

# A Robust Approach to Controller Design for DC-DC Quasi-Resonant Converters

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DC-DC quasi-resonant converters (QRCs) have a highly nonlinear and time-varying behavior as well as other types of DC-DC resonant converters such as conventional and multi-resonant converters. Changing in operating conditions, mainly due to variation in load and line disturbance, leads to significant changes on system dynamics so that desired performance and even stability are lost. Taking into consideration the uprising need for high-quality resonant power converters has opened a new research window on the control of these systems using modern and systematic approaches.

In this paper in order to achieve both stability and desired performance, such as reduced sensitivity to load variations, desired disturbance rejection, reduced output impedance and attenuated transfer from input to output, we have proposed a methodology based on  $\mu$ -synthesis technique for DC-DC QRCs controller design. The  $\mu$ -analysis is used to verify the robustness of the designed controller. The proposed control strategy is applied to a typical zero-current switching QRC and nonlinear simulation is performed using nonlinear model of converter circuit. The simulation results demonstrate the good reference voltage tracking, line disturbance rejection and show that the designed procedure guarantees the robust stability and robust performance for a wide range of load variation.

**Keywords:** robust control, ZCS, quasi-resonant converter,  $\mu$ -synthesis, robust performance

## 1. Introduction

The resonant converters use a resonant circuit for switching the transistors when they are at the zero current or zero voltage point; this reduces the stress on the switching transistors and the radio interference. Due to their reduced switching losses, the resonant converters are currently the object with widespread interests among power conversion applications. This removes 30 to 40 percent of the losses within a comparable conventional switching converter when operated at the same frequency<sup>(1)</sup>.

Recently, a new family of resonant converter, known as Quasi-Resonant Converters (QRCs), has been proposed and attracted much attention. The concept of QRCs was first introduced by F.C. Lee and his collaborators in the mid 1980's<sup>(2)(3)</sup>. QRCs can be considered as a hybrid of resonant and PWM converters. The underlying principle is to replace a power switch in PWM converters with the resonant switch. PWM-based DC-DC switching converters have a better known structure but use a hard switching method where power devices switch at large values of current or voltage that causes switching losses, radio frequency interference (RFI) and

electromagnetic interference at high frequency.

Hard switching refers to the stressful switching behavior of the power electronic devices. On the other hand, resonant converters use a soft switching method where power devices switch at either zero voltage or zero current. QRCs combine the advantages of conventional PWM converters and resonant converters. In QRCs, the switch current or voltage waveforms are forced to oscillate in a quasi-sinusoidal manner, so that Zero Current Switching (ZCS) or Zero Voltage Switching (ZVS) can be achieved<sup>(4)</sup>.

Practical, commercially available QRCs are emerging in the market place today, but the field of resonant structures and its controller is still very much in a state of change. New demands placed on the designers and component manufacturers at these higher frequencies are resulting in improved modeling and control of the converters.

The analysis and controller design for resonant converters, compared to other conventional topologies is difficult. The difficulties in controller design arise from knowing which topologies can offer an optimum solution to the requirements, knowing what reasonable assumptions can be made during the design process, understanding the high frequency behavior of the elements within the design and lack of a suitable dynamics modeling and control strategies because of complexity of their circuits<sup>(5)(6)</sup>.

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Recently, in order to overcome the analysis, modeling and control problems of QRCs some works have been reported<sup>(7)-(15)</sup>. (7)~(9) have developed the small signal model of QRCs using various averaging techniques. (10) introduced the generalized state space averaging technique for modeling the QRCs. The small signal analysis and control of QRCs is given in (11), (12). The nonlinear control of a ZCS QRC using the Lyapunov stability theory is presented in (13). A passivity-based controller for stabilization of QRCs is used in (14).

But the most of control scenarios are concerned mainly with stability only, without considering simultaneous robust stability and robust performance. Some of past controllers for QRCs were designed to keep the output voltage constant in the presence of input perturbations. However load variation, voltage reference change and practical constraint on control action were not considered in these controllers.

Traditionally, the feedback design of resonant converters is carried out using classical control theory where uncertainties and physical constraint are not taken into consideration at the initial design stage. In the best classic methods, subsequent testing of the preliminary design, for example by experimentally or trial and error approach, may lead to design iterations where robustness to small range of uncertainties may be achieved.

This paper addresses the design of robust controller for DC-DC QRCs operating at continuous conduction mode, based on  $\mu$ -synthesis technique. Using the  $\mu$ -synthesis is already reported to design of controller for other switching converters' topologies<sup>(5),(6),(16),(17)</sup>. This paper shows that the  $\mu$ -synthesis can be used for design of DC-DC QRCs controller that guarantees robust stability and robust performance for a wide range of operating conditions.

Our design objective is to robustly output voltage reference tracking in the presence of load uncertainty, while keeping small control actions and short settling time. In order to illustration of proposed methodology and as an example we will design a controller for a typical DC-DC ZCS QRC. It will be shown that the desired performance and physical limit on control action are easily taken into account by introducing appropriate weighting functions (as fictitious uncertainties) in synthesis procedure. The inclusion of these uncertainties is adequately allowed for maximum flexibility in designing controller. The preliminary results of this work are published in (18)~(20).

This paper is organized as follows. Section 2 describes the proposed strategy. In section 3, the synthesis method is applied to a typical DC-DC ZCS QRC. Section 4 demonstrates the effectiveness of proposed scheme by some simulation results by using the obtained controller to nonlinear model.

## 2. Proposed Strategy

We proceed to design a robust controller using the  $\mu$ -synthesis approach. In this paper, our main focus is concentrated on robust performance e.g. minimize the effects of the line disturbances, desired reference tracking and holding stability in presence of load variation.

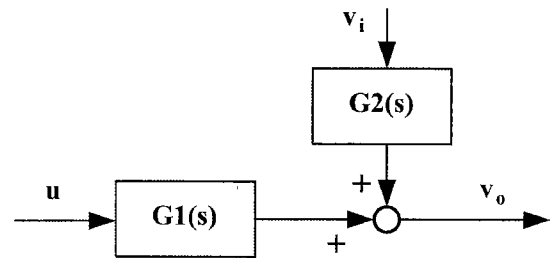


Fig. 1. Simplified block diagram of a resonant converter

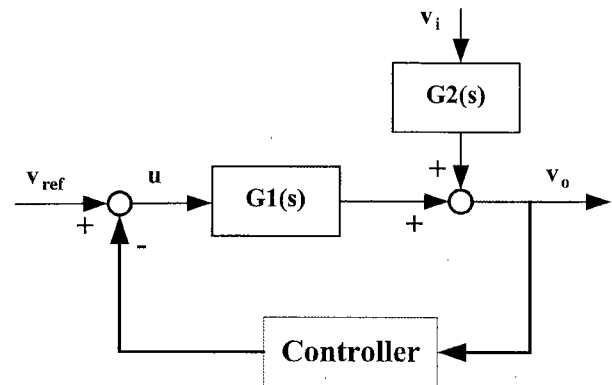


Fig. 2. The diagram for control design

Under steady-state conditions it can be shown that for a QRC operating in the continuous condition mode, depending on possible combination status of circuit's switches (transistor and diode), there are four operation modes in each switching period<sup>(4)</sup>. The generalized averaging technique gives a set of ac small-signal state equations. Using the Laplace transform, we can get the control-to-output  $G_1(s)$  and the input-to-output  $G_2(s)$  transfer functions of QRC, and hence a simple linear open-loop model for a given QRC can be obtained as,<sup>(10)</sup>:

$$v_o = G_1(s)u + G_2(s)v_i \dots\dots\dots (1)$$

Where  $G_1(s)$  and  $G_2(s)$  are the linear nominal transfer functions from control input ( $u$ ) and line voltage ( $v_i$ ) to output voltage ( $v_o$ ) respectively. Fig. 1 shows the block diagram realization of equation (1). Actually, resonant converters have a highly nonlinear and time-varying behavior; however a simplified and linearized model is usually used for linear controller design. The error caused by the simplification and linearization can be considered in robust control strategies (such as  $\mu$ -synthesis).

Voltage regulation in many resonant-type converters, such as QRCs, can be achieved by controlling the switching frequency. The ZCS QRCs require controlled switch-off times (constant on-time). In fact, on-time duration ( $t_{on}$ ) is determined by the parameters of the resonant elements, and hence  $t_{on}$  is constant and uncontrollable, the duty ratio and output voltage are controlled by changing the off-time duration ( $t_{off}$ ). There is a duality relationship between ZCS and ZVS QRCs<sup>(7)</sup>. Therefore the ZVS QRCs require controlled switch-on times (constant off-time). Without less of generality, we can consider the control diagram as shown in Fig. 2.

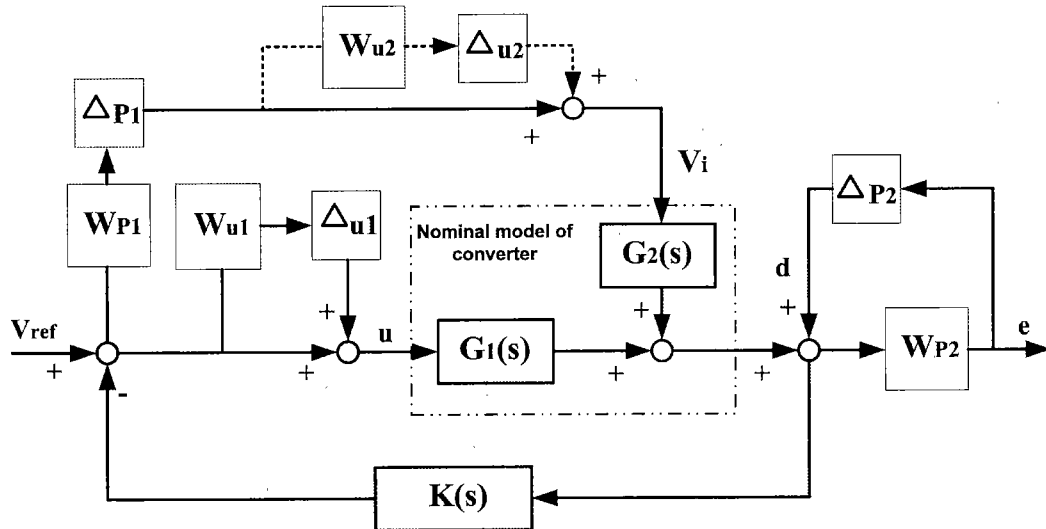


Fig. 3. The block diagram for  $\mu$ -synthesis

To achieve the desired control performance and according to  $\mu$ -synthesis requirements we can redraw the block diagram in Fig. 2 and propose the closed-loop interconnection system as shown in Fig. 3. In fact this figure shows the main framework and synthesis strategy for obtaining desired controller.

It is notable that in the model of resonant converter there are several uncertainties because of parameter variations, model linearization and unmodeled dynamics to due some approximations. However to keep the complexity of the controllers reasonably low, our focus in this paper is concentrated on load uncertainty, which is more narrow but the most important source of uncertainty in QRCs<sup>(18)</sup>. This uncertainty in Fig. 3 is modeled as an unstructured multiplicative uncertainty.

$W_{u1}$  and  $W_{u2}$  represent the fixed weighting functions containing all the information available about the load variation correspond to  $G_1(s)$  and  $G_2(s)$ , respectively.  $\Delta u_1$  and  $\Delta u_2$  are uncertainty blocks associated with  $W_{u1}$  and  $W_{u2}$ .

It can be shown that in full-wave ZCS and ZVS QRCs, the voltage conversion ratio ( $v_o/v_i$ ) is insensitive to load variation, while in half-wave QRCs, this parameter is sensitive to the load variation<sup>(4)</sup>. That is why, the load uncertainty ( $W_{u2}$ ) associated to  $G_2(s)$  is shown by dot-line, and it can be removed from above block diagram in case of full-wave QRCs.

Consider the generalized system interconnection shown in Fig. 4.  $M$  is a stable transfer function matrix (consisting of nominal converter, controller  $K(s)$ ,  $W_{u1}$  and  $W_{u2}$ ) and  $\Delta u$  includes  $\Delta u_1$  and  $\Delta u_2$ . As shown in (21), the necessary and sufficient condition for robust stability of the interconnected system for all perturbations  $\|\Delta u\|_\infty \leq 1$  is that

$$\mu_{\Delta u}(M) < 1 \dots\dots\dots (2)$$

Robust performance can be addressed by recasting the problem in to an augmented robust stability problem by introducing an addition fictitious perturbation block  $\Delta p$ , that for our problem it consists  $\Delta p_1$  and  $\Delta p_2$ . It can be

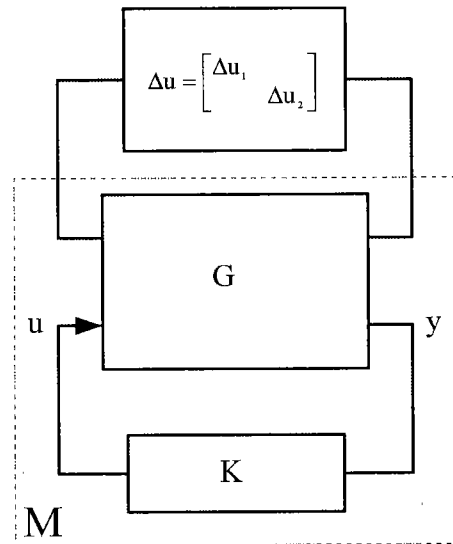


Fig. 4. Robust stability problem

shown (the main loop theorem in (22)) that robust performance is achieved if and only if

$$\sup_{\omega \in R} \mu_{\Delta}(M) < 1 \dots\dots\dots (3)$$

where  $\Delta = \text{diag}\{\Delta u, \Delta p\}$  contains now both the uncertainty and the performance blocks.

According to requirement performance and practical constraint on control action, two fictitious uncertainties  $W_{P1}$  and  $W_{P2}$  are added to resonant model. The  $W_{P1}$  on the control input sets a limit on the allowed control signal to penalize fast change and large overshoot in the control action. This is necessary in order to guarantee implement ability of the resulting controller. The weight  $W_{P2}$  at the output sets the performance goal *e.t.* tracking/regulation on the output voltage signal. Furthermore, it is notable that in order to reject disturbances and to good tracking property,  $W_{P1}$  and  $W_{P2}$  must be such select that singular value of sensitivity transfer function from control input to output voltage be reduced at low frequencies<sup>(6)(23)</sup>.  $\Delta p_1$  and  $\Delta p_2$  are

uncertainty blocks associated with  $W_{P1}$  and  $W_{P2}$  respectively.

The synthesis starts with setting the desired level of stability and performance and chosen uncertainties to achieve robust performance. In order to maintain adequate performance in the face of load variation and disturbances, the appropriate weighting functions must be used. Actually the inclusion of uncertainties is adequately allow for maximum flexibility in designing the converter closed loop characteristics and the demands placed on the controller will increase.

We can redraw the Fig. 3 as a standard  $M-\Delta$  configuration to concluding robust stability (Fig. 4) and robust performance, which is shown in Fig. 5.  $G$  includes the nominal model of converter, associated weighting functions and scaling factors. The block labeled  $M$ , consists of  $G$  and controller  $K$ .

Now, the synthesis problem is designing the robust controller  $K$ . Based on the  $\mu$ -synthesis, the robust stability and performance holds for a given  $M-\Delta$  configuration (Fig. 5), if and only if

$$\inf_K \sup_{\omega \in R} \mu[M(j\omega)] < 1. \dots\dots\dots (4)$$

Using the performance robustness condition and the well-known upper bound for  $\mu$ , the robust synthesis problem reduces to determine

$$\min_K \inf_D \sup_{\omega} \bar{\sigma}(DM(j\omega)D^{-1}),$$

or equivalently

$$\min_{K,D} \|DM(G,K)(j\omega)D^{-1}\|_{\infty},$$

by iteratively solving for  $D$  and  $K$  ( $D$ - $K$  iteration algorithm). Here,  $D$  is any positive definite symmetric matrix with appropriate dimension and  $\bar{\sigma}(\cdot)$  denotes the maximum singular value of a matrix. For deeper insights into the theory, the interested reader is referred to (21), (22), (24).

The controller found by this procedure is typically of

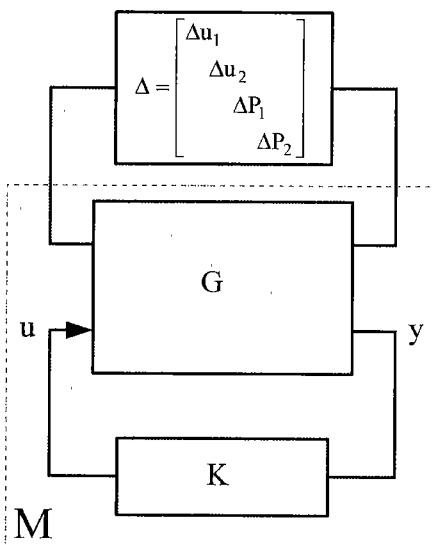


Fig. 5.  $M-\Delta$  configuration

a high order. In order to decrease the complexity of computation, appropriated model reduction techniques might be applied to the obtained controller model. The proposed strategy guarantees the robust performance and robust stability for closed-loop system.

In summary, the proposed method consists of the following steps:

**Step 1:** Calculating the  $G_1(s)$  and  $G_2(s)$  from linear model of given QRC, according to equation (1).

**Step 2:** Identify the uncertainty blocks and associated weighting functions for the given QRC, according to dynamic model, practical limits and performance requirements, as shown in Fig. 3.

**Step 3:** Isolate the uncertainties from nominal model, generate  $\Delta p_1$ ,  $\Delta p_2$ ,  $\Delta u_1$  and  $\Delta u_2$  blocks; and performing  $M-\Delta$  feedback configuration (formulate the robust stability and performance).

**Step 4:** Start the  $D$ - $K$  iteration using  $\mu$ -synthesis toolbox to obtain the optimal controller.

**Step 5:** Reduce the order of result controller by utilizing the standard model reduction techniques and apply  $\mu$ -analysis to closed loop system with reduced controller to check whether or not upper bound of  $\mu$  remains less than one.

It is notable that there isn't any obligate to consider three weighting function, only. Considering the more complete model for the QRC (by including additional performance requirements and other components uncertainties) causes less conservative in synthesis. However the complexity of computations and the order of resulted controller will increase.

### 3. Applied to a ZCS Converter

The controller design approach presented in the previous section is now applied to a DC-DC full-wave buck ZCS QRC as illustrated in Fig. 6(a). Reference (13) has proposed the nonlinear controller for the same circuit. For this converter, it is assumed the resonant frequency is equal to

$$\omega = \sqrt{\frac{1}{L_1 C_1}} = 258200 \text{ (rad/s)} \dots\dots\dots (5)$$

and other parameters are chosen as Table 1, <sup>(13)</sup>.

**3.1 Dynamical Model** For the ZCS-QRC circuit shown in Fig. 6(a), its four operation modes are shown in Fig. 6(b). In this circuit, controlling the duty ratio regulates the output voltage at the load terminal. The switch  $S$  is open and close with the switching fre-

Table 1. Circuit parameters

Parameter	Value	Parameter	Value
$r_i$	$0.025 \Omega$	$V_i$	$30 V$
$r_o$	$5 \Omega$	$L_i$	$5 \mu H$
$r_{Li}$	$0.01 \Omega$	$L$	$0.7 mH$
$r_{Ci}$	$0.04 \Omega$	$L_o$	$4 mH$
$r_L$	$0.02 \Omega$	$C$	$3 mF$
$r_C$	$0.02 \Omega$	$C_i$	$3 \mu F$

quency of  $1/(t_{on} + t_{off})$ , where  $t_{on}$  and  $t_{off}$  are the *on* and *off* durations. When the switch  $S$  is open, the diode  $D$  is forward biased to carry the current  $i_L$ , and the voltage across the capacitor  $C_1$  is zero. When the switch  $S$  is closed, the diode  $D$  remains forward biased while  $i_{L1} < i_L$ , and as  $i_{L1}$  reaches  $i_L$ , the diode turns off. Hence, the switch turns off and on at zero-current. The signal-level switching signal  $u_s$  drives the switch  $S$  (high-frequency transistor). The duty ratio is given as  $d = t_{on}/(t_{on} + t_{off})$  and is bounded to  $d \in [d_{min}, d_{max}]$ .

It is notable that in resonant converters, the frequency is controlled to regulate the output voltage, that is, pulse frequency modulation is used. The  $t_{on}$  duration is uncontrollable; it is determined by the components of resonant switch ( $C_1, L_1$ ). Therefore the duty ratio and output voltage are controlled by changing the  $t_{off}$  duration.

A set of highly coupled differential equations, to model the resonant converter dynamics, is found as

$$\frac{dv_{c1}}{dt} = \frac{1}{C_1}(i_{L1} - i_L)d$$

$$\frac{dv_c}{dt} = \frac{1}{C}(i_L - i_o)$$

$$\frac{di_{L1}}{dt} = \frac{1}{L_1}[-v_{C1} - (r_i + r_{L1} + r_{C1})i_{L1} + r_{C1}i_L + Vi]d$$

$$\frac{di_L}{dt} = \frac{1}{L}[v_{C1} - v_C + r_{C1}i_{L1} - (r_{C1} + r_L + r_C)i_L + r_C i_o]d$$

$$\frac{di_o}{dt} = \frac{1}{L_o}[v_C + r_C i_L - (r_C + r_o)i_o].$$

..... (6)

Due to the multiplication of the state variables  $v_{C1}, v_C, i_{L1}, i_L, i_o$  by the duty ratio  $d$ , nonlinear mathematical model is resulted. The linear state space realization of our example based on nonlinear equation (6) is given by:

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

..... (7)

where,

$$\dot{x} = [\dot{x}_1 \ \dot{x}_2 \ \dot{x}_3 \ \dot{x}_4 \ \dot{x}_5] = [\dot{v}_{C1} \ \dot{v}_C \ \dot{i}_{L1} \ \dot{i}_L \ \dot{i}_o],$$

$$u = d, \text{ and}$$

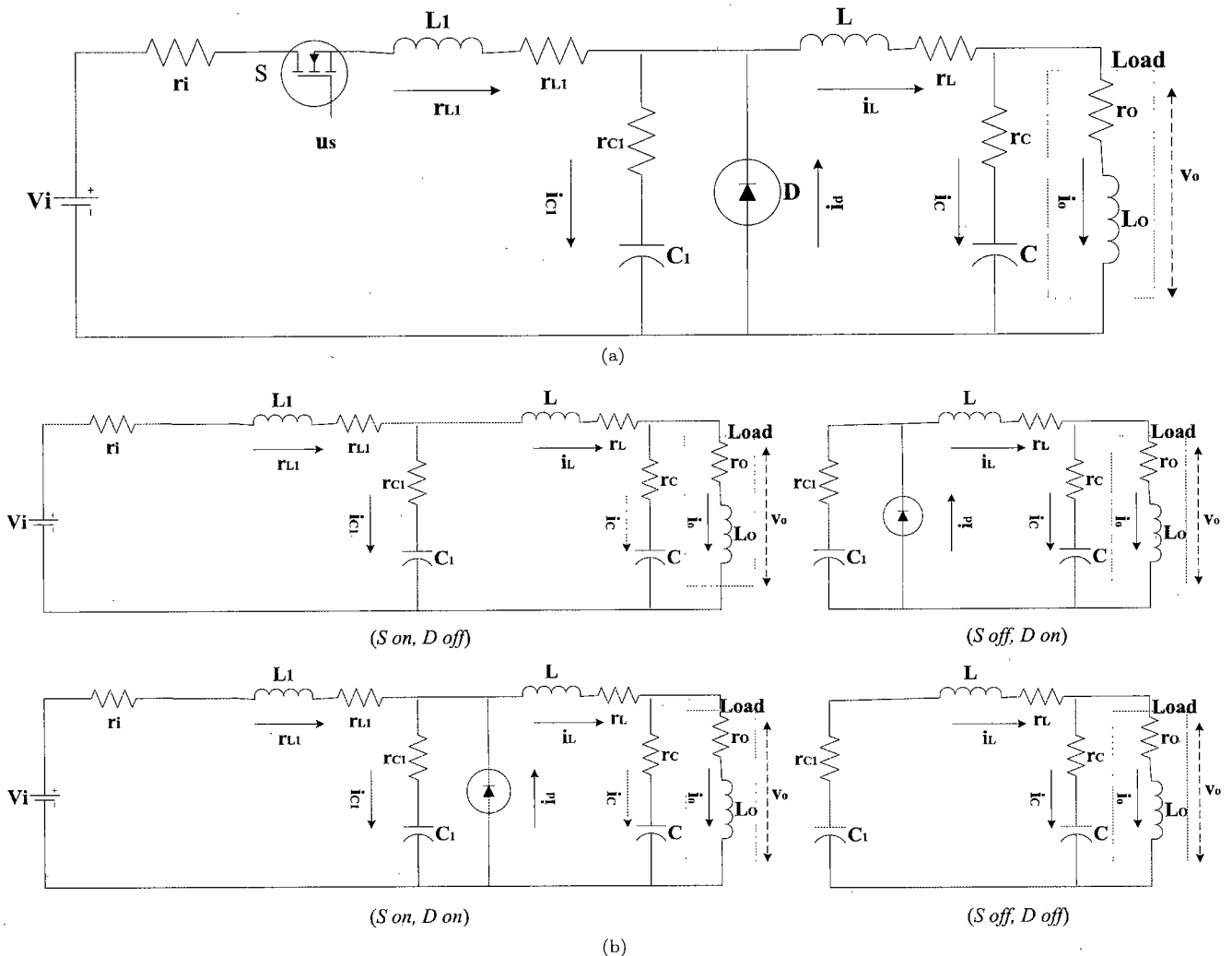


Fig. 6. (a) Quasi-resonant converter with ZCS; (b) Four operation modes of the circuit

$$A = \begin{bmatrix} 0 & 0 & \frac{1}{C_1}u_0 & -\frac{1}{C_1}u_0 & 0 \\ 0 & 0 & 0 & \frac{1}{C} & -\frac{1}{C} \\ \frac{-1}{L_1}u_0 & 0 & -\frac{r_i+r_{L1}+r_{C1}}{L_1}u_0 & -\frac{r_{C1}}{L_1}u_0 & 0 \\ \frac{1}{L}u_0 & -\frac{1}{L}u_0 & \frac{r_{C1}}{L}u_0 & -\frac{r_C+r_L+r_{C1}}{L_1}u_0 & \frac{r_C}{L_1}u_0 \\ 0 & \frac{1}{L_o} & 0 & \frac{r_C}{L_o} & -\frac{r_C+r_o}{L_o} \end{bmatrix}$$

$$B = \begin{bmatrix} \frac{1}{C_1}(x_{30} - x_{40}) \\ 0 \\ \frac{1}{L_1}[-x_{10} - (r_i + r_{L1} + r_{C1})x_{30} + r_{C1}x_{40} + Vi] \\ \frac{1}{L_1}[x_{10} - x_{20} + r_{C1}x_{30} - (r_{C1} + r_L + r_C)x_{40} + r_Cx_{50}] \\ 0 \end{bmatrix}$$

and,

$$C = [0 \ 1 \ 0 \ r_C \ -r_C], D = [0].$$

where  $u_0$  and  $x_{i0}$ ,  $i = 1, 2, \dots, 5$  are nominal values of control action and state variables.

### 3.2 Controller Design

For the given converter, we have set our objectives as follow:

- 1-Holding stability in presence of load variation between  $1 \Omega$  and  $10 \Omega$ ;  $1 \Omega \leq r_o \leq 10 \Omega$ .
- 2-Desired reference tracking for  $10 \text{ V} \leq V_{ref} \leq 25 \text{ V}$ .
- 3-Maintaining settling time less than 5 ms;  $t_s < 5 \text{ ms}$  and acceptable overshoot on output voltage for above cases.
- 4-Set reasonable limit on control action signal;  $u_{\min} \leq u \leq u_{\max}$ .
- 5-Minimizing the effectiveness of line voltage disturbance on output voltage.

Using linearized model (7),  $G_1(s)$  and  $G_2(s)$  in Fig. 2 can be easily calculated.

$$[G_1(s) \ G_2(s)] = \begin{bmatrix} \frac{v_o(s)}{u(s)} & \frac{v_o(s)}{v_i(s)} \end{bmatrix} = C[sI - A]^{-1}B \quad \dots \dots \dots (8)$$

According to Fig. 3, now we must choose necessary uncertainty blocks and associated weighting functions. As it is mentioned in section 2, the load variation is a one of main sources for uncertainties. We can consider this kind of uncertainty in full-wave ZCS QRC as a multiplicative uncertainty ( $W_u = W_{u1}$ ) associated with  $G_1(s)$ . Let  $\hat{G}(s)$  includes  $G_1(s)$  and related multiplicative uncertainty. Then the multiplicative uncertainty can be expressed as

$$\Delta_u = \left| [\hat{G}(s) - G_1(s)]G_1(s)^{-1} \right| \dots \dots \dots (9)$$

Some sample uncertainties for different load are shown in Fig. 7.

In order to reduce the complexity of the controller, we will cover all multiplicative uncertainties due to load changes from  $1 \Omega$  to  $10 \Omega$  with the following first order weight:

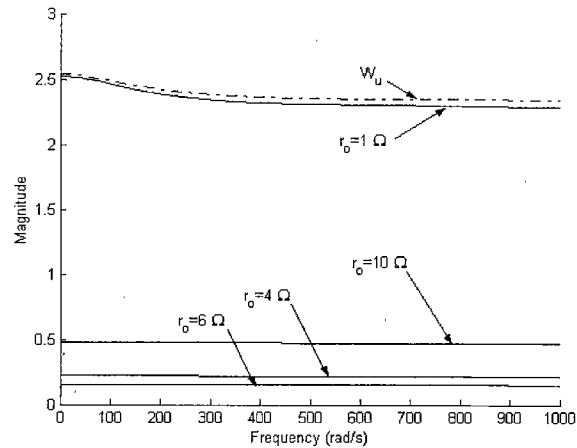


Fig. 7. Multiplicative uncertainty for different loads  $r_o$  and uncertainty weight  $W_u$

$$W_u(s) = \frac{0.07s + 13}{0.03s + 5.12} \dots \dots \dots (10)$$

The robust controller  $K(s)$  must be computed to meet design objectives. An important issue in regard to selection of the weights is the degree to which they can guarantee the satisfaction of design objectives. For the problem at hand a suitable set of performance weighting functions is:

$$W_{P1}(s) = \frac{0.4s + 28000}{s + 0.7} \dots \dots \dots (11)$$

$$W_{P2}(s) = \frac{7 \times 10^{-6}s + 0.07}{s + 0.1} \dots \dots \dots (12)$$

$W_{P1}$  and  $W_{P2}$ , are associated with the control effort and tracking/regulation error, respectively. The selection of  $W_{P1}$  and  $W_{P2}$  entails a trade off among different performance requirements, particularly good regulation versus peak control action. The weight on the control input  $W_{P1}$  was chosen close to a differentiator to penalize fast change and large overshoot in the control input<sup>(25)</sup>. The weight on the error ( $W_{P2}$ ) was chosen close to an integrator at low frequencies in order to get zero steady-state error and good tracking.

Finally, we know that to reject disturbances and to track command signal property, it is required that singular value of sensitivity function be reduced at low frequencies,  $W_{P1}$  and  $W_{P2}$  be such select that this condition satisfied. More details on how these weighting functions are chosen, is given in (18), (23). Fig. 8 shows the magnitude Bode plot of the weighting functions  $W_u$  and inverse of  $W_{P1}$ ,  $W_{P2}$ .

Our next task is to isolate the uncertainties from the nominal plant model and redraw the system in the standard  $M$ - $\Delta$  configuration. We can redraw the Fig. 5 as shown in Fig. 9. As it is mentioned above, the blocks  $\Delta p_1$  and  $\Delta p_2$  are the fictitious uncertainties added to assure robust performance, while the block  $\Delta u$  models the multiplicative uncertainty due to load variation. By using the uncertainty description and performance weights developed in previous section, we get an uncertainty structure  $\Delta$  with a scalar block (corresponding to the uncertainty) and a  $2 \times 2$  block (corresponding

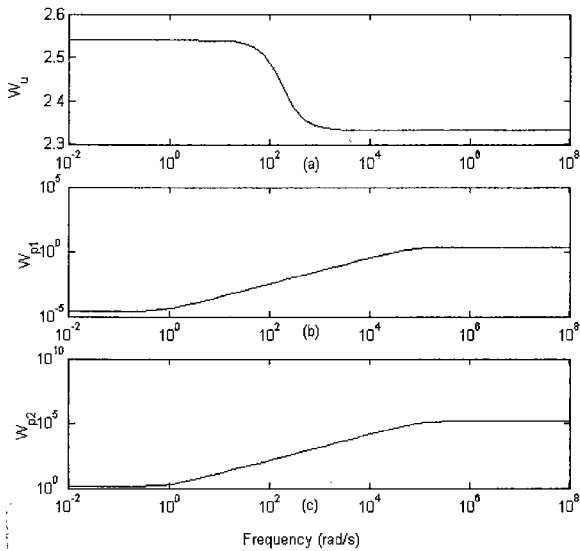


Fig. 8. Magnitude plot of the  
 (a) Multiplicative uncertainty weighting function  $W_u$ ,  
 (b) Inverse of performance weighting function  $W_{P1}$ ,  
 (c) Inverse of performance weighting function  $W_{P2}$ .

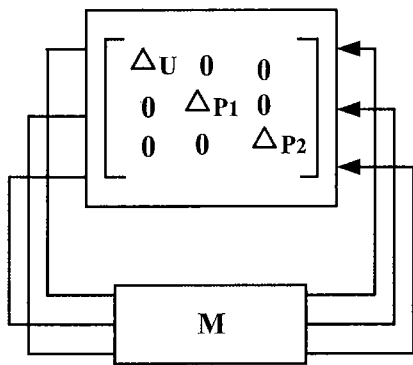


Fig. 9. Standard  $M-\Delta$  block

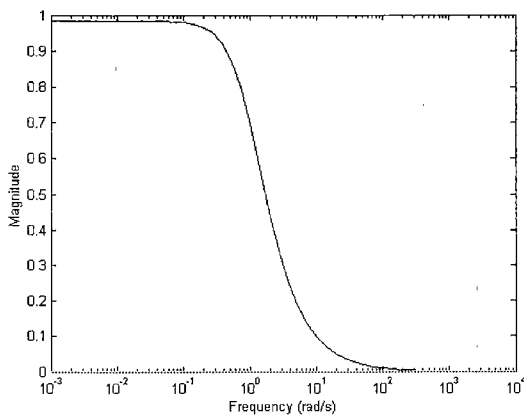


Fig. 10. The  $\mu$  plot of closed-loop system

to the performance). Having setup our robust synthesis problem in terms of the standard  $\mu$ -theory, we used the  $\mu$ -analysis and synthesis toolbox<sup>(26)</sup>, to obtain a solution.

The controller  $K(s)$  is found at the end of the fourth  $D-K$  iteration yielding the value of about 0.9875 on

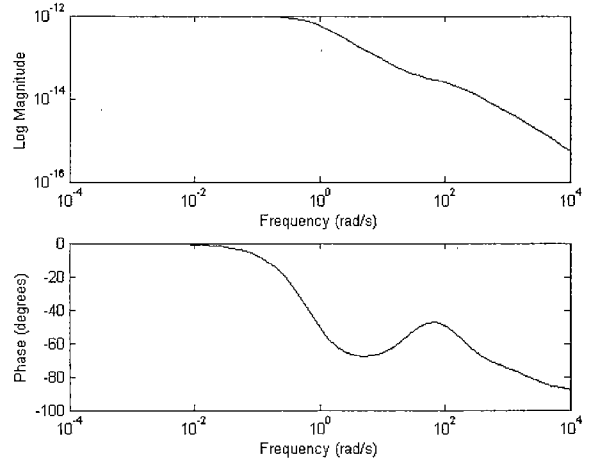


Fig. 11. Bode plots of designed controller

the upper bound on  $\mu$ , thus guaranteeing robust performance (Fig. 10). The order of result controller is 13. The controller is reduced to a 6-th order controller using the Hankel Nom reduction with no performance degradation. The transfer function of the reduced order controller is given as:

$$G_K(s) = \frac{N_K(s)}{D_K(s)} \dots \dots \dots (13)$$

Where,

$$N_K(s) = 6.14 \times 10^{-6} s^5 + 5.04 \times 10^{-3} s^4 + 1.85 \times 10^{-1} s^3 + 1.13 s^2 + 1.77 s + 3.4$$

$$D_K(s) = 10^6 (s^6 + 1301.11 s^5 + 177893.46 s^4 + 1613619.76 s^3 + 3303316.7 s^2 + 6447474.2 s + 3497752.5)$$

The bode plots of result controller are shown in Fig. 11.

#### 4. Simulation Results

Nonlinear simulation is proposed using nonlinear model of given ZCS converter. Figs. 12 to 16 were obtained from simulation to demonstrate the effectiveness of the proposed design. Fig. 12 depicts the waveform for the output voltage. In particular, for the nominal load resistance, we assigned the following reference voltage:

$$V_{ref} = 25 \text{ V}, t \in [0 \ 0.01] \text{ s}$$

$$V_{ref} = 20 \text{ V}, t \in [0.01 \ 0.015] \text{ s}$$

From this figure, it is evident that the tracking error decays to zero; that is, the output voltage converges to the reference one with about 3.75 ms settling time and acceptable overshoot. Fig. 13, shows the corresponded control action signal.

Fig. 14 illustrate the transient behavior for output voltage in nominal value of load, if

$$V_{ref} = 15 \text{ V}, t \in [0 \ 0.009] \text{ s}$$

$$V_{ref} = 25 \text{ V}, t \in [0.009 \ 0.015] \text{ s}$$

These figures show that the proposed controller achieves robust performance in against of reference voltage changing. However a performance variation (an

overshoot) in start-up time is also observed. The proposed method permits us to decrease the overshoot voltage by retuning the performance weights; but it causes a higher order controller. For example, it can be shown that using the following weighting functions, gives about 2% overshoot:

$$W_{p1}(s) = \frac{0.02s + 300}{0.004s + 1}, \quad W_{p2}(s) = \frac{8 \times 10^{-6}s + 1}{0.03s}$$

But in this case the order of resulted controller (after reduction) will be 10 (and settling time from 3.75 ms will increase to 9.6 ms). Hence, the proposed design in section 2 provides a good tradeoff between the specified objectives and complexity of controller.

The analysis of output voltage dynamics in Figs.12 and 14 shows that the achieved settling time (3.75 ms) is significantly reduced in comparison of given results in (13) (7 ms and 5.2 ms corresponding to the proposed nonlinear controllers). Converters need to be operated over a wide load range.

It is known that the behavior of resonant converters is highly dependent upon the load conditions. Fig. 15 illustrate the output waveform if  $V_{ref} = 20$  V,  $r_o = 5 \Omega$  and the load resistance is changed to  $r_o = 1 \Omega$  and  $r_o = 10 \Omega$ , at  $t = 0.007$  s. The dot line corresponds to  $r_o = 1 \Omega$  and dash-dot line correspond to  $r_o = 10 \Omega$ .

The problem of disturbance attenuation was taken into account in the control design. In particular, control laws have been synthesized to guarantee the distur-

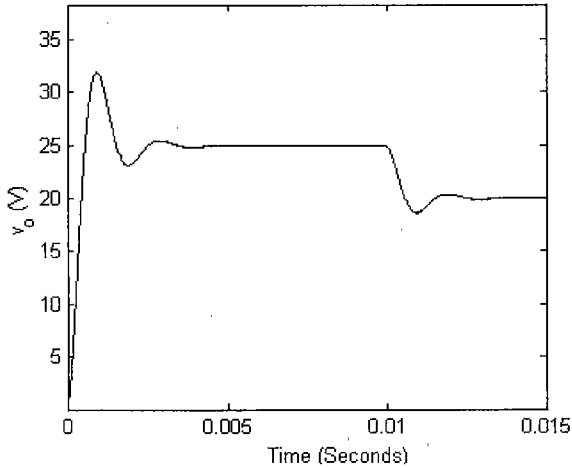


Fig. 12. Output voltage if  $V_{ref} = 25$  V,  $t \in [0 \ 0.01]$  s and  $V_{ref} = 20$  V,  $t \in [0.01 \ 0.015]$  s

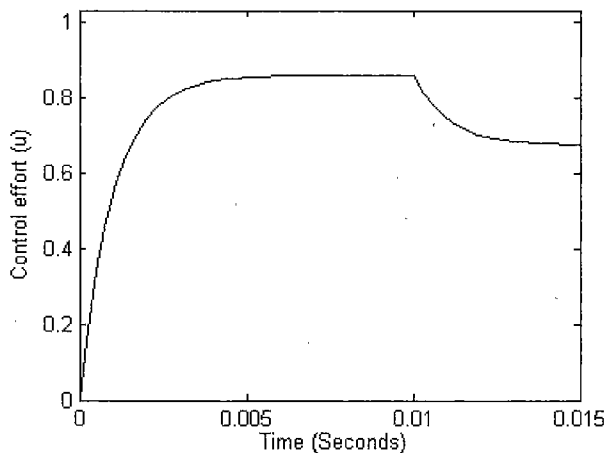


Fig. 13. Control action if  $V_{ref} = 25$  V,  $t \in [0 \ 0.01]$  s and  $V_{ref} = 20$  V,  $t \in [0.01 \ 0.015]$  s

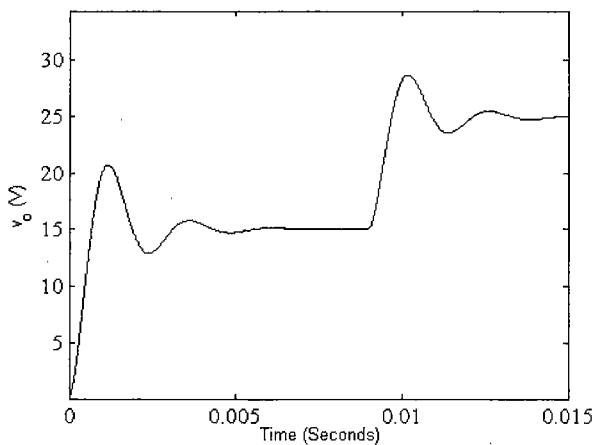


Fig. 14. Output voltage if  $V_{ref} = 15$  V,  $t \in [0 \ 0.009]$  s and  $V_{ref} = 25$  V,  $t \in [0.009 \ 0.015]$  s

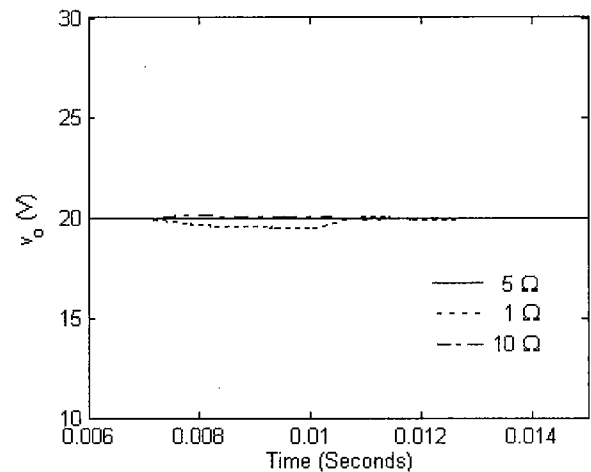


Fig. 15. Output voltage if  $V_{ref} = 20$  V,  $r_o = 5 \Omega$  and the load resistance is changed to  $r_o = 1 \Omega$  and  $r_o = 10 \Omega$ , at  $t = 0.007$  s

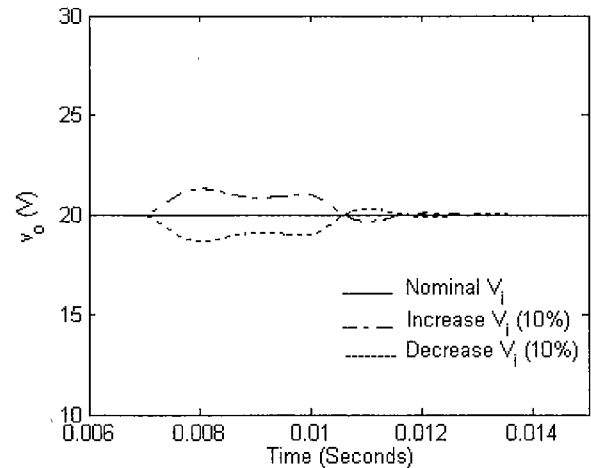


Fig. 16. Output voltage in presence of changing in line voltage at  $t = 0.007$  s



bance attenuation. The output voltage time responses to  $\pm 10\%$  step change in line voltage  $V_i$  are shown in Fig. 16.

$$V_i = 30 \text{ V}, t \in [0 \ 0.007] \text{ s}$$

$$V_i = 30 \pm 3 \text{ V}, t \in [0.007 \ 0.015] \text{ s}$$

The analysis of the results documented in Figs. 15 and 16 allows one to conclude that the steady-state value of the output voltage is 20 V, and the error  $e(t) = V_{ref}(t) - V_o(t)$  tends to zero. Hence, tracking and disturbance attenuation, robustness and stability according to our pre-specified objectives have been proven.

## 5. Conclusions

In this paper, we have addressed and solved the problem of designing of robust controller for DC-DC QRCs. For this purpose a general synthesis frame work based on  $\mu$ -synthesis was proposed. Design strategy includes enough flexibility to setting the desired level of stability and performance, and, considering the practical constraint by introducing appropriate uncertainties.

The proposed method was applied to a DC-DC ZCS converter. Simulation results by using the obtained controller to nonlinear model of converter demonstrated the effectiveness of methodology. It was shown that the designed controller is capable to guarantee the robust stability and robust performance such as precise reference tracking and disturbance attenuation under a wide range of reference voltage and load conditions. In summary because of the main following advantages, the proposed control strategy can be chosen as an appropriate control scenario for QRCs:

- (1) Possibility of components uncertainty consideration,
- (2) Possibility of direct formulation and introduction of performance objectives and physical understanding of QRCs during the design procedure,
- (3) Simultaneous achievement to satisfactory stability and performance under a wide range of load variation,
- (4) Accurate treatment and less conservative design, because of considering uncertainties in the structured form.

The generalization of proposed method to apply for wider range of resonant converters is the next step of this work.

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## References

- (1) R.W. Erickson: Fundamentals of power Electronics, Kluwer Academic Publishers (1999)
- (2) F.C. Lee: "High-frequency quasi-resonant converter techniques", *Proc. IEEE*, Vol.76, No.4, pp.377-390 (1988)
- (3) K.H. Liu, R. Oruganti, and F.C. Lee: "Quasi-resonant converter s-topologies and characteristics", *IEEE Trans. on Power Electronics*, Vol.2, No.1, pp.62-71 (1987)
- (4) M.H. Rashid: Power Electronics Handbook, Academic Press (2001)
- (5) M. Mohan, T. Undeland, and W. Robbins: Power Electronics, Converters, Applications and Design, New York, John Wiley (1995)
- (6) H. Bevrani, M. Abrishamchian, and N. Safari-shad: "Nonlinear and linear robust control of switching power converters", *IEEE Int. Conf. on Control Applications*, USA (1999)
- (7) B.T. Lin and Y.S. Lee: "A unified approach to modeling, synthesizing, and analyzing quasi-resonant converters", *IEEE Trans. on Power Electronics*, Vol.12, No.6, pp.983-992 (1997)
- (8) J.D. Costa and M.M. Silva: "Small-signal model of quasi-resonant converter", *Proc. of IEEE Int. Symposium on Industrial Electronics*, Vol.2, pp.258-262 (1997)
- (9) A. Szabo, M. Kansara, and E.S. Ward: "A unified method for the small-signal modeling of multi-resonant and quasi-resonant converters", *Proc. of IEEE Int. Symposium on Circuits and Systems*, Vol.3, pp.522-525 (1998)
- (10) J. Xu and C.Q. Lee: "A unified averaging technique for the modeling of quasi-resonant converters", *IEEE Trans. on Power Electronics*, Vol.13, No.3, pp.556-563 (1998)
- (11) B. Baha and M.O. Tokhi: "Dynamic modeling and control of resonant switch mode converters", *Control Eng. Practice*, Vol.5, No.11, pp.1533-1542 (1997)
- (12) B. Baha: "Analysis and control of a cross-regulated multi-output forward quasi-resonant converter", *IEE Proc.-Circuits Devices Systems*, Vol.146, No.5, pp.255-262 (1999)
- (13) S.E. Lyshevski: "Resonant converters: nonlinear analysis and control", *IEEE Trans. on Industrial Electronics*, Vol.47, No.4, pp.751-758 (2000)
- (14) X.F. Shi and C.Y. Chan: "A passivity approach to controller design for quasi-resonant converters", *Automatica*, Vol.38, pp.1727-1734 (2002)
- (15) R. Oruganti and F.C. Lee: "Resonant power processing: Part II-Methods of Control", *IEEE Trans. on Ind. Applications*, pp.1461-1471 (1997)
- (16) J. Bu, M. Sznaiier, Z.Q. Wang, and I. Bartarseh: "Robust control design for a parallel resonant converter using  $\mu$ -synthesis", *IEEE Trans. on Power Electronics*, Vol.12, pp.837-853 (1997)
- (17) G.E. Wallis and R. Tymereski: "Generalized approach for  $\mu$ -synthesis of robust switching regulators", *IEEE Trans. on Aerospace and Electronic System*, Vol.36, No.2, pp.422-431 (2000)
- (18) H. Bevrani: "Improvement of stability and performance in DC-DC resonant converters", *Technical research report, ES-FAND1380*, University of Kurdistan, Iran (2002)
- (19) H. Bevrani, Y. Mitani, and K. Tsuji: "Robust control design for resonant converters", *Proc. of IEEE on Industrial Electronics (IECON)*, Spain (2002)
- (20) H. Bevrani, T. Ise, Y. Mitani, and K. Tsuji: "Robust control design for a ZCS converter", *Proc. of IEEE Technical Meeting, SPC-02-96/IEA-02-37*, pp.111-116, Japan (2002)
- (21) J.C. Doyle: "Analysis of Feedback Systems with Structured Uncertainties", *IEE Proceedings*, Vol.129, pp.251-256 (1982)
- (22) A. Packard, J.C. Doyle, and G. Balas: "Linear multivariable robust control with a  $\mu$  perspective", *Trans. ASME*, Vol.115, pp.426-437 (1993)
- (23) P. Lundstrom, S. Skogestad, and Z.Q. Wang: "Performance weight selection for  $H_\infty$  and  $\mu$ -control method", *Trans. on Inst. Meas. Contr.*, Vol.13, No.5, pp.241-252 (1991)
- (24) J.C. Doyle: "Structured uncertainty in control system design", *Proceedings of the 24<sup>th</sup> IEEE Conf. on Decision and Control* (1985)
- (25) M. Abrishamchian, N. Safari-shad, and H. Bevrani: "Nonlinear and robust control of DC-DC switching regulators", *3rd Asia-Pacific Conf. on Control and Measurement (APC CM)*, China (1999)
- (26) G.J. Balas, et al.:  $\mu$ -Analysis and Synthesis Toolbox for use with MATLAB, The MathWorks Inc. (1995)

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