Paper

Development of Matrix Operation Type Protection Relay

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This paper presents a new digital type protection relay based on matrix operations. The basic principle of the matrix operation type protection scheme has been already discussed and investigated. However, further studies have been required especially to confirm the real time capability of the proposed scheme, in addition to evaluate the accuracy of fault detection and the identification of fault location. The off-line pre-calculation has been proposed for solving the time-consuming problem. Furthermore, the interpolation technique based non-iterative solution has been developed, which replaces the traditionally iterative Newton-Raphson's method for saving required calculation time. Combining both of the off-line pre-calculation and the interpolation method enables to ensure the activation time of the proposed protection relay within two cycles after a fault occurrence. Moreover, a high accuracy of fault detection can be achieved. In order to demonstrate the reliability of the proposed scheme, numerical simulations have been performed through using the actual data measured on real power systems with different voltage levels. The compared results indicate the highly precise estimation of the fault locations compared with the inspected fault locations on transmission lines. The required computation time is also estimated with considering types of numerical calculations included in the proposed algorithm. The estimated computation time is short enough for the real time implementation of the proposed scheme.

Keywords: Kirchhoff's laws, matrix operation, fault location, arc resistance, system configuration, single-terminal measurement, interpolation, off-line pre-calculation

1. Introduction

The conventional mechanical type protection relays have been replaced to digital type ones during the last 30 years to achieve the reliable power system protection. However, the basic functions of digital type relays are still the same as just those of the mechanical ones designed on the basis of symmetrical coordinates. Following the quite rapid progress of the computational capabilities on CPUs, the additional functions such as the identification of both the fault location and the fault type can be considered to readily achieve a highly reliable power system operation with the digital protection relay. The existing digital relays are roughly classified into two types: the current differential type relay based on the Kirchhoff's first law and the impedance type relay based on the Kirchhoff's second law.

In this study, through the simultaneous consideration of the Kirchhoff's first and second laws, the integration of two types of relays has been proposed to bring the highly improved protection capability onto power systems. Namely, a matrix operation type protection method [1] has been presented to detect the multifarious and complicated faults in power systems. The proposed digital relay also includes the fault locator based on both of the Kirchhoff's first and second laws [2].

Its highly improved fault-locating accuracy has been successfully demonstrated through the field tests compared with that of the conventional ones [3][4]. However, the additional testing of the fault locator will be required to demonstrate its real time availability of operation. The real time availability might be degraded because of the required relatively heavy computational burden for the iterative calculation to reach the least square solution. For practical usage, the computational time of the proposed protection relay should be within two cycles after the fault occurrence. The most time-consuming calculations included are those of diagonal transformations and matrix inversions. The off-line pre-calculation has been proposed for solving the time-consuming problem. On the other hand, even though the accuracy of the Newton-Raphson's approximate method is quite high, it might require longer computational time. To overcome this situation, the interpolation technique has been introduced to the proposed protection relay scheme. The proposed scheme is basically a non-iterative calculation system, so the computational time is remarkably reduced. Meanwhile the accuracy of the proposed protection scheme is quite high.

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To demonstrate the reliability of the proposed scheme of fault location, numerical simulations have been performed for detecting the fault location based on the actual data of voltages and currents, which are measured on real power system with different voltage levels. The simulation results indicate the highly precise estimation of the fault location compared with the inspected fault location on transmission lines. The required computation time is estimated as well by considering of numerical calculation types included in the proposed algorithm. The estimated computation time is short enough for the real time implementation.

The proposed protection scheme provides the following functions: the identification of fault location, the integrated protection with the main and the back-up protection, the detection of different types of faults such as grounding, short-circuit and other else. In addition, the proposed protection relay is applicable to any power system with different voltage levels and also with different grounding methods.

2. Concepts of the Developed Matrix Operation Type Relay

Several characteristics of the matrix operation type protection relay are summed up as follows:

(1) Both the Kirchhoff's first and second laws are utilized to strictly formulate fault phenomena including mutual impedances between transmission lines.

(2) Generators (G) and loads (L) connected to the same bus are combined together and expressed with the telemeter (TM) as shown in Fig. 1. Meanwhile, the arc resistances at the fault location and the earth electrostatic capacities of each line are considered as well shown in Fig. 2.

(3) Circuit equations are set up for each section of multi-terminal transmission lines.

(4) Circuit equations are represented in the matrix form. With exception of $\boldsymbol{i}_f$, k and $\boldsymbol{R}_f$, all of variables are eliminated by means of the diagonal matrix transformation.

(5) Several solution methods of simultaneous equations are utilized for different states of power system such as the single-circuit fault, the double-circuit fault and the mono-phase grounding fault and etc, also, for different measuring methods such as the single-terminal measurement as shown in Fig. 1 and the double-terminal measurement.

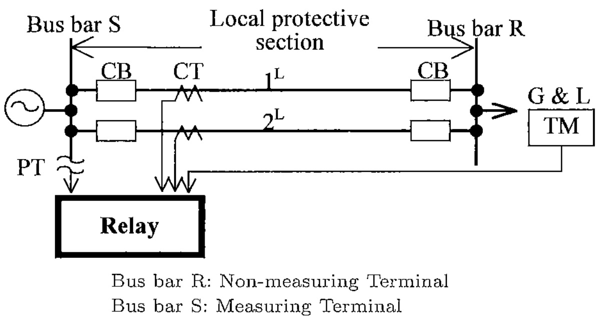

Fig. 1. Configuration of single-terminal measurement type protection scheme

(6) The final calculation results of the matrix operation are not only used for tripping the circuit breakers (CBs) but also for the system maintenance and the system restoration according to the fault location and the fault cause. The fault causes such as snow, lightning, trees, birds and animals are estimated by comparing the arc resistance at fault location with the actual data of past. The estimation is carried out after the detection of fault location.

(7) The off-line pre-calculation is developed for all possible system configurations and for shortening fault detection time. In addition, the off-line pre-calculation is readily done whenever the system configuration is changed accordingly. Meanwhile, all of the calculation results are memorized as the fixed matrices specified for the fault detection.

(8) In the single-terminal measuring method, the state of circuit breakers at the remote terminal is estimated based on the time-series transitional information of the current transformer (CT) and the potential transformer (PT) at the local terminal as shown in Fig. 1.

3. Two Basic Equations for Protection System

In this study, the protective section denotes the portions of transmission lines from one terminal to another one. As shown in Fig. 2, the equivalent circuit illustrates the typical configuration of one protective section with double-circuit transmission lines of three-phase alternating current. For general representation, all of voltages and currents are given by subscripts S or R for representing the sending terminal and the receiving terminal, respectively. The symbol k gives the relative distance between terminal S and the fault point F to the full length of the protective section.

According to the equivalent circuit, the formulations between voltages and currents on the section are drawn based on the Kirchhoff's laws as shown from Eqs. (1) to (7), where the vectors with six ranks are expressed in boldfaced lowercase letters and the matrices with six

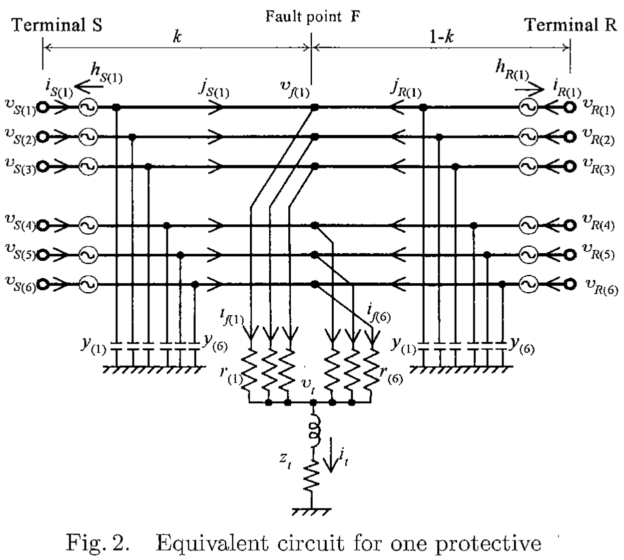

Fig. 2. Equivalent circuit for one protective section

ranks are expressed in boldfaced uppercase letters.

The states of the CBs for each branch are simulated equivalently with the virtual phase shifters ($\boldsymbol{h}$) [5], which is used for reflecting the system configuration.

$$\boldsymbol{v}_S = \boldsymbol{v}_f + k\boldsymbol{Z}\boldsymbol{j}_S + \boldsymbol{h}_S \quad \cdots (1)$$

$$\boldsymbol{v}_R = \boldsymbol{v}_f + (1-k)\boldsymbol{Z}\boldsymbol{j}_R + \boldsymbol{h}_R \quad \cdots (2)$$

$$\boldsymbol{v}_f = \boldsymbol{R}_f \boldsymbol{i}_f + \boldsymbol{v}_t \quad \cdots (3)$$

$$\boldsymbol{i}_S = \boldsymbol{j}_S + \boldsymbol{Y}\boldsymbol{v}_S \quad \cdots (4)$$

$$\boldsymbol{i}_R = \boldsymbol{j}_R + \boldsymbol{Y}\boldsymbol{v}_R \quad \cdots (5)$$

$$\boldsymbol{i}_f = \boldsymbol{j}_S + \boldsymbol{j}_R \quad \cdots (6)$$

$$i_t = \sum_{i=1}^{m} \boldsymbol{i}_{f(i)} \quad \text{with } m = 6 \quad \cdots (7)$$

$$\boldsymbol{v}_t = [v_t, v_t \cdots, v_t]^T$$

$$v_t = z_t \cdot i_t$$

$$\boldsymbol{Z} \equiv \begin{pmatrix} z_{11} & \cdots & z_{16} \\ \vdots & \vdots & \vdots \\ z_{61} & \cdots & z_{66} \end{pmatrix}, \boldsymbol{R}_f \equiv \begin{pmatrix} r_1 & & 0 \\ & \ddots & \\ 0 & & r_6 \end{pmatrix},$$

$$\boldsymbol{Y} \equiv \begin{pmatrix} y_1 & & 0 \\ & \ddots & \\ 0 & & y_6 \end{pmatrix}.$$

z_{ij} : mutual impedance between transmission lines

Through the matrix transformations of above equations mathematically, two basic equations are derived in Eqs. (8) and (9) for the solution of protection relay system. Details of matrix transformations are described in Ref. (1). The derivation of two equations is illustrated in Appendix 1 referentially.

$$\boldsymbol{i}_f = (\boldsymbol{1} + k\boldsymbol{D}_K)^{-1}\boldsymbol{D}_{BK}\boldsymbol{b}_w \quad \cdots (8)$$

$$\boldsymbol{R}_f\boldsymbol{i}_f + \boldsymbol{v}_t = (k\boldsymbol{D}_{BJK} + \boldsymbol{D}_{BJ})\boldsymbol{b}_w - \boldsymbol{D}_J\boldsymbol{i}_f \quad \cdots (9)$$

In the two equations, the first one denotes the relation between the fault current and the fault location, and the second one denotes the relation of arc resistance, root voltage and fault current at fault location, accordingly.

Because these two equations contain all of information about the entire protection system, namely, the constant matrices of $\boldsymbol{D}_K$, $\boldsymbol{D}_{BK}$, $\boldsymbol{D}_{BJK}$, $\boldsymbol{D}_{BJ}$ and $\boldsymbol{D}_J$ reflect system configurations, parameters of the transmission lines, fixed coefficients of generators and loads, respectively. The vector $\boldsymbol{b}_w$ reflects measurements of PT and CT on the transmission lines.

4. Solution of Basic Equations

4.1 Discrimination Method of Fault Lines

In order to ensure a higher accuracy of the fault identification, the sound lines should not be taken into consideration with the fault detecting calculation because the identification accuracy would be affected by the calculation error. Therefore, the discrimination of the fault lines on the protective system is needed before the solution of simultaneous equations.

Generally, it can be roughly determined that which line is a fault line according to the fault current ($\boldsymbol{i}_f$) on each transmission line. So the solution has to be started with the fault current of transmission lines.

From Eq. (8), it is clear that as long as k is given, the fault current of each line would be obtained accordingly. So, an assumed k (eg. $k = 0.5$) is taken first as a temporary constant for fault currents.

Some of importantly estimative approaches below are given for determining the fault lines with the fault current.

(1) Maximum of fault current

$$i_{fmax} = \underset{n=1,\cdots,6}{\text{Max}}|\boldsymbol{i}_{f(n)}|$$

(2) Set value of fault estimation

$$F_{alim} = \alpha \cdot i_{fmax}$$

(3) If the fault current value of some line is larger than the set value, then the line is dealt with the fault line.

$$\underset{n=1,\cdots,6}{|\boldsymbol{i}_{f(n)}|} \geq F_{alim}$$

(4) If the current ratio between the two corresponding phases on double-circuit transmission lines is larger than the unbalance value (β), then the upper line is regarded as the fault line.

$$|\boldsymbol{i}_{f(n)}/\boldsymbol{i}_{f(n+3)}|_{n=1,2,3} \geq \beta$$

Where, the coefficient α and β are given experientially.

4.2 A High-Speed Solution for Non-Linear

It has been confirmed that the orthodox Newton-Raphson's (NR) approximate approach and the least square method [1] are effective means for solving the non-linear problems which has been utilized for the fault locator with quite small errors.

However, a recognized fact is that the kind of iterative method does not keep the computational time not exceeding beyond the practically acceptable time for the protection relay surely. For this reason, a newly high-speed operation method is developed and proposed for the matrix operation type protection relay.

Generally speaking, what much more time consumes are those matrix operations such like diagonal transformations and inverse transformations. To solve this time-consuming problem, the off-line pre-calculation method is developed definitely. Meanwhile, a kind of non-iterative solution method, that is, the interpolation technique for non-linear equation is introduced.

On the other hand, a newly simplified linear approach method has been proposed as well with some assumed works. Details of the methods are given as follows:

(1) Off-line pre-calculation The inverse matrices and diagonal matrices and all fixed matrices used for solving the simultaneous equations are pre-calculated and memorized before the fault occurrence.

Moreover, in order to further shorten the fault detection time, the inverse calculation of the Eq. (8) for $\boldsymbol{i}_f$ have to be prepared and memorized as well as possible with several assumed k beforehand.

(2) An approximately linear approach In the Eq. (9), the voltage at fault location ($\boldsymbol{v}_f$) is also expressed by Eq. (10) or Eq. (11).

$$\boldsymbol{v}_f = \boldsymbol{R}_f \boldsymbol{i}_f + \boldsymbol{v}_t \cdots\cdots\cdots (10)$$

$$\boldsymbol{v}_f = (k\boldsymbol{D}_{BJK} + \boldsymbol{D}_{BJ})\boldsymbol{b}_w - \boldsymbol{D}_J \boldsymbol{i}_f \cdots\cdots\cdots (11)$$

In the fault detection calculation, when a fault occurs, the arc resistance gets so small that the voltage against the ground is approximated close to zero usually except for the short-circuit fault. According to this, an approximately linear solution is obtained from the voltage Eq. (11) by taking the $\boldsymbol{v}_f$ at fault location as zero.

$$k \approx \frac{-\boldsymbol{b}_{jj}}{\boldsymbol{b}_{jk}} \cdots\cdots\cdots (12)$$

Where,

$$\boldsymbol{b}_{jk} \equiv \boldsymbol{D}_{BJK}\boldsymbol{b}_w \cdots\cdots\cdots (13)$$

$$\boldsymbol{b}_{jj} \equiv \boldsymbol{D}_{BJ}\boldsymbol{b}_w - \boldsymbol{D}_J \boldsymbol{i}_f \cdots\cdots\cdots (14)$$

As shown in Eq. (12), owing to $\boldsymbol{i}_f$ is taken as a pre-calculation result for discriminating fault lines first, the definition (13) and (14) are dealt with known numbers there. The mean value (k_p) of Eq. (15) with all of fault lines is dealt with an approximate solution of fault location.

$$k_p = \frac{1}{n}\sum_{i=1}^{n} \frac{-\boldsymbol{b}_{jj(i)}}{\boldsymbol{b}_{jk(i)}} \cdots\cdots\cdots (15)$$

i : number of the fault line,

n : total number of fault lines.

In reality, many simulation results have confirmed that the kind of approximately linear approach is very useful for finding out an appraisal of the fault location with next exact calculation.

(3) Interpolation method with non-consecutive values of k In order to obtain an exactly final solution of fault location within the very limited time, a non-iterative interpolation technique is considered for replacing the Newton-Raphson's iterative calculation. Namely, several non-consecutive k in the protective section are assumed beforehand as shown in Table 1 for an example. After the approximate solution (k_p) determined, several k_i nearest the k_p are picked up from the assumed fault location table. With using these k_i into the interpolation equation, the final solution can be obtained easily.

Table 1. Assumed fault location for interpolation

i	1	2	3	4	5	6	7	8	9	10
k_i	0	0.1	0.3	0.5	0.7	0.8	0.9	0.95	1.0	1.05

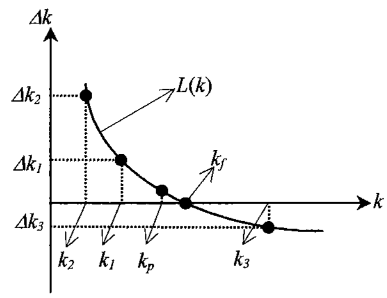

Fig. 3. Lagrange's interpolation for the solution k_f

Where, the interim solution with each k_i is defined as k_s. The error between k_s and k_i is defined as Δk_i shown in Eq. (16). If Δk_i is smaller than a set value, a precision demand (eg. 0.001), the k_i will be regarded as the final solution. Otherwise, with calculating the Δk_i for k_i one by one up to the third one, the final solution (k_f) can be obtained by putting the three pairs of k_i and Δk_i into the interpolation equation $L(k)$ as shown in Fig. 3. Where, $L(k)$ means the Lagrange's interpolation referenced to Appendix 2.

$$\Delta k_i = k_s - k_i \quad (i = 1, 2, 3) \cdots\cdots\cdots (16)$$

(4) The interim solution k_s Eq. (17), which denotes the relationship of k, arc resistance (r_i) of each line and root voltage (v_t), is derived from the Eq. (9) and the definitions (13) and (14). It is seen obviously that the elements of i_f and b_{jj} in the Eq. (17) are considered as constants with the given k_i. Therefore, the interim solution k_s for each fault line can be obtained from Eq. (17). The final solution of the interim solution k_s is given as Eq. (18) with the mean of total number of fault lines. Details of the derivation of Eq. (18) are illustrated in Appendix 3.

$$\begin{array}{c} \\ (1)\\(2)\\(3)\\(4)\\(5)\\(6) \end{array} \begin{array}{|c|c|c|c|c|c|c|} r_1 & r_2 & r_3 & r_4 & r_5 & r_6 & v_t \\ \hline i_{f1} & & & & & & 1 \\ \hline & i_{f2} & & & & & 1 \\ \hline & & i_{f3} & & & & 1 \\ \hline & & & i_{f4} & & & 1 \\ \hline & & & & i_{f5} & & 1 \\ \hline & & & & & i_{f6} & 1 \\ \hline \end{array}$$

$$\begin{array}{cc} k_s & 1 \end{array}$$

$$= \begin{array}{|c|c|} \hline b_{jk1} & b_{jj1} \\ \hline b_{jk2} & b_{jj2} \\ \hline b_{jk3} & b_{jj3} \\ \hline b_{jk4} & b_{jj4} \\ \hline b_{jk5} & b_{jj5} \\ \hline b_{jk6} & b_{jj6} \\ \hline \end{array} \cdots\cdots\cdots (17)$$

$$k_s = \frac{1}{n-2}\sum_{i=1}^{n-2} \frac{b'_{jj(2i)}}{b'_{jk(2i)}} \cdots\cdots\cdots (18)$$

Where, i : number of the fault line,

n : total number of fault lines.

The elements $b'_{jj(2i)}$ and $b'_{jk(2i)}$ express the transformed constant elements respectively.

5. Scheme of the Proposed Relay System

5.1 Off-line Pre-calculation Fig. 4 shows the flow of the off-line pre-calculation. In this part, the fixed matrices including diagonal matrix transformations and inverse matrix calculations with all of assumed values are calculated beforehand and specified for whole system configurations.

Also, all of necessary information of the protection system such as the facilities, system configuration and system application parameters are memorized definitely.

This has to be done only when the system information

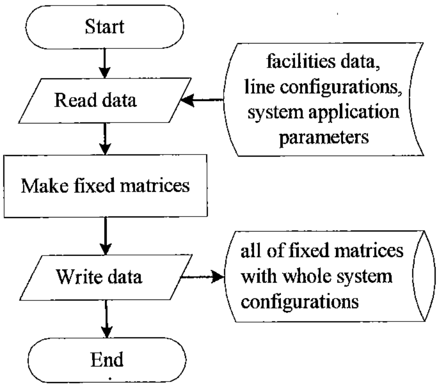

Fig. 4. Flow Diagram of the off-line pre-calculation

is changed initially.

5.2 Periodic System Monitoring In Fig. 5, the periodic system monitoring is shown on the upper part. The measurements of PT and CT at the local side are sampled periodically and changed into vectors for matrix operation.

According to the measurements, the critical lines and the system configuration changes are detected and monitored. Meanwhile, it also does the work of kick relay.

Once the fault lines are detected, the fault locating operation is driven to start running right away. The states of circuit breakers at the non-measuring side are presumed based on the local measurements.

The address of fixed matrices corresponding to the system configuration is pointed to the memory for fault locating operation.

5.3 Fault locating Operation The detective operation for fault location is shown below the periodic system monitoring in Fig. 5. It is driven to start by a kick of the fault occurrence. Based on the measurements and the parameters of system facilities, the state of loads and generators at non-measuring side are presumed equivalently. The subroutine of detecting the fault location utilizes the proposed technique, which is described in the last chapter, for a high-speed and high accuracy solution.

After tripped the circuit breakers of fault lines, the fault resistances are calculated by using the basic Eq. (9) in a time series. Furthermore, according to the fault resistance, the cause of the fault can be specified experientially. Finally, the computation results are recorded on the display or in memories for other else purposes.

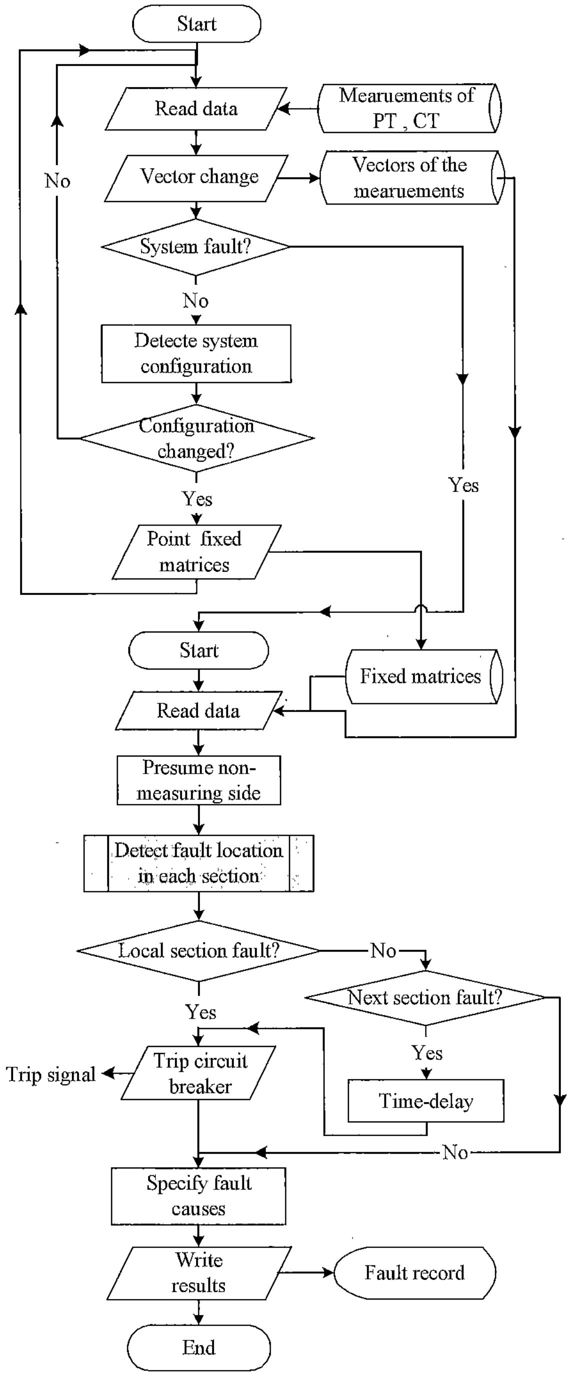

Fig. 5. Simplified flow diagram for on-line operation of proposed protection relay system

6. Practical Use of the Proposed Relay System

6.1 Practical Use of the Proposed Relay System

(1) Accuracy of fault location In order to verify the accuracy of the fault locating with the proposed protection relay, the comparisons have been performed along with thirty-eight cases of actually inspected results to the simulation results. Namely, the verifiable simulations are carried out with the actual measurements of PT and CT for the thirty-eight fault cases.

The compared results are summarized as shown in below tables. The analyzed results are listed with different voltage levels of transmission lines, respectively.

In the tables, the k is expressed with the method of per-unit, that is, the whole distance of the section is taken as 1.0.

On the inspection results, the regional k instead of one point of k expresses that several arc traces are found out at the plural steel towers.

Thus, if the simulation result is within the regional inspection result, the locating error is considered as zero. Otherwise, the side one that is nearest by the simulation result is regarded as the actual fault location for comparing. The locating error is expressed in the

Table 2. Fault location error of single-terminal measurement for 500 kV transmission lines

No.	Section number	Inspection result		Locating error (E)
		Fault line (●)	k (p.u.)	
1	1	1^L ○●● 2^L ○○○	0.267～0.286	0
2	1	1^L ○○○ 2^L ●○●	0.267～0.286	-0.8
3	1	1^L ○○○ 2^L ●○●	0.267～0.286	-0.7
4	1	1^L ○○○ 2^L ●○●	0.267～0.286	-0.6
5	1	1^L ○●● 2^L ○○○	0.267～0.286	0
6	1	1^L ○○○ 2^L ○●●	0.267～0.286	-1.8
7	1	1^L ○●○ 2^L ○○○	0.836	0.8
8	1	1^L ○○○ 2^L ●○○	0.748	-0.6
9	1	1^L ○○● 2^L ○○○	0.733	2.3
10	1	1^L ○○○ 2^L ○○●	0.776	-0.4
Subtotal (E_{ave})				0.8

Table 3. Fault location error of single-terminal measurement for 275 kV transmission lines

No.	Section number	Inspection result		Locating error (E)
		Fault line (●)	k (p.u.)	
11	1	1^L ○●○ 2^L ○○○	0.900～0.905	-0.2
12	1	1^L ●●○ 2^L ○○○	0.872	3.1
13	1	1^L ●●○ 2^L ○○○	0.872	3.8
14	1	1^L ○○○ 2^L ●○●	0.296	2.4
15	1	1^L ○○○ 2^L ●○○	0.208	0
16	1	1^L ○●○ 2^L ●●●	0.110～0.119	1.3
17	1	1^L ○○○ 2^L ●○○	0.833	0.7
18	1	1^L ○○○ 2^L ○●○	0.730	-0.9
19	1	1^L ○●○ 2^L ○○○	0.096～0.100	2.6
20	1	1^L ○○○ 2^L ●○○	0.955	-0.7
21	1	1^L ○○○ 2^L ○●○	0.614	-0.5
22	1	1^L ○●○ 2^L ○●●	0.37	-0.2
23	1	1^L ○○○ 2^L ○●○	0.646	0.5
24	1	1^L ○●○ 2^L ○○○	0.267	-0.2
25	1	1^L ●●○ 2^L ○○○	0.914	-0.7
Subtotal (E_{ave})				1.2

Table 4. Fault location error of single-terminal measurement for 154 kV transmission lines

No.	Section number	Inspection result		Locating error (E)
		Fault line (●)	k (p.u.)	
26	1	1^L ○○○ 2^L ●○●	0.435	-2.5
27	1	1^L ○○○ 2^L ●○○	0.097	-2.8
28	1	1^L ○○○ 2^L ●○○	0.869	0.9
Subtotal (E_{ave})				2.1

difference between the actual value and the simulation value as Eq. (19).

Where, k_{act} is for the inspection result, k_{sim} is for the simulation result. The mean of the absolute locating errors is regarded as the subtotal error (E_{ave}), which is given in Eq. (20). The mean (E_T) of whole locating errors including all of subtotals is given in Eq. (21).

$$E = k_{sim} - k_{act} \quad \cdots\cdots (19)$$

$$E_{ave} = \frac{1}{m}\sum_{i=1}^{m}|E_i| \quad \cdots\cdots (20)$$

$$E_T = \frac{1}{n}\sum_{j=1}^{n}E_{ave,j} \quad \cdots\cdots (21)$$

Where, m: total number of fault with same voltage level

n: total number of voltage level

Table 5. Fault location error of single-terminal measurement for 66 kV transmission lines

No.	Section number	Inspection result		Locating error (E)
		Fault line (●)	k (p.u.)	
29	3	1^L ○○○ 2^L ●○●	0.990	3.3
30	1	1^L ○○● 2^L ○○○	0.148	-1.8
31	2	1^L ○○○ 2^L ●○○	0.998	-4.9
32	2	1^L ○●○ 2^L ○○○	0.125	0.9
33	2	1^L ○●○ 2^L ○○○	0.125	0.2
34	2	1^L ○○● 2^L ○○○	0.960	3.8
35	3	1^L ○○○ 2^L ●○●	0.064	-5.4
36	3	1^L ○●● 2^L ○○○	0.377	-6.2
37	2	1^L ○○○ 2^L ●○○	0.353	-1.4
38	2	1^L ○○○ 2^L ○●○	0.298	3.2
Subtotal (E_{ave})				3.1
Total (E_T)				1.7

From the simulated results of the tables above, it is clear that the lower the voltage level is, the bigger the locating error is.

However, with taking the worst case as a sample concerned about, on the 66 kV systems, the error would be limited within only 300 m even if the length of one section were supposed to be around 10 km.

Moreover, the total error for whole thirty-eight cases has reached the level of being less than 1.7%. So, it is verifiable that the proposed scheme can be put to the practical use in the transmission lines regardless of voltage levels.

On the other hand, comparing the accuracy of the conventional protection relay (near 10%) with the proposed protection relay, it can be concluded that a quite high accuracy has been achieved definitely.

(2) Reason of high accuracy achieved The reason why the matrix operation type protection relay can achieve so high accuracy is explained conceptually as below.

The fault location of impedance type fault locator is expressed by Eq. (22). Square errors of measurements of PT, CT and impedance (Z) of transmission lines are expressed by ε_{PT}, ε_{CT} and ε_Z, so the square error of k can be calculated by Eq. (23).

$$k = \frac{v_{PT}}{i_{CT}} \cdot \frac{1}{Z} \quad \cdots\cdots (22)$$

$$\varepsilon_k = \sqrt{\varepsilon_{PT}^2 + \varepsilon_{CT}^2 + \varepsilon_Z^2} \quad \cdots\cdots (23)$$

If each error assumed as 1.0%, the square error (ε_k) would be near 1.73%. Generally, it is considered that the square error is added while the number of variables increased.

However, it is well known that the fault location for mono-phase grounding of double-circuit can be obtained from Eq. (24), and its high accuracy has been confirmed already.

The reason of high accuracy obtained is that only zero-phase currents (I_{o1} and I_{o2}) on each own circuit are used for the solution of k, which is not affected by PT and line impedance ($\boldsymbol{Z}$) or the load current (i_L) definitely.

On the other hand, the Eq. (24) has been confirmed being tenable from the elements of the proposed protection relay scheme.

$$k = \frac{I_{o2}}{I_{o1} + I_{o2}} \cdots\cdots (24)$$

In addition, though the formulation of basic matrix equations gathers all of system elements, the greater part of variables that affect solution k directly are deleted as well as possible through the diagonal transformation and other integrated transformations.

Therefore, a high accuracy can be considered in the proposed protection relay scheme even if there might be more measuring errors in the system. More details concerning the error analysis are illustrated in Ref. (6).

6.2 Required Computation Time The required computation time is estimated to be 0.33 ms for the protection area covering ten sections. The time is only around one fiftieth of one cycle in the sixty-hertz systems.

Comparing this with the former Newton-Raphson's approximate method, which needs about 10 ms, one thirtieth of the computation time has been shortened greatly.

In addition, the required size of the memory is roughly estimated around 970 kilobytes for all of fixed matrices.

7. Specific Features of Proposed Protection Relay

As described above, the specific features of the proposed protection relay differing from other protection relays are summarized as follows:

(1) Accuracy of fault location

a. The accuracy of detecting the fault location has reached to the level of 2.0% on average. It is much higher than the level of 10%, the conventional fault locating method.

b. Owing to all of arc resistances of each line are taken as independent various as shown in Fig. 2, the various kinds of transmission line faults can be dealt with based on the same principle.

c. The fault detection of mono-phase grounding fault on double-circuit transmission lines with a neutral resistance grounding system, which was considered impossible before, is realized easily.

(2) Functions of the protection relay

a. The protection relay enables to deal with the dynamical change of system configurations by extracting correspondingly the fixed matrices that are specified on the off-line pre-calculation.

b. Differing from the former protection relays, which need their own protection calculation respectively (eg. grounding or short-circuit), the proposed protection relay does not need that because all of fault lines are taken together into consideration for the solution of k.

c. Since the fault location is detected with taking the main protection range as a unit 1.0, the main and the back-up protection can be provided together according to the given k of the back-up protection section.

d. For considering the re-closing failure and the safety, re-closing control of circuit breakers is realized through the specification of the fault cause.

e. The protection relay holding fault locator also makes the same information good use for the fault clearance.

8. Conclusion

Although the previous fault locating scheme has achieved quite accurate identification of the fault location, especially in its practical usage, the principle of fault detection can not be utilized completely for protection relays because of the relatively heavy computational burden with relatively large amounts of consumable time.

To overcome this situation, this paper presents a new high-speed computational method for the new type of digital relays. Namely, an integrated scheme has been proposed for the new protection relay with both of a high detecting accuracy and a high-speed computational ability.

Through out the simulation studies for thirty-eight actual fault cases, the advantages of the proposed scheme have been demonstrated in its utilities and reliabilities on power transmission systems. The major achievements of this study are summarized as below:

(1) A fast solving method for the non-linear problem has been developed based on the interpolation technique. Now, the fault detection is available without the complicated and time-consumed iterative calculation. In the proposed scheme, the iterative calculation has been replaced by off-line pre-operations and the interpolation method. Therefore, the required computation time has reached to 0.33 ms for covering ten protective sections. It is confirmed that the proposed method enables to shorten the computation time for protection relays.

(2) From the verification of the fault locating error for thirty-eight actual fault cases, the averaged absolute error of the fault location is only around 1.7%.

It is proved evidently that the fault detection accuracy has been improved up to a considerably high level compared with the conventional relays, the accuracy of 10%.

(3) The proposed method needs the data of system facilities beforehand. Therefore, whenever the system configuration is changed, the corresponding data should be replaced for the new configuration.

On the other hand, the multi-purpose protection (eg. grounding and short-circuit, main and back-up protections) on power systems can be accomplished by only one protection relay following the comprehensive computation.

Further studies are required for the poly-phase reclosing with single-terminal measurements and for the protection of parallel multi-circuit transmission lines. The feasibility is going to be investigated for the integration of protection relay and fault locator, with the function of re-closing control based on fault causes.

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Appendix

1. Derivation of Simultaneous Equations

The Eqs. (A1) and (A2) are derived from the integrated matrix equation [1], (A1) denotes the relations of variables of fault location (k) and fault current ($\boldsymbol{i}_f$), Eq. (A2) denotes the relations of arc resistance ($\boldsymbol{R}_f$), root voltage ($\boldsymbol{v}_t$) and $\boldsymbol{i}_f$.

$$(\boldsymbol{1}+k\boldsymbol{D}_K)\boldsymbol{i}_f=\boldsymbol{D}_{BK}\boldsymbol{b}_w \quad \cdots\cdots \text{(A1)}$$

$$(k^2\boldsymbol{D}_1+k\boldsymbol{D}_2+\boldsymbol{D}_3+\boldsymbol{R}_f)\boldsymbol{i}_f+\boldsymbol{v}_t = (k\boldsymbol{D}_{BRK}+\boldsymbol{D}_{BR})\boldsymbol{b}_w \quad \cdots\cdots \text{(A2)}$$

Where, in order to change the Eq. (A2) into an equation without the term k^2, some transformations and definitions are performed as follows.

First of all, Eq. (A1) is changed into Eq. (A3), then the term of $(k^2\boldsymbol{D}_1+k\boldsymbol{D}_2+\boldsymbol{D}_3)\boldsymbol{i}_f$ in the Eq. (A2) is transformed with the term of $(\boldsymbol{1}+k\boldsymbol{D}_K)\boldsymbol{i}_f$ as shown in Eq. (A4).

With substituting the Eq. (A4) for Eq. (A2), the Eq. (A5) expressing the voltage at fault location is derived.

$$\boldsymbol{i}_f=(\boldsymbol{1}+k\boldsymbol{D}_K)^{-1}\boldsymbol{D}_{BK}\boldsymbol{b}_w \quad \cdots\cdots \text{(A3)}$$

$$(k^2\boldsymbol{D}_1+k\boldsymbol{D}_2+\boldsymbol{D}_3)\boldsymbol{i}_f=\boldsymbol{D}_1\boldsymbol{D}_K^{-1}k(\boldsymbol{1}+k\boldsymbol{D}_K)\boldsymbol{i}_f +\boldsymbol{D}_{JJ}(\boldsymbol{1}+k\boldsymbol{D}_K)\boldsymbol{i}_f-\boldsymbol{D}_{JJ}\boldsymbol{i}_f+\boldsymbol{D}_3\boldsymbol{i}_f \quad \cdots \text{(A4)}$$

$$\boldsymbol{R}_f\boldsymbol{i}_f+\boldsymbol{v}_t=(k\boldsymbol{D}_{BJK}+\boldsymbol{D}_{BJ})\boldsymbol{b}_w-\boldsymbol{D}_J\boldsymbol{i}_f \quad \cdots\cdots \text{(A5)}$$

Where, some constant matrices are defined as follows.

$$\boldsymbol{D}_{JJ}\equiv(\boldsymbol{D}_1\boldsymbol{D}_K^{-1}-\boldsymbol{D}_2)\boldsymbol{D}_K^{-1} \quad \cdots\cdots \text{(A6)}$$

$$\boldsymbol{D}_J\equiv\boldsymbol{D}_3+\boldsymbol{D}_{JJ} \quad \cdots\cdots \text{(A7)}$$

$$\boldsymbol{D}_{BJK}\equiv\boldsymbol{D}_{BRK}-\boldsymbol{D}_1\boldsymbol{D}_K^{-1}\boldsymbol{D}_{BK} \quad \cdots\cdots \text{(A8)}$$

$$\boldsymbol{D}_{BJ}\equiv\boldsymbol{D}_{BR}+\boldsymbol{D}_{JJ}\boldsymbol{D}_{BK} \quad \cdots\cdots \text{(A9)}$$

2. Lagrange's interpolation $L(k)$ with three points (k_1, k_2, k_3)

$$L(k)=a_1(k-k_1)(k-k_2)+a_2(k-k_1)(k-k_3) + a_3(k-k_1)(k-k_2) \quad \cdots\cdots \text{(A10)}$$

Where, $a_1\equiv\dfrac{\Delta k_1}{(k_1-k_2)(k_1-k_3)}$,

$a_2\equiv\dfrac{\Delta k_2}{(k_2-k_1)(k_2-k_3)}$,

$a_3\equiv\dfrac{\Delta k_3}{(k_3-k_1)(k_3-k_2)}$.

3. Derivation of interim solution $\boldsymbol{k}_s$

For the purpose of derivation, it is assumed that all of transmission lines are in fault.

First of all, in order to delete the variable of v_t, the Eq. (17) is changed into Eq. (A11) through the subtractions between every two rows.

Then, dividing the complex elements into two parts of real and imaginary, the matrix equation is transformed into Eq. (A12) with all of real numbers.

	r_1	r_2	r_3	r_4	r_5	r_6
(1)	i_{f1}	$-i_{f2}$				
(2)		i_{f2}	$-i_{f3}$			
(3)			i_{f3}	$-i_{f4}$		
(4)				i_{f4}	$-i_{f5}$	
(5)					i_{f5}	$-i_{f6}$

=

k_s	1
$b_{jk1}-b_{jk2}$	$b_{jj1}-b_{jj2}$
$b_{jk2}-b_{jk3}$	$b_{jj2}-b_{jj3}$
$b_{jk3}-b_{jk4}$	$b_{jj3}-b_{jj4}$
$b_{jk4}-b_{jk5}$	$b_{jj4}-b_{jj5}$
$b_{jk5}-b_{jk6}$	$b_{jj5}-b_{jj6}$

$\cdots\cdots$ (A11)

	r_1	r_2	r_3	r_4	r_5	r_6
(1)	i^r_{f1}	$-i^r_{f2}$				
(2)	i^i_{f1}	$-i^i_{f2}$				
(3)		i^r_{f2}	$-i^r_{f3}$			
(4)		i^i_{f2}	$-i^i_{f3}$			
(5)			i^r_{f3}	$-i^r_{f4}$		
(6)			i^i_{f3}	$-i^i_{f4}$		
(7)				i^r_{f4}	$-i^r_{f5}$	
(8)				i^i_{f4}	$-i^i_{f5}$	
(9)					i^r_{f5}	$-i^r_{f6}$
(10)					i^i_{f5}	$-i^i_{f6}$

=

k_s	1
$b^r_{jk1}-b^r_{jk2}$	$b^r_{jj1}-b^r_{jj2}$
$b^i_{jk1}-b^i_{jk2}$	$b^i_{jj1}-b^i_{jj2}$
$b^r_{jk2}-b^r_{jk3}$	$b^r_{jj2}-b^r_{jj3}$
$b^i_{jk2}-b^i_{jk3}$	$b^i_{jj2}-b^i_{jj3}$
$b^r_{jk3}-b^r_{jk4}$	$b^r_{jj3}-b^r_{jj4}$
$b^i_{jk3}-b^i_{jk4}$	$b^i_{jj3}-b^i_{jj4}$
$b^r_{jk4}-b^r_{jk5}$	$b^r_{jj4}-b^r_{jj5}$
$b^i_{jk4}-b^i_{jk5}$	$b^i_{jj4}-b^i_{jj5}$
$b^r_{jk5}-b^r_{jk6}$	$b^r_{jj5}-b^r_{jj6}$
$b^i_{jk5}-b^i_{jk6}$	$b^i_{jj5}-b^i_{jj6}$

$\cdots\cdots$ (A12)

Therefore, the resultant equation is derived as Eq. (A13) through further transformations.

From the Eq. (A13), the interim solution (k_s) can be obtained by taking the average of k in each even row as Eq. (18).

On the resistance (r_i) of each fault line, it is clear that the odd rows of Eq. (A13) are used for solving it once the final resolution (k_f) is given.

In addition, Eq. (A13) is required only in the case of more than two fault lines.

	r_1	r_2	r_3	r_4	r_5	r_6
(1)	1					
(2)		0				
(3)		1				
(4)			0			
(5)			1			
(6)				0		
(7)				1		
(8)					0	
(9)					1	
(10)						1

k_s	1
$b'_{jk(1)}$	$b'_{jj(1)}$
$b'_{jk(2)}$	$b'_{jj(2)}$
$b'_{jk(3)}$	$b'_{jj(3)}$
$b'_{jk(4)}$	$b'_{jj(4)}$
$b'_{jk(5)}$	$b'_{jj(5)}$
$b'_{jk(6)}$	$b'_{jj(6)}$
$b'_{jk(7)}$	$b'_{jj(7)}$
$b'_{jk(8)}$	$b'_{jj(8)}$
$b'_{jk(9)}$	$b'_{jj(9)}$
$b'_{jk(10)}$	$b'_{jj(10)}$

= ………………… (A13)

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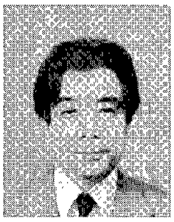

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