An Effective and Practical Method for Solving Hydro-Thermal Unit Commitment Problems Based on Lagrangian Relaxation Method

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This paper presents an effective and practical method based on the Lagrangian relaxation method for solving hydro-thermal unit commitment problem in which operational constraints involve spinning reserve requirements for thermal units and prohibition of simultaneous unit start-up/shut-down at the same plant. This method is processed in each iteration step of LRM that enables a direct solution. To improve convergence, this method applies an augmented Lagrangian relaxation method. Its effectiveness demonstrated for a real power system.

Keywords: system operation, unit commitment, Lagrangian relaxation method, spinning reserve requirements, prohibition of simultaneous unit start-up/shut-down at the same plant

1. Introduction

To achieve economical power system operation, it is important to solve the unit commitment. Many methods have been studied, such as Dynamic Programming (DP), Mixed Integer Programming (MIP) and Lagrangian Relaxation Method (LRM). With the expansion of modern power systems, LRM has become a commonly used method for solving large-scale unit commitment problems. LRM dualizes system constraints such as system supply and demand balance and system reserve constraints, and decomposes the problem into a sequence of dynamic programming sub-problems [1] [2].

However, it cannot easily handle generator-side constraints that couple plural generation operation decisions over the time hori-zon. For example, the constraint of prohibition of simultaneous unit start-up/shutdown at the same plant (CPSS) cannot be optimized globally in the standard dynamic programming sub-problem without enlarging the dynamic programming state space.

As an alternative to LRM, one method has been proposed for preparing practical operational schedules using an expert system (ES) [3]. Another both Genetic Algorithms (GA) and LRM [4]. Although ES can take

practical system operational constraints into account, it often has difficulty in dealing with large-scale problems within a realistic computation time. The proposed method in Ref. (5) efficiently utilizes advantages of both GA and LRM. However, it requires large computation time due to plural execution of LRM.

This paper presents a practical method. To handle CPSS in LRM, sub-problems of LRM for units at the same plant are paired, and an approximate constraint is introduced for solving sub-problems over plural units. This enables a direct solution considering CPSS by LRM.

In addition, the Hokkaido Electric Power Co. (HEPCO) requires for thermal unit spinning a reserve constraint (CSRT) as well as one for the total system aimed at reliable power system operation. This comes from basic operation philosophy for an islanding power system, although HEPCO is now interconnected with other systems in Japan via HVDC. This method incorporates CSRT to LRM in the same fashion as for other constraints. However, when considering many constraints, conventional LRM has rather poor convergence. To improve convergence, this method applies the augmented Lagrangian relaxation method (ALRM) proposed in Ref. (6) (7). Moreover, this method introduces a heuristic algorithm for CSRT in which re-dispatching of thermal units' operational maximal power is processed in each iteration step of ALRM.

Let us summarize the major characteristics of the proposed method. It enables the direct solution for CPSS and CSRT in the framework of ALRM to obtain

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a practical generation schedule.

The method's effectiveness has been demonstrated for a real power system with 13 thermal units, 10 hydro units and 4 pumped-storage hydro units. This method reduced operation costs by 0.657% on average compared to the operation schedule prepared by operation planners.

The paper consists of five sections. The hydro and thermal unit commitment problem is formulated in Section 2 and Section 3 in the manner of ALRM. The proposed method is described in Section 4. Section 5 demonstrates test results.

2. Problem Formulation

2.1 Notations   In the problem formulation, we make use of the following notations.

- t: index of time periods;
- T: total hours of study periods;
- i: index of thermal units;
- I: number of thermal units;
- j: index of hydro and pumped-storage hydro units;
- J: number of hydro and pumped-storage hydro units;
- l: index of fuel stockpile base;
- $I(l)$: number of thermal units which consume fuel in stockpile base l;
- L: number of fuel stockpile base;
- m: index of reservoir;
- $J(m)$: number of hydro or pumped-storage hydro units which consume water in reservoir m;
- M: number of reservoir;
- $u_{i,t}$: zero-one decision variable indicating whether unit i is up or down in time period t;
- $PT_{i,t}$: generation of thermal unit i during time period t;
- $PT_{i,t}^{\max}$: operational maximum output of thermal unit i during time period t;
- $PT_{i,t}^{\min}$: operational minimum output of thermal unit i during time period t;
- $PH_{j,t}$: generation of hydro or pumped-storage hydro unit j during time period t;
- $PH_{j,t}^{\max}$: operational maximum output of hydro or pumped-storage hydro unit j at time period t;
- $PH_{j,t}^{\min}$: operational minimum output of hydro or pumped-storage hydro unit j during time period t;
- $f_i(PT_{i,t})$: fuel costs for operating unit i at output level $PT_{i,t}$ during time period t;
- $w_j(PH_{j,t})$: water consumption for hydro or pumped-storage hydro unit j at output level $PH_{j,t}$ during time period t;
- s_i: start up cost associated with turning on unit i
- D_t: system load demand during time period t;
- R_t: system spinning reserve requirement during time period t;
- $e1_t$: slack variables to equalize constraint of system spinning reserve requirement during time period t,
- SPT_t: spinning reserve requirement for thermal units during time period t;
- $e2_t$: slack variables to equalize constraint of spinning reserve requirement for thermal units at time

period t;
- FL_l: fuel consumption limit of stockpile base l;
- FC_l: fuel unit cost at stockpile base l;
- WR_m: water consumption constraint of reservoir m;
- λ_t: Lagrangian multiplier for system power balance constraint;
- μ_t: Lagrangian multiplier for system spinning reserve constraint;
- ν_t: Lagrangian multiplier for constraint of spinning reserve requirement for thermal units
- γ_l: Lagrangian multiplier for fuel consumption constraint;
- δ_m: Lagrangian multiplier for water consumption constraint.

2.2 Objective   The objective of this method is to minimize thermal operating costs including fuel and start up costs.

$$C(PT, u) = \sum_{t=1}^{T} \sum_{i=1}^{I} C_{i,t}(PT_{i,t}, u_{i,t}, u_{i,t-1}) \to \min$$
$$\cdots\cdots\cdots\cdots (1)$$

Here we define,

$$C_{i,t}(PT_{i,t}, u_{i,t}, u_{i,t-1}) \overset{\leftharpoonup}{=} f_i(PT_{i,t}) + s_i u_{i,t}(1 - u_{i,t-1})$$
$$\cdots\cdots\cdots\cdots (2)$$

2.3 Constraints

System Constraints
- system power balance

$$\sum_{i=1}^{I} PT_{i,t} + \sum_{j=1}^{J} PH_{j,t} = D_t \quad (t = 1, \cdots, T)$$
$$\cdots\cdots\cdots\cdots (3)$$

- system spinning reserve requirements

$$\sum_{i=1}^{I} u_{i,t} PT_{i,t}^{\max} + \sum_{j=1}^{J} PH_{j,t}^{\max} \geq R_t \quad (t = 1, \cdots, T)$$
$$\cdots\cdots\cdots\cdots (4)$$

Spinning reserve requirement for thermal units

$$\sum_{i=1}^{I} u_{i,t}(PT_{i,t}^{\max} - PT_{i,t}) \geq SPT_t \quad (t = 1, \cdots, T)$$
$$\cdots\cdots\cdots\cdots (5)$$

Note that this constraint is limited to thermal units.

Generator Constraints
- thermal units' generation limits

$$PT_{i,t}^{\min} \leq PT_{i,t} \leq PT_{i,t}^{\max} \quad (i = 1, \cdots, I, \ t = 1, \cdots, T)$$
$$\cdots\cdots\cdots\cdots (6)$$

- hydro units' generation limits

$$PH_{j,t}^{\min} \leq PH_{j,t} \leq PH_{j,t}^{\max} \quad (j = 1, \cdots, J, \ t = 1, \cdots, T)$$
$$\cdots\cdots\cdots\cdots (7)$$

Consumption constraints

fuel consumption from the same fuel stockpile base.

$$\sum_{i=1}^{I(l)} \sum_{t=1}^{T} [f_i(PT_{i,t}) + s_i u_{i,t}(1 - u_{i,t-1})] \Big/ FC_l = FL_l$$

$$\cdots\cdots\cdots\cdots\cdots\cdots (8)$$

A start up cost is considered as fuel cost for unit warming-up.

water utilization from the same reservoirs

$$\sum_{j=1}^{J(m)} \sum_{t=1}^{T} w_j(PH_{j,t}) = WR_m \quad (m = 1, \cdots, M)$$

$$\cdots\cdots\cdots\cdots\cdots\cdots (9)$$

The reservoir capacity is not considered.

Unit operating constraints

The start-up and shut-down schedule should meet the operating constraints stated below.

- minimum up time and minimum down time constraint
- must-run and must-stop constraint
- unit output change pattern for start-up and shutdown
- constraint of prohibition of simultaneous unit start-up/shutdown at the same plants

3. Augmented Lagrangian Relaxation Method

3.1 Augmented Lagrangian Relaxation Function

We apply the augmented Lagrangian relaxation method (ALRM) using the decomposition and coordination method. ALRM combines the penalty method and conventional LRM, and overcomes the disadvantages of both methods. Quadratic penalty terms associated with constraints presented by linear functions are added to the objective function to improve the convergence property.

Hence, the augmented Lagrangian function for this problem is defined as:

$$L(PT, PH, u, \lambda, \mu, \nu, \gamma, \delta)$$
$$= C(PT, u)$$
$$- \sum_{t=1}^{T} \left[\lambda_t \left(\sum_{i=1}^{I} PT_{i,t} + \sum_{j=1}^{J} PH_{j,t} - D_t \right) \right.$$
$$\left. -c \left(\sum_{i=1}^{I} PT_{i,t} + \sum_{j=1}^{J} PH_{j,t} - D_t \right)^2 \right]$$
$$- \sum_{t=1}^{T} \left[\mu_t \left(\sum_{i=1}^{I} u_{i,t} PT_{i,t}^{\max} + \sum_{j=1}^{J} PH_{j,t}^{\max} - R_t \right) \right.$$
$$\left. -c \left(\sum_{i=1}^{I} u_{i,t} PT_{i,t}^{\max} + \sum_{j=1}^{J} PH_{j,t}^{\max} - R_t \right)^2 \right]$$

$$- \sum_{t=1}^{T} \left[\nu_t \left(\sum_{i=1}^{I} u_{i,t} \left(PT_{i,t}^{\max} - PT_{i,t} \right) - SPT_t \right) \right.$$
$$\left. -c \left(\sum_{i=1}^{I} u_{i,t} \left(PT_{i,t}^{\max} - PT_{i,t} \right) - SPT_t \right)^2 \right]$$
$$- \sum_{l=1}^{L} \gamma_l \left(\left(\sum_{t=1}^{T} \sum_{i=1}^{I(l)} C_{i,t} \left(PT_{i,t}, u_{i,t}, u_{i,t-1} \right) \right) - FL_l \right)$$
$$+ \sum_{m=1}^{M} \delta_m \left(\left(\sum_{t=1}^{T} \sum_{j=1}^{J(m)} w_j \left(PH_{j,t} \right) \right) - WR_m \right).$$

$$\cdots\cdots\cdots\cdots\cdots\cdots (10)$$

Here c is a positive number (0.0001).

Defining the augmented Lagrangian function, the original problem is returned to the dual problem defined as:

$$\max_{\lambda, \mu, \nu, \gamma, \varsigma, \delta} \left\{ \min_{PT, PH, u} L(PT, PH, u, \lambda, \mu, \nu, \gamma, \varsigma, \delta) \right\}.$$

$$\cdots\cdots\cdots\cdots\cdots\cdots (11)$$

3.2 Sub-problems

The dual problem can be decomposed into $(I + J)$ sub-problems. The sub-problems for thermal units of iteration index $(k+1)$ are expressed as:

$$\min_{PT_{i,t}, u_{i,t}} \left\{ \sum_{t=1}^{T} \begin{bmatrix} (1 - \gamma_l^k) C_{i,t}(PT_{i,t}, u_{i,t}, u_{i,t-1}) \\ -\lambda_t^{k+1/2} PT_{i,t} - \mu_t^{k+1/2} u_{i,t} PT_{i,t}^{\max} \\ -\nu_t^{k+1/2} u_{i,t} PT_{i,t}^{\max} + \nu_t^{k+1/2} PT_{i,t} \\ +\alpha/2 \left(PT_{i,t} - PT_{i,t}^k \right)^2 \\ +\alpha/2 \left(u_{i,t} PT_{i,t}^{\max} - u_{i,t}^k PT_{i,t}^{\max} \right)^2 \\ +\alpha/2 \left(u_{i,t} PT_{i,t}^{\max} - PT_{i,t} \right. \\ \left. -(u_{i,t}^k PT_{i,t}^{\max} - PT_{i,t}^k) \right)^2 \end{bmatrix} \right\}$$

$$(i = 1, \cdots, I), \cdots\cdots\cdots\cdots\cdots\cdots (12)$$

and the sub-problems for hydro units or pumped-storage hydro units of iteration index $(k + 1)$ are expressed as:

$$\min_{PH_{j,t}} \left\{ \delta_m^k W_j(PH_{j,t}) - \lambda_t^{k+1/2} PH_{j,t} \right.$$
$$\left. +\alpha/2 \left(PH_{j,t} - PH_{j,t}^k \right)^2 \right\}. \cdots\cdots\cdots\cdots (13)$$

Here, k is an iteration index, and α is a positive number, and we define:

$$\lambda_t^{k+1/2} = \lambda_t^k + c \left(D_t - \sum_{i=1}^{I} u_{i,t}^k PT_{i,t} - \sum_{j=1}^{M} PH_{j,t} \right)$$
$$(t = 1, \cdots, T) \cdots\cdots\cdots\cdots\cdots\cdots (14)$$

$$\mu_t^{k+1/2}$$
$$= \mu_t^k + c \left(R_t - \sum_{i=1}^{I} u_{i,t}^k PT_{i,t}^{\max} - \sum_{j=1}^{M} PH_{j,t}^{\max} + e1_t^k \right)$$
$$(t = 1, \cdots, T) \cdots\cdots\cdots\cdots\cdots\cdots (15)$$

$$\nu_t^{k+1/2}$$
$$= \nu_t^k + c \left(SRT_t - \sum_{i=1}^{I} u_{i,t}^k PT_{i,t}^{\max} + \sum_{i=1}^{I} PT_{i,t}^k + e2_t^k \right)$$
$$(t = 1, \cdots, T) \cdots\cdots\cdots\cdots\cdots\cdots\cdots (16)$$

where, "$k + 1/2$" is the index of an iteration step which exists between k and $k + 1$.

3.3 Multipliers Updating   The search process is an iterative procedure that solves relaxed sub-problems and updates Lagrangian multipliers and slack variables according to the extent of the violation of each constraint.

$$e1_t^{k+1} = \max \left\{ \left(e1_t^k - \frac{\mu_t^{k+1/2}}{\alpha} \right), 0 \right\}$$
$$(t = 1, \cdots, T) \cdots\cdots\cdots\cdots\cdots\cdots (17)$$

$$e2_t^{k+1} = \max \left\{ \left(e2_t^k - \frac{\nu_t^{k+1/2}}{\alpha} \right), 0 \right\}$$
$$(t = 1, \cdots, T) \cdots\cdots\cdots\cdots\cdots\cdots (18)$$

$$\lambda_t^{k+1} = \lambda_t^k + c \left(D_t - \sum_{i=1}^{I} PT_{i,t}^{k+1} - \sum_{j=1}^{J} PP_{j,t}^{k+1} \right)$$
$$(t = 1, \cdots, T) \cdots\cdots\cdots\cdots\cdots (19)$$

$$\mu_t^{k+1/2} = \mu_t^k + c \left(\max \left(R_t - \sum_{i=1}^{I} u_{i,t}^k PT_{i,t}^{\max} \right.\right.$$
$$\left.\left. - \sum_{j=1}^{M} PH_{j,t}^{\max} + e1_t^k, 0 \right) \right)$$
$$(t = 1, \cdots, T) \cdots\cdots\cdots\cdots\cdots (20)$$

$$\nu_t^{k+1} = \nu_t^k + c \left(\max \left(SP_t - \sum_{i=1}^{I} u_{i,t} PT_{i,t}^{\max} \right.\right.$$
$$\left.\left. + \sum_{i=1}^{I} PT_{i,t} + e2_t^{k+1}, 0 \right) \right) \quad (t = 1, \cdots, T)$$
$$\cdots\cdots\cdots\cdots\cdots (21)$$

$$\gamma_l^{k+1} = \gamma_l^k - \left(\sum_{t=1}^{T} \sum_{i=1}^{I(l)} C_{i,t}(PT_{i,t}, u_{i,t}, u_{i,t-1}) - FL_l \right)$$
$$(l = 1, \cdots, L) \cdots\cdots\cdots\cdots\cdots (22)$$

$$\delta_m^{k+1} = \delta_m^k - \left(\sum_{t=1}^{T} \sum_{j=1}^{J(m)} w_j(PH_{j,t}) - WR_m \right)$$
$$(m = 1, \cdots, M) \cdots\cdots\cdots\cdots\cdots (23)$$

4. Proposed Method

4.1 Constraint of Prohibition of Simultaneous Unit Start-up/Shut-down at the Same Plamnt

We consider the constraint of prohibition of simultaneous unit start-up/shutdown at the same plant (CPSS). This constraint is defined as a limitation on the number of units that can be started up or shutdown in the same plant within a given time period. This constraint is required due to some technical or personnel reasons

Table 1.   Start-up time lag between unit G1 and unit G2 when G1 precedes G2

		Unit G2		
	Start-up Mode	Hot	Warm	Cold
Unit G1	Hot	2 H	2 H	2 H
	Warm	2 H	2 H	3 H
	Cold	2 H	3 H	4 H

Table 2.   Start-up time lag between unit G1 and unit G2 when G2 precedes G1

		Unit G1		
	Start-up Mode	Hot	Warm	Cold
Unit G2	Hot	1 H	1 H	2 H
	Warm	2 H	2 H	4 H
	Cold	3 H	3 H	4 H

Table 3.   State transition condition

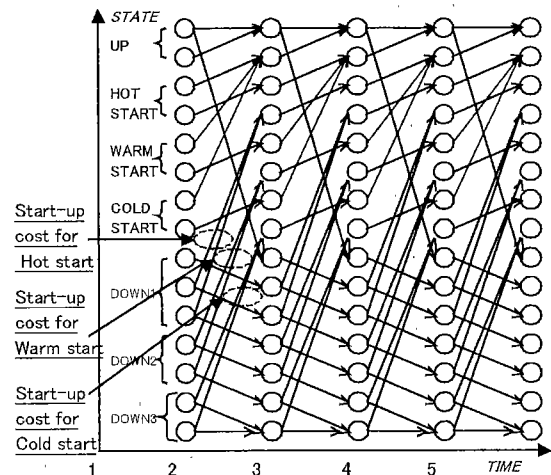|	
minimum up time	2
minimum down time	2
off-line time period of Hot start (Down1)	~3
off-line time period of Warm start (Down2)	4~5
off-line time period of Cold start (Down3)	6~
time period required for AGC after Hot Start	2
time period required for AGC after Warm Start	2
time period required for AGC after Cold Start	2

Fig. 1.   State transition graph of unit G1 and unit G2

at a plant to bring more than one unit on-line simultaneously. Such a constraint is not applied to a plant with more than three units. As an example, this constraint is expressed in the following two tables (Table 1 and Table 2).

Here, "Hot", "Warm" and "Cold" are the kinds of start-up mode, which correspond to the units' off-line time periods. From Table 1 and Table 2, we see that the time lag depends on the start-up modes and the preceding unit.

Suppose that unit G1 and G2 have the same state transition condition shown in Table 3 and Figure 1. In Figure 1, each node illustrated by "〇" represents a state of the unit. The nodes are connected by arrows, which represent possible transitions. The operational cost at a

node, which indicates that the unit is on-line, becomes a solution of the following problem.

$$(1 - \gamma_l^k) f_i(PT_{i,t}) - \lambda_t^{k+1/2} PT_{i,t} - \mu_t^{k+1/2} PT_{i,t}^{\max}$$
$$-\nu_t^{k+1/2} PT_{i,t}^{\max} + \nu_t^{k+1/2} PT_{i,t}$$
$$+\alpha \left(PT_{i,t} - PT_{i,t}^k\right)^2 + \alpha \left(PT_{i,t}^{\max} - u_{i,t}^k PT_{i,t}^{\max}\right)^2$$
$$+\alpha(PT_{i,t}^{\max} - u_{i,t}^k PT_{i,t}^{\max})(PT_{i,t} - PT_{i,t}^k)$$
$$\rightarrow \min \quad (t = 1, \cdots, T) \cdots\cdots\cdots\cdots\cdots (24)$$

The starting cost is given along an arrow which connects from a down state node to a up state node. The solution to sub-problem (12) means finding the path with the least cost from one of the nodes at the initial time to one of the nodes at the last time.
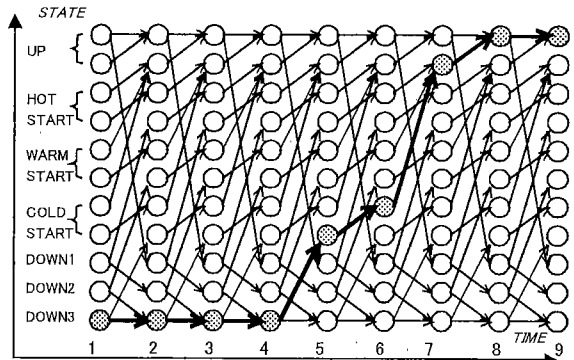
Fig. 2.   Resulting route of unit G1's sub-problem
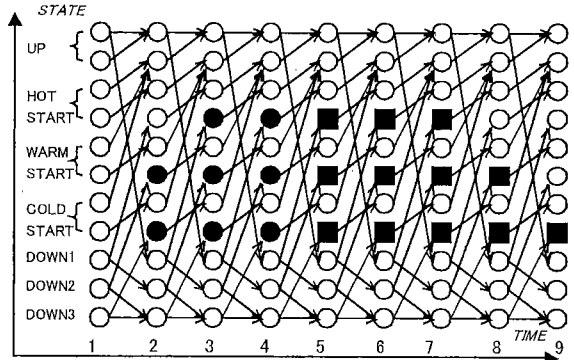
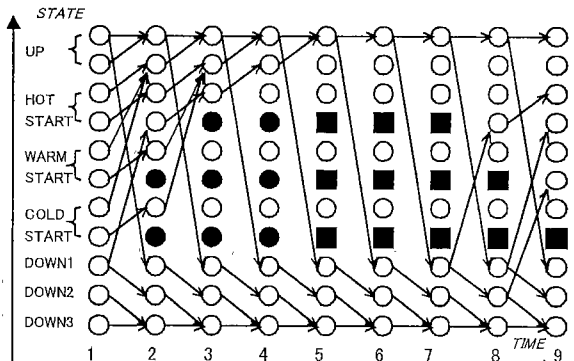Fig. 3.   Infeasible state in state transition for unit G2

Fig. 4.   Feasible state transitions for unit G2

Suppose that the unit G1's sub-problem is solved and the resulting route is given by the bold arrows shown in Figure 2. (Note that down states are simplified in this figure.)

Figure 2 shows that unit G1 starts-up during time period 5 in the cold mode. Taking Table 1 into consideration, unit G2 cannot take states '■' illustrated in Figure 3. Simirally, considering Table 2, unit G2 cannot take states '●'.

Thus, the feasible state transitions for unit G2 is restricted as shown in Figure 4.

This state transition graph shows that the resulting route of unit G2's sub-problem satisfies CPSS.

It is clear that the processes we have outlined by example can be generalized to the algorithm for CPSS. The process are summarized below.

（1） Group the sub-problems for units at the same plant. Suppose these sub problems are denoted as l-th sub-problem and $(l+1)$-th sub-problem.

（2） Solve the l-th sub-problem and record the resulting route and cost, and restrict the state transition graph of $(l+1)$-th sub-problem regarding this route.

（3） Solve the $(l+1)$-th sub-problem and record the resulting route and cost.

（4） Solve the $(l+1)$-th sub-problem newly after cancelling the restriction of state transition graph of $(l+1)$-th sub-problem and record the resulting route and cost of this sub-problem, and restrict the state transition graph of (l)-th sub-problem regarding this route.

（5） Solve the (l)-th sub-problem and record the resulting route and cost.

（6） Compare the sum of cost of step 2–3 and sum of cost of step 4–5 and select the lower cost solution.

Note that this procedure is done in each iteration of ALRM and the restriction of the state transition graph is cleared after solving the sub-problems.

4.2 Spinning Reserve Reauirements for Thermal Units   We use ALRM to improve convergence. Even so, the constraint of spinning reserve requirements for thermal units (CSRT) is rather difficult to satisfy. To understand why, we show a table of effects of changes of Lagrange multipliers on the unit commitment and the power outputs.

We see from Table 4 that λ and ν have counter effects on power dispatching. For example, if the supply is insufficient for the demand during time period t, λt would be increased to meet the demand. Then, some thermal units would increase the power output and CSRT would

Table 4.   Effect of Lagrange multipliers

Changes of the Lagrange multiplier	Effect	
	Commitment	Power Outputs
λ ↑	On	Increase
λ ↓	Off	Decrease
μ ↑	On	No Change
μ ↓	Off	No Change
ν ↑	On	Decrease
ν ↓	Off	Increase

be violated. Therefore, it is necessary to incorporate other methods to overcome these difficulties and obtain feasible solutions.

There are two heuristic ways to satisfy CSRT.

(a) Committing the off-line thermal units in the time periods during which CSRT is violated.

(b) Increasing the power output of on-line hydro units or committing the off-line hydro units in the time periods during which CSRT is violated.

Solution (a) is reliable but economically unprofitable. On the other hand, solution (b) is economically profitable, but it is limited by water conservation constraints. We introduce a heuristic method which aims to reduce the power output of the committed thermal units in order to satisfy the CSRT and compensate the reduction by hydro units. It is realized by reducing the operational maximum outputs of the thermal units in order to satisfy the CSRT within some time periods in which the thermal units do not change the commitment status during some iterations. This method makes the CSRT satisfied without modification, which would remove the counter action of λ and ν. Moreover, the power imbalance brought by reduction of the thermal units output would be compensated by hydro units.

The heuristic algorithm for CSRT is summarized as below.

(1) At every iteration k, the set of committed thermal units within each time period t, which we denote $SCU^k(t)$, are recorded.

(2) If $SCU^k(t) = SCU^{k+1}(t)$ within some time period t, introduce the unchanging counter on t, which is denoted by $UCC(t)$, and if $SCU^k(t) \neq SCU^{k+1}(t)$, clear the $UCC(t)$.

(3) If $UCC(t)$ becomes greater than a threshold (= 20) within some time period t, recalculate the operational maximum output of thermal units in $SCU^k (t)$ to satisfy subsequent conditions.

$$\min \sum_{i \in SCU^k(t)} (1 - \gamma_i^k)f(P_{i,t}^{\max'}) \cdots\cdots\cdots\cdots (25)$$

$$s.t. \sum_{i \in SCU^k(t)} P_{i,t}^{\max'} = \sum_{i \in SCU^k(t)} P_{i,t}^{\max} - SPT_t$$
$$\cdots\cdots\cdots\cdots\cdots (26)$$

$$P_{i,t}^{\min} \le P_{i,t}^{\max'} \le P_{i,t}^{\max} \quad (i \in SCU^k(t))$$
$$\cdots\cdots\cdots\cdots\cdots (27)$$

If step 3 is processed in a time period during an iteration step k, equation (26) is satisfied. That is, CSRT is satisfied after iteration step k. Thus, the counter action of λ and ν would not occur after iteration step k during that time period. Moreover, hydro units would be committed or dispatched during that time period more appropriately than before and the surplus water would be re-dispatched for other time periods.

4.3 Algorithm Flow   We show the proposed method in the flow chart in Figure 5. This figure shows that the proposed method is incorporated in the iteration process of ALRM.

5. Test Results

We have applied the proposed method to a generation system of the Hokkaido Electric Power Co. (HEPCO). This system has 13 thermal units, 10 hydro units and 4 pumped-storage hydro units. The study horizon is one week, split into 168 h time periods.

For comparison, we selected 6 real cases prepared by operation planners and compared them with the results obtained by the proposed method under the same condition of load curve, fuel consumptions, water consumption and other operational constraints. Table 5 summarizes the comparison results, from which we see that the proposed method reduced the operational cost in each case.

To illustrate the proposed method, we show the unit operation schedules of case 6 obtained by the proposed method (Figure 7) and the one prepared by operation planners (Figure 6). In these figures, TA1 to TF2 are the thermal units in which A or F indicates a plant name and the figure indicates a unit number. PA1 to PB2 are the pumped-storage hydro units. HA1 to HE2 are the hydro units. Note that the vertical axes of the graphs
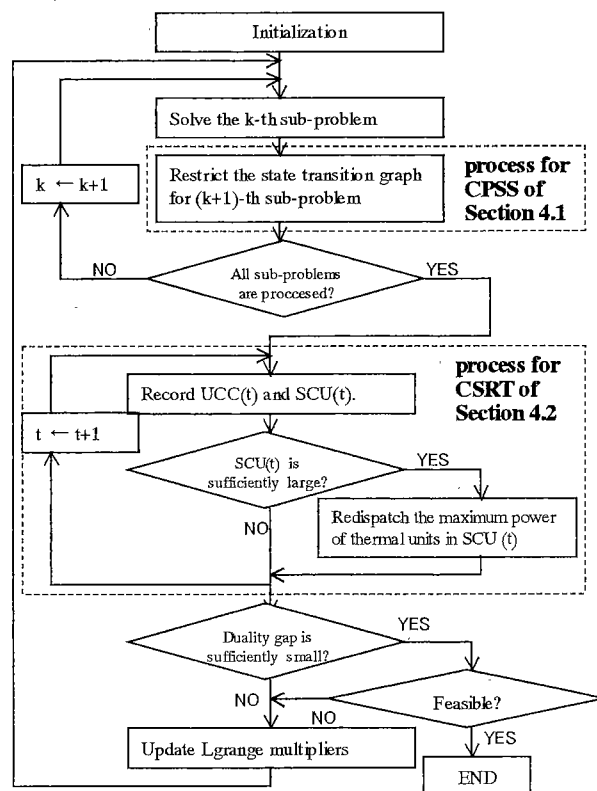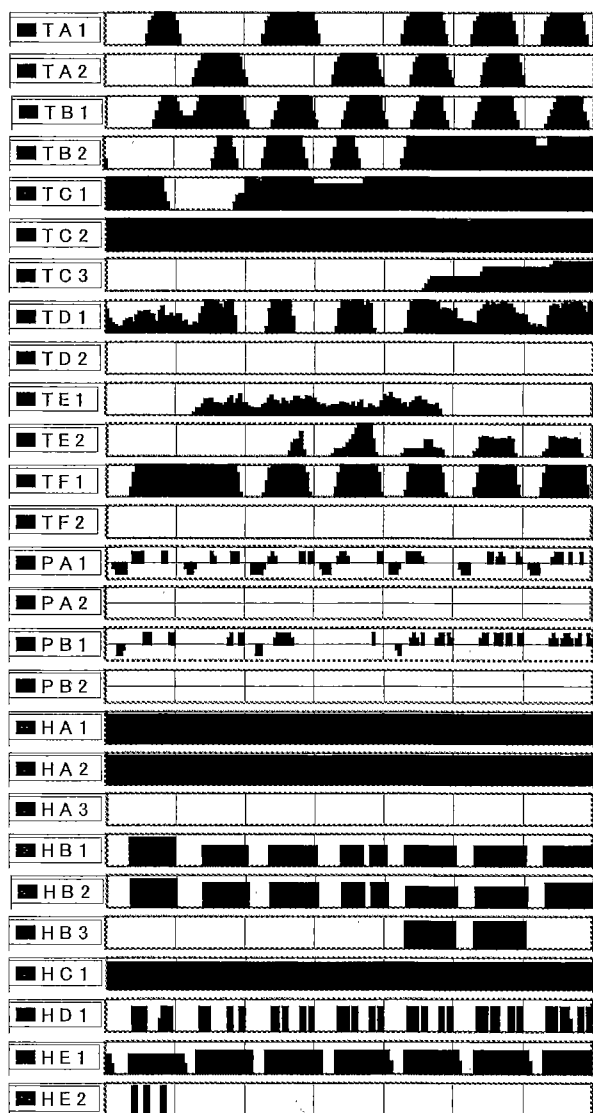

Fig. 5. Algorithm flow

Table 5. Test cases and comprison results

Case No. (Season)		Reduced cost (%)
Case 1	(April '98)	1.435
Case 2	(November '98)	0.592
Case 3	(February '99)	0.093
Case 4	(April '99)	0.012
Case 5	(July '99)	1.094
Case 6	(August '99)	0.717

(Unit Type T: Thermal P: pumped-storage hydro H: Hydro)

Fig. 6.   Unit schedule of Case 6 prepared by operational planners (Real Schedule)

Fig. 7.   Unit schedule of Case 6 obtained using proposed method

show 0% to 100% of the maximum power output of the units. The horizontal axes of the graphs are the time periods.

We see from the Figure 7 that there is no simultaneous unit start-up or shutdown in any thermal plant. In other words, CPSS is satisfied. Moreover, we see that during each time period, there exist thermal units whose power output is not reached at the maximum power, which leads to CSRT satisfaction.

Figure 8 shows the convergence proterty comparison between conventional LRM and ALRM in Case 1. Here total demand error is an absolute summation of ratio of power imbalance to demand. Figure 8 shows that ALRM has a good convergence property.

Through these numerical tests, it it proved that all these unit commitment problems can be solved with CPU time about 10 minutes when it is run on a Pentium 366 MHz machine. These results therefore demonstrate that the proposed method is effective.
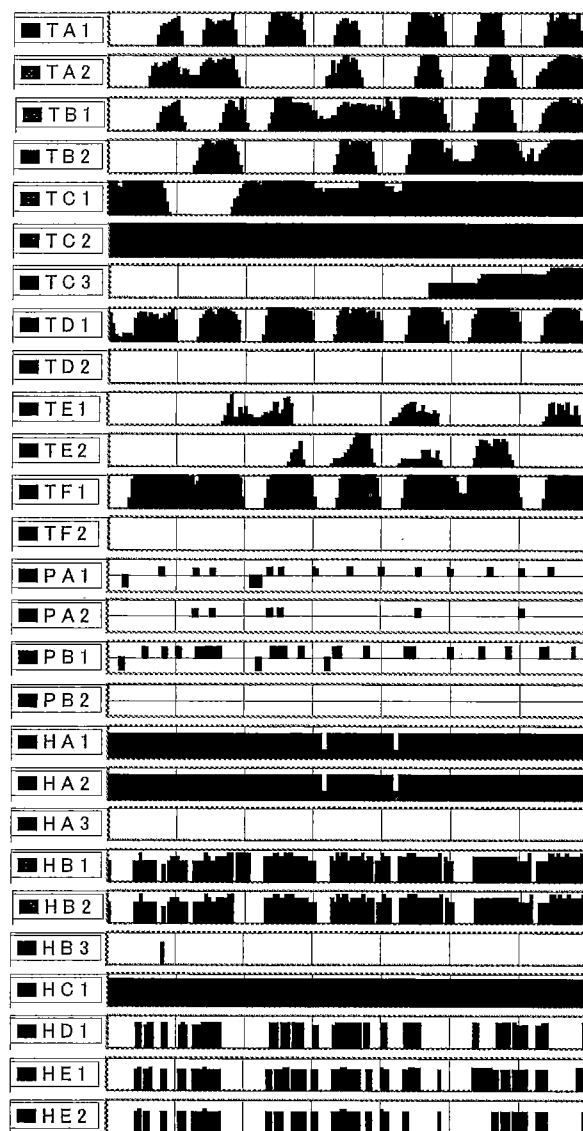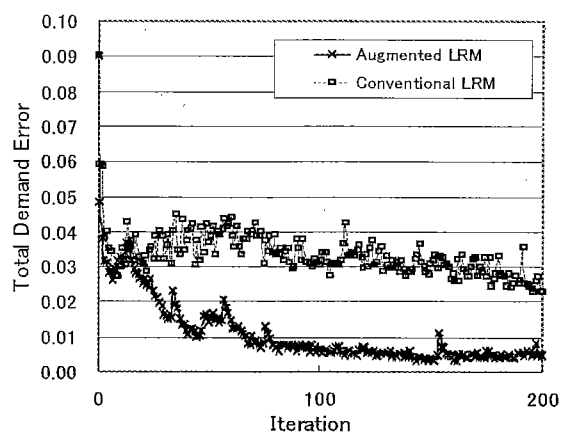

Fig. 8.   Convergence process of Case1

6. Conclusion

This paper has described a practical method based on a Lagrangian relaxation method for solving the hydro-thermal unit commitment problem. This method is processed during each iteration step of ALRM that enables a direct solution considering CPSS and CSRT, and it is easy to implement. Numerical results have shown that the proposed method can effectively and automatically generate practical unit commitment and can reduce the operation costs compared to the unit commitment schedule prepared by operation planners. This method is expected to help operational planners to achieve more economical system operations.

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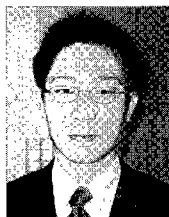
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