Numerical Aspects in the Calculation of the Transient Lightning Electromagnetic Radiation Over Lossy Ground

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The propagation of lightning electromagnetic radiation over a lossy ground has been investigated numerically with the combined use of a newly developed computer algorithm for Sommerfeld integrals and the fast numerical inverse Laplace transform. The transmission line model has been adopted as the lightning current model and we have dealt with lightning discharges with sharp initial peak. The dependences of the radiation fields (especially horizontal electric field) on ground conductivity, distance etc. have been extensively studied, with a special reference to its comparison with previous approximations by Zenneck and Cooray and Rubinstein. Based on these comparisons, we have found that our numerical methods are effective when calculating the radiation field from the lightning for any combinations of the parameters including ground conductivity, distance etc.

Keywords: Lightning discharge, transient radiation, lossy ground

1. Introduction

The direct coupling of external electromagnetic waves like lightning discharges to the transmission line is a well-known fundamental problem in EMC [see some latest papers by Diendorfer (1990), Omid et al. (1997a), Rachidi et al. (1996) and references therein]. In addition to this direct coupling, there exists another type of electromagnetic coupling; that is, the induction by a lightning discharge struck very close to the power transmission line, which is becoming serious in power engineering field. This problem reduces essentially to that of propagation of transient lightning radiation over an imperfectly conducting ground. Furthermore, this problem also attracts attention in the field of lightning research because of the presence of sub-microsecond phenomena of initial peak of the electric field change due to the lightning return stroke (Weidman and Krider, 1980).

The study of the effect of finite ground conductivity on the electromagnetic radiation from a dipole was first published by Sommerfeld (1909). Then Banos (1966) treated the complete problem of the electromagnetic radiation of a dipole by determining the solution of Maxwell's equations for both air and ground media with taking into account the boundary condition. The resulting equations are obtained in frequency domain and are given in terms of slowly converging integrals (Sommerfeld integrals). Different numerical techniques (e.g., Kuo and Mei, 1978; Mosig, 1989; Ichikawa and Karasudani, 1989; Omid et al., 1997b) and analytical approximations (Norton, 1937; Bannister, 1984) for the Sommerfeld integrals have been proposed, but we have to note that such a formulation requires a prohibitive computer time. Zeddam and Degauque (1990) have discussed some sophisticated approximations to the rigorous theory, in particular the Norton's and the Bannister's approaches and have defined their validity limits in terms of frequency and distance of the observation point to the lightning channel. Recently, Cooray (1992) and Cooray and Scuka (1998) have made the extensive study on the comparison of different approximations. However it is highly desirable for us to estimate the numerical integrations for any combinations of arbitrary parameters (frequency, distance, ground conductivity etc.) (Ichikawa and Karasudani, 1989). This paper reports on the exact numerical evaluation of the electromagnetic fields over a lossy ground for a realistic lightning current source (transmission line model) by improving the computational methods and we present the numerical results as a function of different parameters. Then we compare our numerical computational results with previous approximations (Zenneck, and Cooray-Rubinstein (Cooray, 1992; Rubinstein, 1996; Cooray and Scuka, 1998) approximations) and we discuss their validity limit. Finally we recommend you to use the present numerical computations for this problem because we have reduced computer time by means of newly developed computation algorithms described in this paper.

2. Formulation and Numerical Computations of Electromagnetic Fields Radiated from a Lightning Discharge above a Finely Conducting Ground

Fig.1 illustrates the geometry used in the field calculations, in which the lightning channel is a vertical dipole with the top height of H. We use the cylindrical coordinates (r, φ, z), and z=0 is the boundary between the two media; the space of z>0 (medium 1) is free space (dielectric constant ε₁=ε₀, permittivity μ₁=μ₀, propagation constant k₁), and z<0 (medium 2), the finely conducting ground (ε₂=ε₀εᵣ (εᵣ: relative dielectric constant), μ₂=μ₀, conductivity σ₂, propagation constant k₂) The electromagnetic fields detected at the observing point P(height =h) are the summation of the elementary fields due to the small electric dipoles over the whole lightning channel. We here show the expressions of the vector potential induced at the observing point P due to an elementary short dipole located between z and z+dz (Sommerfeld, 1909) as follows;

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Fig. 1. Geometry of the problem (A lightning discharge is of the transmission line model, and an observing point is located at the point P. Medium 1 is free space, while medium 2 is the conducting ground)

\[ d\mathbf{A}_t = d\mathbf{A}_t^s + d\mathbf{A}_t^c \]  \hspace{1cm} (1)

where \( d\mathbf{A}_t^s \) is the vector potential when the ground is a perfect conductor and \( d\mathbf{A}_t^c \) is the correction term due to the presence of the finitely conducting ground. These two terms are expressed as follows,

\[ d\mathbf{A}_t^s = \frac{\mu_0 J(z)dz}{2\pi} \left[ \exp(-jkz/R) - \exp(-jkR'/R') \right] \]  \hspace{1cm} (2)

\[ d\mathbf{A}_t^c = \frac{\mu_0 J(z)dz}{2\pi} \int_0^\infty J_0(\omega r) \exp(-\omega(z+h)/\omega \cdot u_\omega) \frac{\kappa^2 u_\omega}{\kappa^2 u_\omega + \kappa^2 u_\omega} \, du \]  \hspace{1cm} (3)

where \( k^2 = -\epsilon_0 \mu_0 \omega^2 (s: complex frequency) \), \( \kappa^2 = -\epsilon_0 \mu_0 \omega^2 + \sigma_0 \mu_0 \omega \), \( u_\omega^2 = u_\omega^2 + \kappa^2 \), \( u_\omega^2 = u_\omega^2 + \kappa^2 \), \( R^2 = (z-h)^2 + r^2 \), \( R^2 = (z+h)^2 + r^2 \) (see Fig 1) and \( J_0 \) is the first-kind Bessel function with 0th order.

The electromagnetic fields at the point \( P \) can be deduced from the vector and scalar potentials by using the following relationships,

\[ E = -\nabla \phi - sA \]  \hspace{1cm} (4)

\[ B = \nabla \times A \]  \hspace{1cm} (5)

\[ \nabla \times A + \epsilon_0 s \phi = 0 \] (Lorentz's condition)  \hspace{1cm} (6)

Then, we can derive the expressions of electromagnetic fields induced at the point \( P \),

\[ d\mathbf{B} = (d\mathbf{B}_t^s + d\mathbf{B}_t^c) n_\varphi \]  \hspace{1cm} (7)

\[ d\mathbf{E} = (d\mathbf{E}_t^s + d\mathbf{E}_t^c) a_\varphi + (d\mathbf{E}_t^s + d\mathbf{E}_t^c) a_z \]  \hspace{1cm} (8)

where \( a_\varphi \) and \( a_z \) are the unit vectors in \( \varphi \) and \( z \) directions. The superscript \( s \) corresponds to the term for the perfectly conducting ground, and the superscript \( c \) indicates the correction term due to the finite ground conductivity, leading to the dispersive effect of the ground medium. Each term in Eqs. (7) and (8) is given as follows (Sommerfeld, 1909; Ichikawa and Karasudani, 1989):

\[ d\mathbf{B}_t^c = \frac{\mu_0 J(z)dz}{2\pi} \left[ \frac{k^2}{R^2} + \frac{1}{R^2} \exp(-kz/R) + \frac{k^2}{R^2} + \frac{1}{R^2} \exp(-kR) \right] \]  \hspace{1cm} (9)

\[ d\mathbf{B}_t^c = \frac{\mu_0 J(z)dz}{2\pi} \int_0^\infty J_0(\omega r) \exp(-\omega(z+h)/\omega \cdot u_\omega) \frac{\kappa^2 u_\omega}{\kappa^2 u_\omega + \kappa^2 u_\omega} \, du \]  \hspace{1cm} (10)

\[ d\mathbf{E}_t^s = \frac{\mu_0 J(z)dz}{2\pi} \left[ \frac{k^2}{R^2} + \frac{3k^2}{R^2} \right] \exp(-kz/R) \]  \hspace{1cm} (11)

\[ d\mathbf{E}_t^s = \frac{\mu_0 J(z)dz}{2\pi} \int_0^\infty J_0(\omega r) \exp(-\omega(z+h)/\omega \cdot u_\omega) \frac{\kappa^2 u_\omega}{\kappa^2 u_\omega + \kappa^2 u_\omega} \, du \]  \hspace{1cm} (12)

\[ d\mathbf{E}_t^s = \frac{\mu_0 J(z)dz}{2\pi} \left[ \frac{k^2}{R^2} + \frac{3k^2}{R^2} \right] \exp(-kR) \]  \hspace{1cm} (13)

\[ d\mathbf{E}_t^s = \frac{\mu_0 J(z)dz}{2\pi} \int_0^\infty J_0(\omega r) \exp(-\omega(z+h)/\omega \cdot u_\omega) \frac{\kappa^2 u_\omega}{\kappa^2 u_\omega + \kappa^2 u_\omega} \, du \]  \hspace{1cm} (14)

So, the total fields at the observing point \( P \) can be obtained by integrating each term over the whole height range from the ground \((z=0)\) to the cloud height \((H)\).

The above expressions of electromagnetic fields are given in terms of complex frequency, so that we have to take the following procedure to estimate the transient electromagnetic fields; (1) Estimation of induced fields due to a short dipole \( u \) integral : integral of the integrand including Bessel functions (Eqs. (9)∼(14)) over the whole \( u \) range from \( u=0 \) to \( u=\infty \) and \( 2 \) integral (integral contribution over the whole lightning channel), and (3) inverse Laplace transform. Several numerical methods have been proposed for the \( u \) integral as described in the Introduction, but we have adopted the numerical method developed by Ichikawa and Karasudani (1989). It is found that there is a peak at a particular \( u \) value in the integrand in Eqs. (10), (12) and (14). The appearance of such a peak in the integrands is apparent due to the following factor,
It is found that \( |k_0\nu_0| \ll |k_0\nu_1| \), so that the reason for having a peak is entirely due to \( \nu_1 \). By using the definition of \( \nu_1 \), this \( \nu_1 \) will be zero for a particular \( u = x_p \). Next, the convergence due to \( u \) of the integrands in Eqs. (10), (12) and (14) is mainly dependent on the exponential factor \( \exp(-u_1) \). This exponential factor shows different behaviors around \( u=x_p \). The definition of \( u_1 \) indicates that \( u_1 \) is purely imaginary for \( u \ll x_p \), so that the exponential term shows an oscillation without damping. While \( u_1 \) is real when \( u \gg x_p \), so the exponential term indicates only damping to zero without any oscillation. By taking into account these behaviors of the integrands with respect to \( u \), we divide the whole \( u \) range into three characteristic regions. The first and second boundaries in \( u \) are defined as the \( u \) indicating the starting point of the peak (located at \( u=x_p \)) \( (u=x_1) \) and then as the ending point of the peak \( (u=x_2) \). Then, \( u=x_2 \) is defined as the \( u \) value where the integrand value is nearly zero. So the first \( u \) area is from \( u=0 \) to \( u=x_1 \), and the third \( u \) area is from \( u=x_2 \) to \( u=x_3 \). The second \( u \) area including the peak is from \( u=x_1 \) to \( u=x_2 \). The details of determining these \( x_2 \) and \( x_3 \) are based on the estimation of the gradient of the integrand (Ichikawa and Karasudani (1989)). In the following computations, each \( u \) region is divided into 10,000 equal divisions, which means that we have sufficiently small divisions in the second \( u \) area with the peak in the numerical integration. We have confirmed that this way of division in the numerical computations is sufficient to have relative error less than 1%.

Next step is to perform the inverse Laplace transform and we have used a numerical method (fast inversion of Laplace transform (FILT)) developed by Hosono (1981) for our inverse Laplace transform. We assume a very realistic lightning current waveform as follows (Master and Uman, 1984):

\[
I(t) = I_0 \left( e^{-\alpha t-\beta t^2} - e^{-2\beta t^2} \right)
\]

This is called "the transmission line" model which is based on the Bruce and Golde model (double exponential model) and which takes into account the upward propagating group speed of the current (\( \nu \) : velocity). This model is recently well accepted, and we assume the following values for the parameters: \( I_0 = 10^6 \, \text{kA} \), \( \alpha = 3 \times 10^5 \, \text{sec}^{-1} \), \( \beta = 1 \times 10^6 \, \text{sec}^{-1} \), and \( \nu = 1.1 \times 10^8 \, \text{m} \, \text{sec}^{-1} \). The initial waveform of a lightning discharge is strongly dependent on the two parameters (\( \alpha \) and \( \beta \)). Nickolaenko and Hayakawa (2002) have summarized the experimental \( \alpha \) and \( \beta \) values, but the \( \alpha \) and \( \beta \) values assumed in this paper are taken to simulate the sub-microsecond phenomena mentioned in the Introduction (Weidman and Krider, 1980) (the rise time (defined as the time interval between the points of 10\% and 90\% of the peak) being 0.21 \( \mu \)s). While the conventional lightning discharge has the rise time of a few \( \mu \)s to a few tens of \( \mu \)s. This sharp rise of initial peak suggests the presence of much higher frequency components than usual, which are strongly influenced by the ground dispersive effect.

We present the computational results for the horizontal electric field, \( E_h \), for different values of ground conductivity, because \( E_h \) is the quantity very sensitive to finite conductivity of the ground. Fig.2 illustrates an example of the calculated waveforms of horizontal electric field \( E_h \) for a particular combination of parameters (\( \sigma = 10^{-5} \, \text{S/m} \), \( r = 100 \, \text{m} \), and \( h = 6 \, \text{m} \)). The cloud height is always assumed to be \( H = 4000 \, \text{m} \). A dash-dot line indicates the result on the assumption of the perfectly conducting ground, and a dotted line indicates the correction term due to the finiteness of ground conductivity. Finally, the full line indicates the total wave field as the sum of these two terms. Fig.3 illustrates the effect of ground conductivity on the \( E_h \) waveforms, in which the ground conductivity (\( \sigma \)) is widely varied (\( \sigma \) from \( 1 \times 10^{-5} \, \text{S/m} \) to \( 1 \times 10^{-3} \, \text{S/m} \)). \( \sigma = 1 \, \text{S/m} \) corresponds to the sea water, while \( \sigma = 10^{-2} \, \text{S/m} \), the wet ground. This figure suggests clearly that the ground conductivity plays an important role in the waveform distortion, as has already been found before.

Next we want to compare our computational results with some previous approximations. It is known that Zenneck approximation (wave tilt approximation) is valid for the distance more than a few kilometers (\( r > a \) few kilometers) (Rachidi et al., 1997), and is not valid for short distances. Then Cooray (1992) and Cooray and Scuca (1998) and Rubinstein (1996) (The Cooray-Rubinstein expression has been discussed by Wait (1997)) have proposed their approximations for short and intermediate distances (200m \( \leq r \leq 1 \, \text{km} \)), and so this Cooray and Rubinstein approximation is compared with our computations in order to find out the usefulness of our computational method.
and also to find out the validity limit to their approximation. The Zenneck approximation is well known, but we have to briefly describe the Cooray-Rubinstein approximation. Their formula on the horizontal electric field over a lossy ground is given by,

\[ E_x(r, \varphi, h, s) = E_x^c(r, \varphi, h, s) - B_x^c(r, \varphi, 0, s) \frac{c}{\sqrt{\epsilon_r + \sigma h / \sigma}} \]

where the observing point P is located at \((r, \varphi, h)\) as in Fig.1, and the quantities with the superscript \(\infty\) are those for the perfectly conducting ground. So that the Cooray-Rubinstein approximation is again using only the computed \(E_x^\infty\) and \(B_x^\infty\) for the perfectly conducting ground, with taking into account the ground parameters \((\epsilon_r, \sigma)\). Fig.4 is the result of comparison for the case of \(\sigma = 10^2 \text{ S/m}\). Our result is given in a dotted line (while the full line refers to the assumption of perfect conducting ground). The Zenneck approximation is indicated by a cross (+), and the Cooray-Rubinstein approximation is given by a square with a dot inside. In Fig.4 we assume a constant conductivity, \(\sigma = 10^2 \text{ S/m}\) and \(h = 6.0 \text{ m}\), but the propagation distance \((r)\) is widely changed from \(r = 100\) to \(500\)m:\((a)\ r = 100\), \((b)\ r = 200\)m and \((c)\ r = 500\)m.

As is known, the Zenneck approximation is useful for long distances \((r \geq \text{ a few kilometers})\), so that the general waveforms by this approximation are found to deviate a lot from our computations. However, if we increase the propagation distance \((r)\) up to a few kilometers, we will be able to get good agreement with the result by the Zenneck approximation. Our main interest is the comparison with Cooray-Rubinstein approximation.

First of all, looking at the results for three propagation distances \((r=100, 200,\text{ and } 500\)m\) in Figs.4 (a), (b) and (c), it is clear that the agreement between ours and their approximation is rather good. Of course, we notice small discrepancies between the two; especially small differences in the initial part of the waveforms for all propagation distances. However, when the conductivity becomes lower, we can find a significant difference in the waveforms computed by our method and by the Cooray and Rubinstein approximation. Fig.5 is such an example, in which the ground conductivity \((\sigma)\) is decreased \((\sigma = 10^3 \text{ S/m})\), and the distance \((r) = 100\)m. Although the general tendency is the same as ours in dotted line, there are a lot of differences in the initial part of the waveform and also some visible differences in the middle and tail parts of the waveforms.

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**Fig. 4.** Comparison of our computational results with previous approximations. (Our computational results are given by dotted lines (while the full lines indicate the results on the assumption of perfect conducting ground). Then, crosses (+) refer to the approximation by Cooray and Rubinstein, while the Zenneck approximation is given by circles with a dot at the center. The propagation distance \((r)\) is changed; \((a)\ r = 100\)m, \((b)\ r = 200\)m and \((c)\ r = 500\)m and \(\sigma = 10^2 \text{ S/m}\) and \(h = 6.0 \text{ m}\).)

**Fig. 5.** Comparison between our computation and Zenneck and Cooray-Rubinstein approximations for a lower ground conductivity \((\sigma = 10^3 \text{ S/m})\) \((r = 100\)m and \(h = 6.0 \text{ m}\)\)
3. Conclusion

We have proposed a new analysis algorithm for the numerical computations of electromagnetic fields from a lightning discharge over a lossy ground. The essential point of our computations is the useful combined use of the fast convergent integral method for Sommerfeld integrals and a fast inverse numerical Laplace transform. By adopting the conventional "transmission line model" for a lightning current waveform, we have succeeded in estimating the transient electromagnetic fields in an exact way. We think that our numerical method would be useful for the computations of propagation of transient lightning electromagnetic fields for "any" combinations of different parameters (propagation distance, receiver height, ground conductivity). We have treated the lightning current with a sharp rise-time, so that this study would be useful not only for EMC problem, but also for the physics of lightning (especially submicosecond risetime lightning). Comparisons of our numerical computations with previous approximations by Zenneck and Cooray-Rubinstein have been performed, but we have extensively compared with the latter approximation which seems to be effective even for short and intermediate distances (completely ineffective by Zenneck approximation). As the result, the Cooray-Rubinstein approximation seems to be effective for higher conductivity ($\sigma \geq 10^2$ $S/m$) and in the intermediate distance ($100m < r < 1km$). However, when the ground conductivity becomes smaller ($\sigma < 10^2$ $S/m$), we have found significant differences in the waveform between our exact numerical solution and their approximation. We recommend you to use this numerical estimation for this problem.

Acknowledgement

We are grateful to Dr. S. Ichikawa of Kyoto University for his useful suggestions.

(Manuscript received April 30, 2003, revised July 9, 2003)

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