Paper

Robust Model Matching for Polytopic LPV Plant

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Abstract the present paper investigates attachable robust compensators design for polytopic linear parameter varying (LPV) plants using prior or real-time knowledge of the parameter. Based on each knowledge, this paper explores both robust LPV system and robust gain scheduled system design method. Quadratic L_2 gain performance framework and robust model matching (RMM) strategy are combined to develop compensators. Namely, a RMM design method, that is, a practical approach to the design of attachable robust compensators for the linear time invariant (LTI) plant, is extended for application to the LPV plant. A design example and simulation results are presented in order to demonstrate the proposed method.

Keywords : robust control, gain-scheduled control, linear parameter-varying system, polytopic system, linear matrix inequalities

1. Introduction

In practical, nonlinear and/or time-variant dynamical systems are widespread, and a certain class of them can be represented as linear time varying (LTV) systems. Basic analysis and synthesis of control systems for LTV systems has been examined in previous studies(1)~(3). Recently LTV system design, including tracking, stabilization, optimization and robust control, has been investigated comprehensively in several studies(4)~(9). However, unlike the linear time invariant (LTI) systems, few powerful tool or algebraical frequency-domain description exists to solve LTV problems. As such, a realistic and practical control system design method for the general LTV system has not yet been completed.

Shamma & Athans(10)(11) formalized a certain type of nonlinear system as a linear parameter varying (LPV) system. It is well known that many practical LTV systems can be translated into LPV form. For example, servos with time-varying parameters, aircraft flying in various situations etc. are described as LPV. A certain type of nonlinear system also can be translated into LPV form by Jacobian linearization or quasi-LPV modeling. Shamma & Athans also succeeded in developing a control strategy for this system based on classical gain scheduled methodology. Basically, this LPV control system design method, known as the frozen parameter method, deals with only parameters that vary slowly with time. Recently, significant progress has been made in this area, and a unified H-infinity approach is being developed that is reducible, to a linear matrix inequality (LMI) optimization problem(6)(12)~(17). Compared to the classical gain scheduled method, these approaches take into consideration the time-varying nature of plants and grow out of ad-hoc interpolation. During the last couple of years, tutorial paper and special publications concerning this problem have appeared(18)~(21). The recent gain scheduled method assures a quadratic H-infinity property and robust stability for all possible parameter trajectories. In contrast to LTV systems, gain scheduled approaches are applicable under assumption that the dependent parameters can be measured on-line instead of prior knowledge.

In LPV control systems, nominal parameter trajectory can be derived from prior or real-time knowledge, respectively, in LTV or gain scheduled viewpoint. However, the nominal trajectory differs from real trajectory because of modeling error, observation error and so on. Thus, a robust control technique is needed to compensate for this error.

Turning now to robust control design method, a practical approach to the design of attachable robust compensators has been developed(22)~(27), for the LTI plant. The principle behind this method is robust model matching (RMM), which adjust 'a real plant with a robust compensator' to 'a nominal plant' by equivalent-disturbance attenuation without changing desirable response to reference in two-degree-of-freedom control scheme.

In the present paper, RMM has been developed for application to LPV plants in combination with quadratic L_2 gain performance framework. A novel RMM is a unified approach for robust LPV system and robust gain scheduled system, based on nominal LPV plant derived from prior or real-time knowledge of the dependent parameter. Since the additional robust compensator is designed without information of previously designed controllers, moreover, the robust compensator is constructed separately with the previous controllers; novel RMM is applicable for any existing control systems. Methods to test robust stability of the overall system for feasible trajectories are also shown. A design example and simulation results are presented in order to illustrate the proposed method.

2. Plant Description

The notation used in this paper is as follows:
$w \in \Re^p$: exogenous inputs (reference, disturbance, etc.)
$x \in \Re^l$: state vector,
$y \in \Re^g$: measurable outputs,
$u \in \Re^q$: control inputs,
$z \in \Re^m$: controlled outputs,
$\theta(t) = \left[\theta_1(t),\ \theta_2(t), \cdots, \theta_r(t)\right]^T \in \Re^r$:time-varying parametric

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uncertainty,

$d \in \Re^g$: equivalent disturbances representing influence on the controlled outputs due to trajectory error between the real dependent parameters and the nominal ones,

I_k : $k \times k$ unit matrix,

0_k, $0_{a\times b}$: respectively, $k \times k$, $a \times b$ zero matrix,

Co : convex hull.

Consider an LPV plant: $P(\theta(t))$ described by state space equations as:

$$\begin{bmatrix} \dot{x}(t) \\ z(t) \\ y(t) \end{bmatrix} = \begin{pmatrix} A(\theta_0(t)) & B_w(\theta(t)) & B_u \\ C_z(\theta(t)) & D_{wz}(\theta(t)) & D_{uz} \\ C_y & D_{wy} & 0_{g\times q} \end{pmatrix} \begin{bmatrix} x(t) \\ w(t) \\ u(t) \end{bmatrix} \cdots\cdots\cdots\cdots (1)$$

Here state-space matrices have compatible dimensions. Moreover we have the following assumptions.

(1) The state-space matrices $A(\theta), B_w(\theta), C_z(\theta), D_{wz}(\theta)$ depend affinely on $\theta(t)$.

(2) The real parameter $\theta(t)$ is not real-time measurable but nominal one $\theta_0(t)$ can be known in advance or on-line. Both θ and θ_0 vary in the same polytope Θ of vertices $\omega_1, \omega_2, \cdots, \omega_N$, $N = 2^r$; they can be expressed respectively as:

$$\theta(t) \in \Theta := Co\{\omega_1, \omega_2, \cdots, \omega_N\} = \left\{ \sum_{i=1}^{N} \alpha_i(t)\omega_i : \alpha_i(t) \geq 0, \sum_{i=1}^{N} \alpha_i(t) = 1 \right\}$$

$$\theta_0(t) \in \Theta := Co\{\omega_1, \omega_2, \cdots, \omega_N\} = \left\{ \sum_{i=1}^{N} \alpha_{0i}(t)\omega_i : \alpha_{0i}(t) \geq 0, \sum_{i=1}^{N} \alpha_{0i}(t) = 1 \right\} \cdots\cdots\cdots (2)$$

(3) The pair $(A(\theta), C_y)$ and $(A(\theta), B_u)$ are quadratically detectable and quadratically stabilizable over Θ, respectively.

With above assumptions, the LPV plant is called polytopic when it ranges in a matrix polytope. Namely, rewriting (1) with (2), the nominal LPV polytopic plant $P(\theta_0)$ can be expressed as:

$$\begin{pmatrix} A(\theta_0) & B(\theta_0) \\ C(\theta_0) & D(\theta_0) \end{pmatrix} = \sum_{i=1}^{N} \alpha_{0i}(t) \begin{pmatrix} A_i & B_i \\ C_i & D_i \end{pmatrix},$$

$$\alpha_{0i}(t) \geq 0, \sum_{i=1}^{N} \alpha_{0i}(t) = 1 \cdots\cdots\cdots (3)$$

Here, $\begin{pmatrix} A_i & B_i \\ C_i & D_i \end{pmatrix} := \begin{pmatrix} A(\omega_i) & B(\omega_i) \\ C(\omega_i) & D(\omega_i) \end{pmatrix}$.

Also, the real plant $P(\theta)$ can be expressed as:

$$\begin{pmatrix} A(\theta) & B(\theta) \\ C(\theta) & D(\theta) \end{pmatrix} = \sum_{i=1}^{N} \alpha_i(t) \begin{pmatrix} A_i & B_i \\ C_i & D_i \end{pmatrix},$$

$$\alpha_i(t) \geq 0, \sum_{i=1}^{N} \alpha_i(t) = 1 \cdots\cdots\cdots (4)$$

3. Controllers Design

We present methods for designing controllers consist of two components. The one is a nominal controller that is designed according to the nominal LPV plant to obtain stability and desirable reference response property (We can also exploit existing controller for this one). The other is a robust compensator that is added in order to reduce the influence of parameter perturbation due to the real parameter's deviation from the nominal one based on the method shown in reference (26).

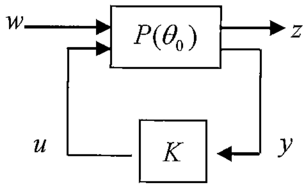


Fig. 1. Nominal control scheme.

3.1 Nominal Controller Design Consider control scheme of Fig.1. Here, $P(\theta_0)$ is weighted nominal LPV plant. A nominal controller: K is designed to satisfy the following control objectives:

1) Desirable property to reference for $P(\theta_0)$,

2) Stability for $P(\theta_0)$.

For example, the quadratic L_2 gain performance strategy can be applied to design the nominal controller. Design of this controller is reduced to solve LMI optimization problem similarly formulated by the method proposed in section five of reference (14) for nominal plant $P(\theta_0)$. We can obtain vertex state space matrices of the controller, and then the resulting continuous controller is led as:

$$\begin{pmatrix} A_k(\theta_0) & B_k(\theta_0) \\ C_k(\theta_0) & D_k(\theta_0) \end{pmatrix} = \sum_{i=1}^{N} \alpha_{0i}(t) \begin{pmatrix} A_k(\omega_i) & B_k(\omega_i) \\ C_k(\omega_i) & D_k(\omega_i) \end{pmatrix}. \cdots\cdots\cdots (5)$$

Remark:

It should be noted, as to unstable LPV plants (1), we can adopt gain or dynamic output feedback to make them stable due to quadratic stabilizability[6]*. These stable LPV plants are regarded as controlled plants to design nominal controller and robust compensators.*

3.2 Robust Compensator Design Because the real trajectory $\theta(t)$ will differ from nominal ones: $\theta_0(t)$, the LPV control system that consists of real plant of $\theta(t)$ and nominal controller of $\theta_0(t)$ may not satisfy the desired specifications mentioned above. A robust compensator should be added into the control system to recover the specifications.

In this subsection, we introduce the principle of robust model matching (RMM) method briefly, and develop this method to apply for LPV systems.

3.2.1 Principle of RMM We see the robust compensator have structures separate from nominal control system compared Fig.1 with Fig.2. Here, the augmented plant is composed of a real plant : P and a robust compensator: R.

The philosophy of RMM is to make input-output property of the augmented plant approaches to the nominal model, Namely, the low sensitivity to external disturbance and modeling error. This objective is achieved by means of rejecting the equivalent

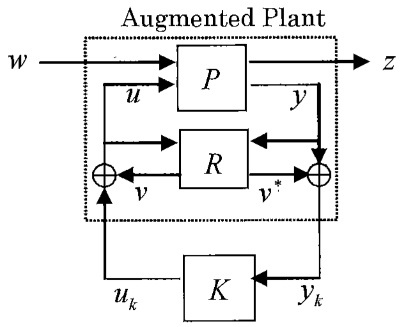


Fig. 2. RMM control scheme.

disturbance that represents the modeling errors. It must be noted that unlike Fig.1, P, K and R are LTI system, and $y \in \Re^g$ is composed of $z \in \Re^m$ and $\bar{z} \in \Re^{g-m}$: measurable outputs except z, since we treat equivalent disturbance rejection to z by feedback control with measurable output y.

The robust compensator: $R(s)$ consists of following elements:

1.Observer of equivalent disturbances: $R_o(s)$,

2.Zeroing matrix: $R_z(s)$,

2.Robust filter: $R_f(s)$.

The observer calculates equivalent disturbances from measurable variables, y and u. Zeroing matrix cancels the effect of plant's changes by minimizing transfer matrix of over all system from equivalent disturbances to measurable outputs. Because the R_o multiplied by R_z is not always proper matrix, a low-pass filter R_f called robust filter should eliminate differentiators in them. Another purpose of the robust filter is to consist disturbance rejection with robust stability.

Now we develop the RMM strategy to apply for LPV systems, and explain design procedure.

3.2.2 Robust Compensator Design for Polytopic LPV Systems Because there is no algebraic transfer function like LTI system, unlike conventional RMM, we propose a robust compensator based on state-space expression.

1) Observer: $R_o(\theta_0)$

The real signals around the plant can be expressed with the nominal plant and disturbances as:

$$y = P(\theta)u = P(\theta_0)u + d \quad \cdots\cdots (6)$$

The vector $d \in \Re^g$ represents the influence of trajectory error on the measurable outputs, and called equivalent disturbance of LPV plants.

The state space equation of the observer $R_o(\theta_0)$ can be derived from substituting (1) into (6) as the following:

$$\begin{bmatrix} \dot{x}_o(t) \\ d(t) \end{bmatrix} = \begin{pmatrix} A(\theta_0) & (B_u, & 0_{l\times g}) \\ -C_y & (0_{g\times q}, & I_g) \end{pmatrix} \begin{bmatrix} x_o(t) \\ \begin{pmatrix} u(t) \\ y(t) \end{pmatrix} \end{bmatrix} \quad \cdots\cdots (7)$$

Besides, $x_o \in \Re^o$ stands for the states of the observer.

2) Zeroing element: $R_z(\theta_0)$

In RMM control scheme (Fig.2), robust compensator is constructed as shown in Fig.3. The role of the zeroing element is to cancel the influence of the equivalent disturbance: d on both controlled output: z and measurable output: y_k.

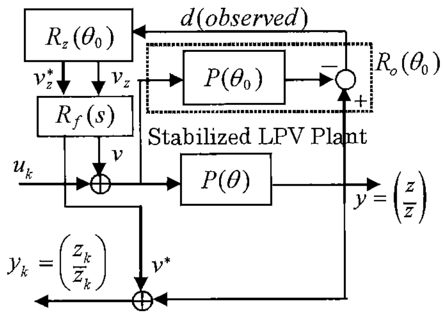


Fig. 3. LPV plant with robust compensator.

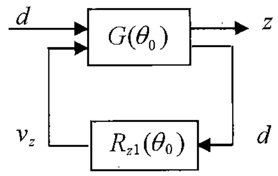


Fig. 4. Rearranged figure of Fig.3 except the observer.

Furthermore, corresponding output v_z and v_z^*, $R_z(\theta_0)$ consists of two elements $R_{z1}(\theta_0)$ and $R_{z2}(\theta_0)$. The $R_{z1}(\theta_0)$ eliminates the influence of d to z and the $R_{z2}(\theta_0)$ eliminates the influence of d to y_k.

To do so, first, we design $R_{z1}(\theta_0)$ to satisfy the following bounded input/output map of the augmented LPV plant for all possible trajectories as:

$$\|z\|_2 \le \gamma \|d\|_2 \quad \cdots\cdots (8)$$

This problem can be illustrated simply to LFT (Linear Fractional Transformation) structure as in Fig.4.

Here, state-space expression of $G(\theta_0)$ is expressed as:

$$\begin{bmatrix} \dot{x}(t) \\ z(t) \\ d(t) \end{bmatrix} = \begin{pmatrix} A(\theta_0) & 0_{l\times g} & B_u \\ C_z & (I_m, 0_{m\times(g-m)}) & 0_{m\times q} \\ 0_{g\times l} & I_g & 0_{g\times q} \end{pmatrix} \begin{bmatrix} x(t) \\ d(t) \\ v_z(t) \end{bmatrix}. \quad \cdots\cdots (9)$$

Above $G(\theta_0)$ is produced from unweighted nominal plant, but if necessary we can use weighting function that can be selected based on a frozen time analysis and follows the same way as conventional H-infinity synthesis.

Similarly to nominal controller design mentioned in subsection 3.1, we obtain vertex state space matrices of the compensator $R_{z1}(\theta_0)$, and then the resulting continuous compensator is led as:

$$\begin{pmatrix} A_{z1}(\theta_0) & B_{z1}(\theta_0) \\ C_{z1}(\theta_0) & D_{z1}(\theta_0) \end{pmatrix} = \sum_{i=1}^{N} \alpha_{0i}(t) \begin{pmatrix} A_{z1}(\omega_i) & B_{z1}(\omega_i) \\ C_{z1}(\omega_i) & D_{z1}(\omega_i) \end{pmatrix} \quad \cdots\cdots (10)$$

concerning input d, outputs v and state $x_{z1}(t) \in \Re^{z1}$.

After deriving R_{z1}, another zeroing element R_{z2} that produce signal v^* added to y is considered. We see that the R_{z2} with the following state space equation makes the influence of equivalent disturbance d on y_k completely zero.

$$\begin{bmatrix}\dot{x}_{z1}\\ \dot{x}_o\end{bmatrix}=\begin{bmatrix}A_{z1}(\theta_0) & 0_{z1\times l}\\ B_uC_{z1}(\theta_0) & A(\theta_0)\end{bmatrix}\begin{bmatrix}x_{z1}\\ x_o\end{bmatrix}+\begin{bmatrix}B_{z1}(\theta_0)\\ B_uD_{z1}(\theta_0)\end{bmatrix}d,$$

$$v_z^*=\begin{bmatrix}0_{g\times z1} & -C_y\end{bmatrix}\begin{bmatrix}x_{z1}\\ x_o\end{bmatrix}-d\,. \quad\cdots\cdots(11)$$

Finally, zeroing element R_z that is composed of R_{z1} and R_{z2} can be derived as:

$$\begin{bmatrix}\begin{pmatrix}\dot{x}_{z1}\\ \dot{x}_o\end{pmatrix}\\ \begin{pmatrix}v_z\\ v_z^*\end{pmatrix}\end{bmatrix}=\begin{bmatrix}\begin{pmatrix}A_{z1}(\theta_0) & 0_{z1\times l}\\ B_uC_{z1}(\theta_0) & A(\theta_0)\end{pmatrix} & \begin{pmatrix}B_{z1}(\theta_0)\\ B_uD_{z1}(\theta_0)\end{pmatrix}\\ \begin{pmatrix}C_{z1}(\theta_0) & 0_{m\times l}\\ 0_{g\times z1} & -C_y\end{pmatrix} & \begin{pmatrix}D_{z1}(\theta_0)\\ -I_g\end{pmatrix}\end{bmatrix}\begin{bmatrix}\begin{pmatrix}x_{z1}\\ x_o\end{pmatrix}\\ d\end{bmatrix}$$

$$\cdots\cdots(12)$$

Here, $x_z=\begin{bmatrix}x_{z1}^T & x_o^T\end{bmatrix}^T\in\Re^z$ stands for states of the zeroing element. To convenient notation, we have following equations:

$$A_z=\begin{pmatrix}A_{z1}(\theta_0) & 0_{z1\times l}\\ B_uC_{z1}(\theta_0) & A(\theta_0)\end{pmatrix},\ B_z=\begin{pmatrix}B_{z1}(\theta_0)\\ B_uD_{z1}(\theta_0)\end{pmatrix},$$

$$C_z=\begin{pmatrix}C_{zv}\\ C_{zv^*}\end{pmatrix}=\begin{pmatrix}C_{z1}(\theta_0) & 0_{m\times l}\\ 0_{g\times z1} & -C_y\end{pmatrix},\text{and}$$

$$D_z=\begin{pmatrix}D_{zv}\\ D_{zv^*}\end{pmatrix}=\begin{pmatrix}D_{z1}(\theta_0)\\ -I_g\end{pmatrix}.$$

Remark:

The signal d does not always belong to L_2 signals. However, it is well known that performance (8) is also effective to finite power signal. Thus, design based on (8) is useful for most practical cases.

3) Robust filter

In order to consist disturbance rejection with robust stability and keep the closed-loop state-space matrix affine dependent on $\theta(t)$ or $\theta_0(t)$, a transfer function matrix called robust filter is used. To satisfy these requirements, the robust filter should have adequate band-width and be strictly proper as the form of

$$\begin{bmatrix}\dot{x}_f\\ v\\ v^*\end{bmatrix}=\begin{pmatrix}A_f & \begin{pmatrix}B_{fv}, & B_{fv^*}\end{pmatrix}\\ \begin{pmatrix}C_{fv}\\ C_{fv^*}\end{pmatrix} & 0_{q+g}\end{pmatrix}\begin{bmatrix}x_f\\ v_z\\ v_z^*\end{bmatrix}.$$

$$\cdots\cdots(13)$$

4. Robust Stability Analysis of Whole Closed-loop System

Since robust compensator has a capability to make augmented plant approaches to $P(\theta_0)$, it is reasonable to suppose that over-all system including the robust compensator is robust stable.

The overall system is composed of double loop structure as shown in Fig.5. Here, $R(\theta_0)=R_f(\theta_0)R_z(\theta_0)$.

State-space function of $\tilde{P}(\theta,\theta_0)$ consists of real plant $P(\theta)$ and observer $R_o(\theta_0)$ can be derived as:

$$\begin{bmatrix}\dot{x}\\ \dot{x}_o\end{bmatrix}=\begin{bmatrix}A(\theta) & 0\\ 0 & A(\theta_0)\end{bmatrix}\begin{bmatrix}x\\ x_o\end{bmatrix}+\begin{bmatrix}B_u\\ B_u\end{bmatrix}u,$$

$$\begin{bmatrix}y\\ d\end{bmatrix}=\begin{bmatrix}C_y & 0\\ C_y & -C_y\end{bmatrix}\begin{bmatrix}x\\ x_o\end{bmatrix}.$$

$$\cdots\cdots(14)$$

It must be noted that influence of d on y_k is completely eliminated, thus, the lower loop consists of $\tilde{P}(\theta,\theta_0)$ and K works unaffected by $R(\theta_0)$. Therefore, if the robust stability of upper loop (augmented plant) is assured, the robust stability of the overall system is also assured.

The rest of this section, first we derive state-space expression of the augmented plant. Then we show two ways to test the stability of the augmented plant following the way described in the reference (16).

To convenient notation, the above equation also can be written as:

$$\dot{x}_p=A_p(\tilde{\theta}(t))x_p+B_pu_c,\ y=C_px_p\,. \quad\cdots\cdots(16)$$

Here, $x_p=\begin{bmatrix}x^T & x_o^T & x_z^T & x_f^T\end{bmatrix}^T,\ \tilde{\theta}=[\theta^T,\theta_0^T]^T$.

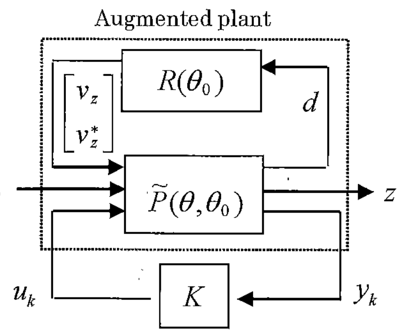


Fig. 5. Structure of whole system.

$$\begin{bmatrix}\dot{x}\\ \dot{x}_o\\ \dot{x}_z\\ \dot{x}_f\end{bmatrix}=\begin{bmatrix}A(\theta) & 0_{l\times l} & 0_{l\times z} & B_uC_{fv}\\ 0_{l\times l} & A(\theta_0) & 0_{l\times z} & B_uC_{fv}\\ B_z(\theta_0)C_y & -B_z(\theta_0)C_y & A_z(\theta_0) & 0_{z\times f}\\ [B_{fv}D_{zv}(\theta_0)+B_{fv^*}D_{zv^*}(\theta_0)]C_y & -[B_{fv}D_{zv}(\theta_0)+B_{fv^*}D_{zv^*}(\theta_0)]C_y & B_{fv}C_{zv}(\theta_0)+B_{fv^*}C_{zv^*}(\theta_0) & A_f\end{bmatrix}\begin{bmatrix}x\\ x_o\\ x_z\\ x_f\end{bmatrix}+\begin{bmatrix}B_u\\ B_u\\ 0_{z\times q}\\ 0_{f\times q}\end{bmatrix}u_c$$

$$y=\begin{bmatrix}C_y & 0_{g\times l} & 0_{g\times z} & 0_{g\times f}\end{bmatrix}\begin{bmatrix}x\\ x_o\\ x_z\\ x_f\end{bmatrix} \quad\cdots\cdots(15)$$

Then we show two ways to test the stability of this system. Note that $A_p(\tilde{\theta}(t))$ is affinely dependent on both $\theta(t)$ and $\theta_0(t)$.

4.1 Quadratic Stability The system is said to be quadratically stabilizable via an dependent parameter if there exists a $(2l+z+f)\times(2l+z+f)$ positive definite matrix P such that

$$PA_p(\tilde{\theta})+A_p^T(\tilde{\theta})P<0. \quad\cdots(17)$$

Inequality (17) is reduced to be similar problem as after-mentioned (19) with common matrix P.

4.2 Parameter-dependent Lyapunov Functions Less conservative sufficient conditions for stability over the entire polytope are as follows:

Since $A_p(\tilde{\theta})$ varies in the convex envelope of a set of LTI models as:

$$A_p(\tilde{\theta})\in Co(A_1,\cdots,A_{N^2})=\left\{\sum_{i=1}^{N^2}\beta_i(t)A_i:\beta_i(t)\geq 0,\sum_{i=1}^{N^2}\beta_i(t)=1\right\},$$

$$\cdots(18)$$

we seek a parameter-dependent Lyapunov function of the form $V(x,\beta)=x^T\mathrm{P}(\beta)^{-1}x$ where $P(\beta)=\beta_1P_1+\cdots+\beta_{N^2}P_{N^2}$.

If there exists symmetric matrices $\mathrm{P}_1,\cdots,\mathrm{P}_{N^2}$, and scalars $t_{ij}=t_{ji}$ such that

$$A_iP_j+P_jA_i^T+A_jP_i+P_iA_j^T<2t_{ij}I_{(2l+z+f)},$$

$$P_j>I_{(2l+z+f)}, \quad\cdots(19)$$

$$\begin{bmatrix} t_{11} & \cdots & t_{1(N^2)} \\ \vdots & \ddots & \vdots \\ t_{(N^2)1} & \cdots & t_{(N^2)(N^2)} \end{bmatrix}<0,$$

for all $i,j\in\{1,2,\cdots,N^2\}$, then the Lyapunov function $V(x,\beta)$ establishes stability of this system.

Remark:

In order to check the stability of $A_p(\tilde{\theta})$, it is not necessary to derive parameters: $\beta_i(t)$. Instead, we should find out P_is that satisfy (19) with given A_is.

5. Example

A classical example of parameter-varying unstable plant that can be viewed as a mass-spring-damper system with time-varying spring stiffness is considered. The state space equation of this unstable unweighted LPV plant is as follows

$$A(\theta)=\begin{bmatrix} 0 & 1 \\ -0.5-0.5\theta & -0.2 \end{bmatrix}, B_w=\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \ B_u=\begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

$$C_z=\begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix}, \ C_y=\begin{bmatrix} 1 & 0 \end{bmatrix}, \ D_{wz}=\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \ D_{uz}=\begin{bmatrix} 0 \\ 1 \end{bmatrix}, \ D_{wy}=0.$$

$$\cdots(20)$$

Here the scope of nominal time-varying parameter $\theta(t)$ is in polytopic spaces $\Theta:=Co\{-1,1\}$. Also, the nominal trajectory of dependent parameter $\theta_0(t)$, $\alpha_{01}(t)$ and $\alpha_{02}(t)$ are assumed as:

$$\theta_0=\cos(0.05t),$$

$$\alpha_{01}(t)=(1-\theta_0)/2, \quad\cdots(21)$$

$$\alpha_{02}(t)=1-\alpha_{01}(t).$$

5.1 Transferring to Stable LPV System Because the plant is not stable, firstly, it should be stabilized. After some try and errors, we found an output-feedback $u=-6y$ to make the above plant to a stable LPV plant as:

$$A(\theta)=\begin{bmatrix} 0 & 1 \\ -6.5-0.5\theta & -0.2 \end{bmatrix}, \ B_w=\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \ B_u=\begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

$$C_z=\begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix}, \ C_y=\begin{bmatrix} 1 & 0 \end{bmatrix}, \ D_{wz}=\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \ D_{uz}=\begin{bmatrix} 0 \\ 1 \end{bmatrix}, \ D_{wy}=0.$$

$$\cdots(22)$$

5.2 Nominal Controller Design To enforce the performance and robustness requirements, we treat L_2 gain of the map from w to z_1, z_2 less than γ_n as the following inequality (23) and global asymptotic stability for all feasible parameter trajectories θ_0 in the polytopic space Θ.

$$\left\|\begin{matrix} W_1S \\ W_2SK \end{matrix}\right\|\leq\gamma_n \quad\cdots(23)$$

Here S denotes maps from w to z_1.

The weighting functions were chosen in sense of frozen time method as follows:

$$W_1=\frac{0.9s+0.25}{s+0.0001}, \ W_2=0.0045. \quad\cdots(24)$$

The gain diagram of the weighs is shown in Fig. 7.

Using standard software from the Matlab LMI toolbox[16], we got controller vertex matrices as:

$$A_{k1}=\begin{bmatrix} 16.9 & 6.74 & 8904 \\ -27.7 & -9.49 & -14136 \\ -2.23 & -0.016 & -940.3 \end{bmatrix}, \ B_{k1}^T=\begin{bmatrix} -0.13 & -0.09 & 0.22 \end{bmatrix},$$

$$C_{k1}=\begin{bmatrix} 28.6 & 9.17 & 16723 \end{bmatrix}, D_{k1}=0,$$

$$A_{k2}=\begin{bmatrix} 17.3 & 6.94 & 8905 \\ -28.3 & -9.81 & -14137 \\ -2.26 & -0.037 & -940.3 \end{bmatrix},$$

$$B_{k2}^T\approx B_{k1}^T, C_{k2}\approx C_{k1}, D_{k2}=0 \quad\cdots(25)$$

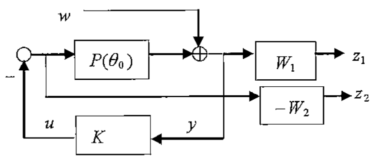


Fig. 6. Block diagram of the nominal control system.

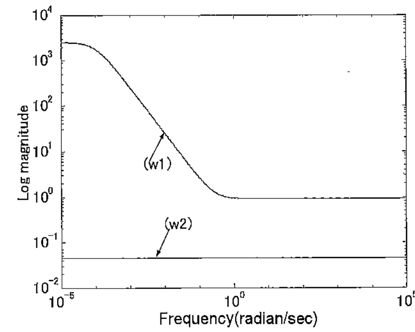


Fig. 7. Gain diagram of weighting functions.

Then the nominal controller can be constructed as:

$$\begin{pmatrix} A_k & B_k \\ C_k & D_k \end{pmatrix} = \alpha_{01}(t)\begin{pmatrix} A_{k1} & B_{k1} \\ C_{k1} & D_{k1} \end{pmatrix} + \alpha_{02}(t)\begin{pmatrix} A_{k2} & B_{k2} \\ C_{k2} & D_{k2} \end{pmatrix}, \quad \cdots\cdots(26)$$

The H-infinity norm of the above optimal problem γ_n is 0.92 after 27 iterations of the algorithm.

5.3 Design of Robust Compensator

1) Observer

Observer of the base-equivalent disturbance R_o is derived from substituting (22) into (7) as

$$\begin{bmatrix} \dot{x}_o \\ d \end{bmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -6.5-0.5\theta_0 & -0.2 & 1 & 0 \\ -1 & 0 & 0 & 1 \end{pmatrix}\begin{bmatrix} x_o \\ \begin{pmatrix} u \\ y \end{pmatrix} \end{bmatrix}. \quad \cdots\cdots(27)$$

2) Zeroing element

According to the subsection 3.2.2, we consider the minimization problem as (8). After some try and errors, the following weighting function: $k(s)$ was used in both d to d and d to z relation of $G(\theta_0)$ in Fig.4 to obtain feasible LMI.

$$k(s) = \frac{1}{s+0.1} \quad \cdots\cdots(28)$$

Using Matlab LMI Toolbox, solving for the zeroing element yielded a performance level of $\gamma = 1.03$ after 13 iterations of the algorithm, we got the optimization result as

$$A_{RZ11} = \begin{bmatrix} -9.5e-2 & -2.3e-2 & -1.46 & -23.5 \\ 0.15 & 1.18 & -13.5 & -149 \\ 0.21 & 1.45 & -4.76 & -55.3 \\ 8.3e-2 & 9.2e-2 & 0.76 & -32.2 \end{bmatrix}, \quad B_{RZ11} = \begin{bmatrix} 0.69 \\ -0.13 \\ 0.13 \\ -3.5 \end{bmatrix}$$

$$C_{RZ11} = \begin{bmatrix} -0.04 & 0.28 & 8.76 & 160.8 \end{bmatrix}, \quad D_{RZ11} = 0.$$

And, $A_{Rz12} \approx A_{Rz11}, B_{Rz12} \approx B_{Rz11}$,

$$C_{Rz12} = \begin{bmatrix} -8.5e-3 & 0.62 & 7.86 & 161 \end{bmatrix}, D_{Rz12} = 0. \quad \cdots\cdots(29)$$

Consequently, R_{z1} whose inputs and outputs are respectively d (out put of the observer) and v_z is given as:

$$\begin{bmatrix} A_{Rz1} & B_{Rz1} \\ C_{Rz1} & D_{Rz1} \end{bmatrix} = \alpha_{01}(t)\begin{bmatrix} A_{Rz11} & B_{Rz11} \\ C_{Rz11} & D_{Rz11} \end{bmatrix} + \alpha_{02}(t)\begin{bmatrix} A_{Rz12} & B_{Rz12} \\ C_{Rz12} & D_{Rz12} \end{bmatrix} \quad \cdots\cdots(30)$$

Substituting (30) to (12), we can get Zeroing element R_z whose inputs and outputs are respectively d and (v_z, v_{z^*}) as:

$$A_z = \begin{bmatrix} -9.5e-2 & -2.3e-2 & -1.46 & -23.5 & 0 & 0 \\ 0.15 & 1.18 & -13.5 & -149 & 0 & 0 \\ 0.21 & 1.45 & -4.76 & -55.3 & 0 & 0 \\ 8.3e-2 & 9.2e-2 & 0.76 & -32.2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -0.025+0.016\theta_0 & 0.45+0.17\theta_0 & 8.3-0.45\theta_0 & 160+0.12\theta_0 & -6.5-0.5\theta_0 & -0.2 \end{bmatrix}$$

$$B_z = \begin{bmatrix} 0.69 \\ -0.13 \\ 0.13 \\ -3.5 \\ 0 \\ 0 \end{bmatrix},$$

$$C_z = \begin{bmatrix} -0.025+0.016\theta_0 & 0.45+0.17\theta_0 & 8.3-0.45\theta_0 & 160+0.12\theta_0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \end{bmatrix},$$

$$D_z = \begin{bmatrix} 0 \\ -1 \end{bmatrix}. \quad \cdots\cdots(31)$$

3) Robust filter

In this case, after some try and errors, the robust filter was chosen to tune control performance as:

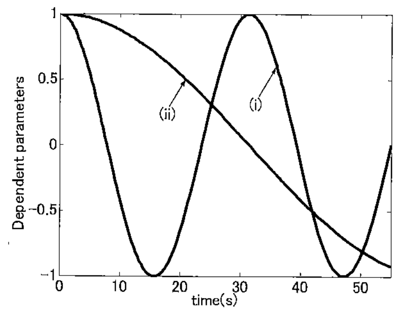
$$\dot{x}_f = \begin{bmatrix} -1.5 & 0 \\ 0 & -1.5 \end{bmatrix}x_f + \begin{bmatrix} 1.5 & 0 \\ 0 & 1.5 \end{bmatrix}\begin{bmatrix} v_z \\ v_z^* \end{bmatrix}, \begin{bmatrix} v \\ v^* \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}x_f \quad \cdots\cdots(32)$$

5.4 Stability Test of Whole System The common $P > 0$ satisfying (17) was not found for the $A_P(\tilde{\theta}(t))$ in this case, but parameter-dependent P_i s satisfying (19) were found. Each P_i was 12×12 positive definite matrix and omit here due to lack of space.

5.5 Simulation Results The proposed method is illustrated by indicial responses. Proposed control systems are compared with nominal control system designed based on reference (14).

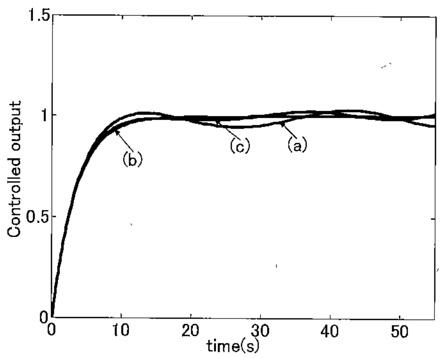
Here, real parameter changes more quickly than the nominal one, which is tight situation for classical gain scheduling (frozen parameter method).

Even if parameter changes quickly and deviates from nominal ones, the proposed method shows near response with desirable ones. From Fig.9, the response of proposed method has almost the same property as nominal system.



(i) Trajectory of real dependent parameter $\theta(t) = 2.5 + \cos(0.2t)$
(ii) Trajectory of nominal dependent parameter

Fig. 8. Parameter trajectories.



(a) Control system without robust compensator; (b) control system with robust compensator, (c) nominal system.

Fig. 9. Indicial responses.

6. Conclusions

In the present paper, a new attachable compensator design method that deals with the following issues has been proposed:
(1) Robust LPV system design,
(2) Robust gain scheduled system design.

The *Robust Model Matching* has been expanded to solve above both problems for polytopic LPV plant.

The problems are reduced to an optimization problem having LMI constraints for the vertex matrix derived from LPV plants. The robust compensator is designed using only information from the nominal plant, so the robust compensator can be attached to any types of existing control system. Methods to test robust stability of the overall system with LPV plants for feasible trajectories also have been shown. The design procedure has been demonstrated in an example design, and the performance of the proposed method has been examined.

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References

(1) L. A.Zadeh and C. A Deser: Linear System Theory. McGraw-Hill (1963)
(2) A. Stubberud: Analysis and Synthesis of Linear Time-variable Systems. University of California press (1964)
(3) H. D'Angelo: Linear Time-Varying Systems: Analysis and Synthesis. Allyn and Bacon (1970)
(4) B. R. Barmish: "Necessary and Sufficient conditions for quadratic stabilizability of an uncertain linear systems", *J. Optimiz. Theory Appl.*, Vol.46, No.4, pp.399-408 (1985)
(5) K. G. Arvanitis and P. P. N. Araskevopoulos: "Uniform exact model matching for a class of linear time-varying analytic systems", *Systems & Control Letters*, Vol.19, pp 313-323 (1992)
(6) S. Boyd, L. E. Ghaoui, E. Feron, and V. Balakrishnan: "Linear Matrix Inequality in Systems and Control Theory", *SIAM Studies in Applied Mathematics*, Vol.15, SIAM, Philadelphia (1994)
(7) M. Chen. "A tracking controller for linear time-varying systems", *ASME J. of Synamic Systems, Measurement, and Control,* Vol.120, pp.111-116 (1998)
(8) A. Feintuch: "Optimal robust disturbance attenuation for linear time-varying systems", *Systems & control Letters*, Vol.46, pp.353-359 (2002)
(9) A. Ichikawa and K Katayama: Linear time varying systems and sampled-data systems. Lecture notes in control and information sciences 265, Springer (2001)
(10) J. S. Shamma and M. Athans: "Analysis of nonlinear gain-scheduled control systems". *IEEE Trans on Autom.Control*, **35**, pp.898- 907 (1990)
(11) J. S. Shamma and M. Athans: "Guaranteed properties of gain scheduled control for linear parameter-varying plants", *Automatica*, **27**, pp.559-564 (1991)
(12) G. Becker, A. Packard, D. Philbrick, and G. Blas: "Control of Parametrically-dependent linear systems. a single quadratic Lyaponov approach", *Proc. American Control Conf., San Francisco,* pp.2795-2799 (1993)
(13) A. Packard: "Gain-scheduling via linear fractional transformation", *Systems & control Letters*, Vol.22, pp.79-92 (1994)
(14) P. Apkarian, P. Gahinet, and G Becker. "Self-scheduled H infinity control of linear parameter varying systems. a design example", *Automatica,* Vol.**31**, No 9, pp.1251-1261 (1995)
(15) P. Apkarian and P. Gahinet: "A convex characterization of gain-scheduled H infinity controllers", *IEEE Trans. Autom.Control*, **40**, 853-864 (1995)
(16) P. Gahinet, A. Nemirovskii, A J. Laub, and M. Chilali: LMI Control Toolbox. Natick, MA: Mathworks (1995)
(17) F. Wang and V. Balakrishnan: "Improved stability analysis and gain-scheduled Controller synthesis for parameter-dependent systems", *IEEE Trans on Autom. control*, Vol.47, No.5, pp.720-734 (2002)
(18) W. J. Rugh and J. S. Shamma: "Research on gain scheduling", *Automatica*, 36, pp.1401-1425 (2000)
(19) D. J. Leith and W. E. Leithead: "Survey of gain-scheduling analysis and design", *Int. J. Control*, Vol 73, No.11, pp.1001-1025 (2000)
(20) F.Wu. "A generalized LPV system analysis and control synthesis framework", *INT.J. Control*, Vol.74, No.7, pp 745-759 (2001)
(21) *International Journal of Robust and Nonlinear Control,* Vol.12, Issue 9, Special Issue on Gain Scheduling (2002)
(22) T. Kimura, E. Tokuda, M. Takahama, and R. Tagawa: "Design of the robust flight control system by realizable linear compensator", *AIAA Guidance and control conference*, pp.334-341 (1985)
(23) R. Tagawa: "Robust Model Matching", reprint from 8th Society of Instrument and Control Engineers Symposium on Dynamical System Theory, 91-96 (1985) (in Japanese)
(24) T. Eisaka, Y. S. Zhong, S. Bai, and R. Tagawa: "Evaluation of robust model-matching for the control of a DC servo motor", *INT J. Control,* Vol.50, No 2, pp.182-187 (1985)
(25) Y. S. Zhong "Robust model matching control system design for MIMO plants with large perturbations", *Reprint from 13th IFAC World Congress*, 1, pp.387-392 (1996)
(26) A. R. Yali and T. Eisaka. "Robust compensator design for exploiting existing control system", *IEE Proc.-Control Theory Appl.*, Vol.147, No.1, pp 71-79 (2000)
(27) Y. S. Zhong: "Robust output tracking control of SISO plants with multiple operating points and with parametric and unstructured uncertainties", *INT. J. CONTROL*, VOL.75, No.4, pp.219-241 (2002)

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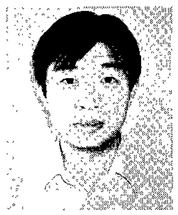

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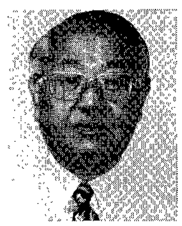

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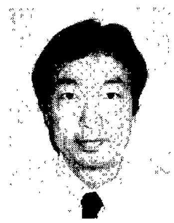

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