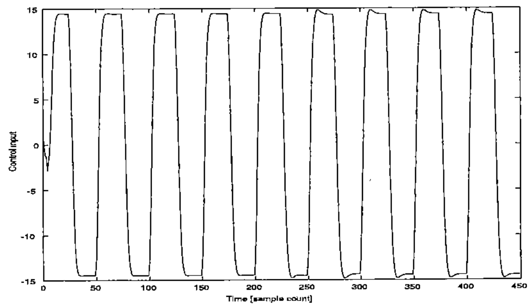


(c) Convergence of the estimated plant parameters

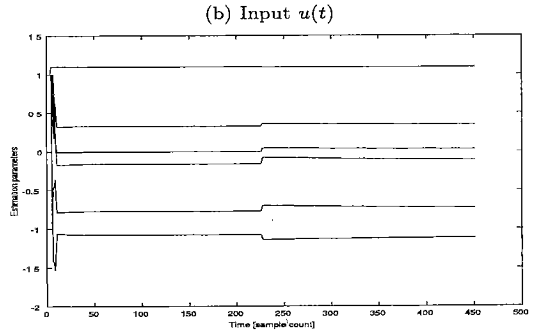
Fig. 4. Results of robust MRAC.



(a) Plant output $y(t)$ (—) and reference output $y_m(t)$ $(\cdots)$



(b) Input $u(t)$



(c) Convergence of the estimated plant parameters

Fig. 5. Results of robust MRAC under parameter variation.

$$H(z) = L(z) = 1 + 0.01z^{-1}$$
$$F_2(z) = \frac{0.863}{F_{2d}(z)}$$
$$F_{2d}(z) = 1.01 + 0.8987z^{-1} + 0.8895z^{-2} + 0.008806z^{-3}$$
$$N_1(z) = 1 - 0.08z^{-1} + 0.0007z^{-2}$$
$$\rho = 0.99, \quad \rho_1 = 0.001$$
$$\eta = 0.001, \quad \lambda = \sigma = 1$$

The reference model $G_m(z)$ is given by

$$G_m(z) = \frac{z^{-3}(1 + 0.5z^{-1} + 0.06z^{-2})}{1 - 0.2z^{-1} - 0.25z^{-2} + 0.05z^{-3}}$$

The gains of $\Delta G(e^{j\omega})$ and $W(e^{j\omega})$ in this case are shown in Fig. 2. Looking at Fig. 2, it is seen that $W(z)$ satisfies $|\Delta G(e^{j\omega})| \leq |W(e^{j\omega})|$ for all $z = e^{j\omega}$.

The plant output $y(t)$ and the reference output $y_m(t)$ are plotted in Fig. 3 (a) and the convergence of the plant parameters that were estimated is plotted in Fig. 3 (c). Fig. 3 shows that the plant output $y(t)$ can converge to the desired output $y_m(t)$, when the unmodeled dynamics exists. And Fig. 3 (c) shows that the estimated parameters can stably converge.

The robust adaptation control when $F(z)$ is not introduced(the conventional adaptation control(1)), the result is shown in Fig.4. As shown in Fig.3 and Fig.4, we see that better results are obtained by using the proposed method.

In the case when the plant parameters vary suddenly to $\tilde{G}_{1num}(z)$ and $\tilde{G}_{1den}(z)$ during operation, the simulation results become as shown in Fig.5. The polynomials $\tilde{G}_{1num}(z)$ and $\tilde{G}_{1den}(z)$ are described by the following equations:

$$\tilde{G}_{1num}(z) = 1.1 - 1.25z^{-1} + 0.365z^{-2} + 0.1005z^{-3} - 0.04536z^{-4}$$
$$\tilde{G}_{1den}(z) = 1 + 0.6z^{-1} - 0.12z^{-2} - 0.146z^{-3} - 0.0309z^{-4} - 0.0018z^{-5}$$

As shown in Fig. 5, we obtain good control results regardless of the plant parameter variations during operation. The estimated parameters also converge to their true values fast.

7. Conclusion

We have proposed a new technique to design a robust MRAC for discrete-time systems in the presence of unmodeled dynamics, such that the robust stability of the discrete-time system can be assured in presence of unmodeled dynamics. Furthermore, the results of computer simulation have shown that the plant output $y(t)$ can converge to the desired output $y_m(t)$, when the unmodeled dynamics exists. Sufficient condition for the robust stability of the system has been derived.

(Manuscript received Nov. 29, 2001,
revised Nov. 22, 2002)

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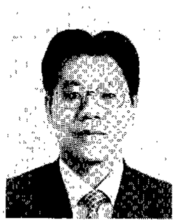

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