

Adaptation of Condensed Node Spatial Network for Vector and Scalar Potentials to Analysis of Near Field caused by Changing Charges with Discharge Process

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“Electromagnetic Interference (EMI)” has become serious problems as development of the digital technology in downsizing in devices and performing high-speed operation. In the EMI problem such as Electro-Static Discharge (ESD), not only the solenoidal field from currents but also the non-solenoidal field due to charges are included. For this problem, the combined numerical analysis of both fields in the three-dimensional space and on the time domain is inevitable. I have already proposed the condensed node Spatial Network Method for the vector and scalar potential fields based on the gauge condition. In this paper, the fundamental field properties near the changing charges including both charging and discharge processes are presented by the method.

Keywords : vector potential, scalar potential, gauge condition, electro-static discharge

1. Introduction

“Electromagnetic Interference (EMI)” has becomes serious problems as development of the digital technology such as downsizing in devices and performing high-speed operation. In this EMI problems, it is known widely that the Electro-Static Discharge (ESD) phenomena often gives severe influences on many kinds of electronic devices⁽¹⁾⁽²⁾. But, the field properties that bring about serious influences have not been clear yet. This difficulty is due to the mechanism of field generation of this ESD phenomena. The discharge process are critically related to environment parameters and temporary conditions. But, the characteristics of ESD phenomena, especially in the near field, is given by not only the solenoidal vector field from the discharge current and the displacement currents but also the non-solenoidal scalar field due to the changing charges. So far, the analyses and experiments of ESD phenomena have been almost concerned with the solenoidal field generated by the discharge current. For this complicated problem including both fields, it has been known that the combined numerical analysis by using the electromagnetic potential variables, that are, the vector and scalar potential ones are very effective in the three-dimensional space and on the time domain.

I have already proposed the condensed node “Spatial Network Method (SNM)” for the vector and scalar potential fields⁽³⁾⁻⁽⁶⁾. The network for the scalar potential is derived by using the Lorenz gauge condition and is related to the network for the vector potential by the equivalent current sources at every nodes. Therefore, the total electromagnetic fields including both the solenoidal and non-solenoidal components are simulated in the vector potential network. As a results, the time variation of the total fields due to the charging and discharge processes can be simulated in the vector potential spatial network in the three-dimensional space.

In this paper, it is shown that the basic field properties near the

changing space charge pair including discharge process are presented by the method.

2. Analysis Method

2.1 Spatial Network for the Vector Potential Field In this method, to utilize the conventional iterative computing procedure as same as that in the SNM method using both the electric field and magnetic field variables in the three-dimensional time-dependent analysis, the characteristic equations can be defined as follows by using the magnetic vector potential “**A**” and the electric vector potential “**S**”⁽³⁾. Here, “**S**” is supposed to have the opposite sign to “**A**” defined by Stratton⁽⁷⁾.

$$\nabla \times \mathbf{A} = \sigma \mathbf{S} + \mu_0 \frac{\partial \mathbf{S}}{\partial t} \quad (\equiv \mathbf{B}) \dots\dots\dots (1a)$$

$$\nabla \times \mathbf{S} = -\sigma^* \mathbf{A} + \epsilon_0 \frac{\partial \mathbf{A}}{\partial t} \quad (\equiv \mathbf{D}) \dots\dots\dots (1b)$$

here “ σ ” and “ σ^* ” is the conductivities for the electric current and the hypothetical magnetic current, respectively. In each node in the spatial network shown in Fig.1, each component of both vector potential variables is arranged to satisfy the above equations. The nodes are classified into the two types, that is, the electric nodes and the magnetic nodes shown as black nodes and white ones in Fig. 1, respectively. In the former, the magnetic vector potential corresponds to the equivalent voltage variable and the electric vector potentials correspond to the equivalent current variables. On the other hand, in the magnetic node, the opposite correspondences are defined. In this network, the vector potential wave field satisfying the following wave equation, for example, of the magnetic vector potential “**A**” is simulated.

$$\begin{aligned} \nabla^2 \mathbf{A} - \epsilon_0 \mu_0 \frac{\partial^2 \mathbf{A}}{\partial t^2} - (\mu_0 \sigma + \epsilon_0 \sigma^*) \frac{\partial \mathbf{A}}{\partial t} - \sigma \sigma^* \mathbf{A} \\ = \nabla \nabla \cdot \mathbf{A} \quad (\equiv \mu_0 \mathbf{J}_s) \dots\dots\dots (2) \end{aligned}$$

The wave field for “**S**” is also simulated simultaneously.

2.2 Spatial Network for the Scalar Field In the above

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equation, the right hand side term is conventionally neglected because of supposition of the Coulomb's gauge condition ($\nabla \cdot \mathbf{A}=0$) to treat only the solenoidal field due to the external current source \mathbf{J}_0 . But, in the case in which the scalar potential field must be considered, this term has to be introduced by using the Lorenz gauge condition. To simulate the scalar potential field by the same iterative procedure as similar as that for the vector potential field on the time-domain, I have proposed the following characteristic equations⁽⁴⁾⁽⁵⁾.

$$-\nabla F = \varepsilon_0 \frac{\partial \mathbf{E}_s}{\partial t} + \sigma \mathbf{E}_s \quad (3a)$$

$$-\nabla \mathbf{E}_s = \mu_0 \frac{\partial F}{\partial t} - \frac{\rho}{\varepsilon_0} \quad (3b)$$

here the "F" function is defined as follows by using the Lorenz gauge condition.

$$\nabla \cdot \mathbf{A} = -\varepsilon_0 \mu_0 \frac{\partial \phi}{\partial t} - \mu_0 \sigma \phi \quad (\equiv -\mu_0 F) \quad (4a)$$

$$\therefore F = \varepsilon_0 \frac{\partial \phi}{\partial t} + \sigma \phi \quad (4b)$$

On the other hand, " $\mathbf{E}_s (\equiv -\nabla \phi)$ " is the conventional electric field for the scalar potential field, that is, the non-solenoidal field. For the above characteristic equation, the equivalent circuit of the node in the spatial network for the scalar potential field is given as shown in Fig. 2, in which, the "F" function corresponds to the equivalent voltage variable and the electric field " \mathbf{E}_s " corresponds to the equivalent current variables as defined to coincide the each component of the field with the current of the same direction. Also, from the second term in the right hand side of Equ. (3b), the charge density divided by the permittivity corresponds to the equivalent current source at each node. Therefore, in each node, the current continuity law is realized between the currents of every directions and the current source. In this spatial network for the scalar potential, the next wave equation according to the voltage variables, that is, "F" function is given.

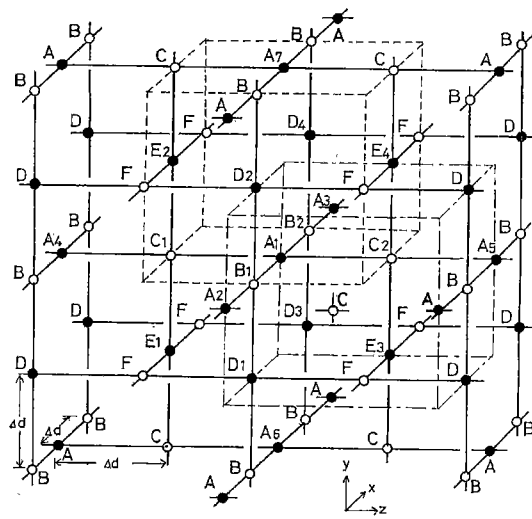


Fig. 1. Spatial network for the vector potential

$$\nabla^2 F - \varepsilon_0 \mu_0 \frac{\partial^2 F}{\partial t^2} - \mu_0 \sigma \frac{\partial F}{\partial t} = \nabla \cdot \mathbf{J} \quad (5)$$

In the above equation, the right hand side is related to the following current and charge continuity equation.

$$\nabla \cdot \mathbf{J} \equiv -\frac{\partial \rho}{\partial t} - \frac{\sigma \rho}{\varepsilon_0} \quad (6)$$

Therefore, this spatial network for the scalar potential field can support the field excited by the time-dependent variation of the spatial charge density. Also, the wave equation for the equivalent current, that is, the electric field is simultaneously simulated in this network

2.3 Connection of the Scalar Field to the Vector Field

In this analysis, to clarify the correspondence between each node in the both spatial networks for the vector and scalar potential fields, the condensed node expression shown in Fig. 3 is used as the spatial network for the vector potential field, in which every field components are included in each node.⁽⁸⁾⁻⁽¹¹⁾ As the cube shown by dashed lines in Fig.3 corresponds to that in Fig. 2, the gradient divergence term in right hand side Equ. (2) of the vector potential can be defined in this cube and can be given as the equivalent current term as follows by using Equ. (3a) with the variables in the node of the corresponding cube for the spatial network for the scalar potential field.

$$\begin{aligned} \nabla \nabla \cdot \mathbf{A} &= -\mu_0 \nabla F = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}_s}{\partial t} + \mu_0 \sigma \mathbf{E}_s \\ &= \mu_0 \mathbf{J}_s \end{aligned} \quad (7)$$

By connecting this current source to every nodes in the spatial network for the vector potential, the scalar field, that is, the non-solenoidal field can be also simulated simultaneously in the spatial network for the vector potential field, which originally supports only the solenoidal field. In this case, the wave equation, for example, of the magnetic vector potential is given as follows to be excited by both the conventional external current source \mathbf{J}_0 which generates the solenoidal (rotational) field and the

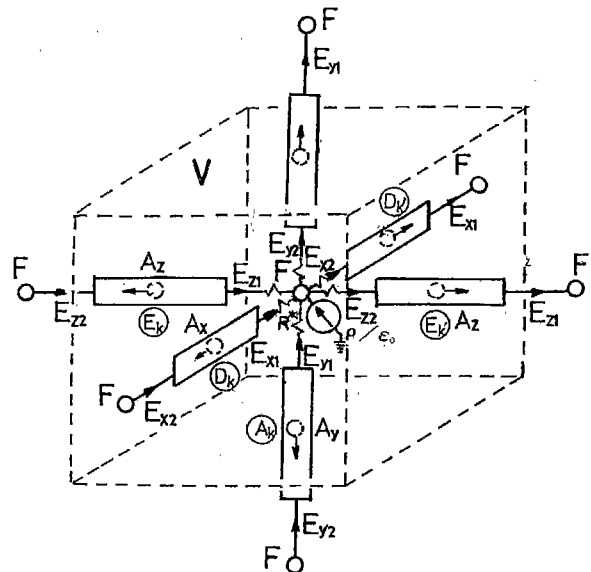


Fig. 2. Equivalent circuit of a node in the spatial network for the electric scalar potential field

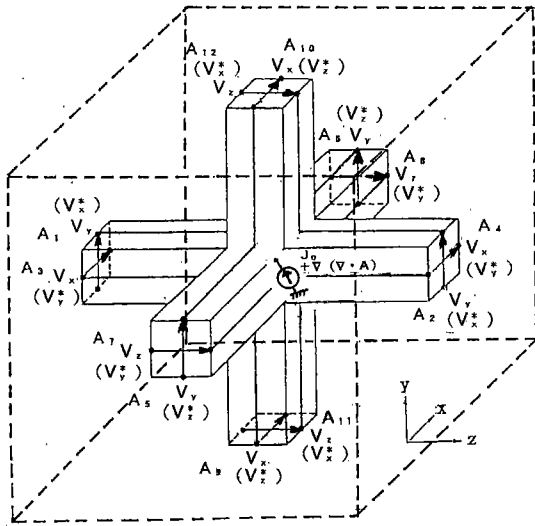


Fig. 3. Conceptual condensed node expression

above-mentioned equivalent current source \mathbf{J}_s which generates both the solenoidal and non-solenoidal (ir-rotational) field.

$$\nabla^2 \mathbf{A} - \epsilon_0 \mu_0 \frac{\partial^2 \mathbf{A}}{\partial t^2} - (\mu_0 \sigma + \epsilon_0 \sigma^*) \frac{\partial \mathbf{A}}{\partial t} - \sigma \sigma^* \mathbf{A} = \mu_0 \mathbf{J}_0 + \mu_0 \mathbf{J}_s \quad (8)$$

This formulation has many advantages in simulating the ESD phenomena including simultaneously both the retarded scalar potential field due to the induced changing charges and the solenoidal field due to the discharge current and the electric dipole current.

2.4 Formulation of the Discharge Process In the discharge process, as the conductivity of the discharge region changes, the wave equation becomes nonlinear and difficult to be solved analytically. But, the iterative computation on the time domain is effective in treating such the nonlinear process by supposing the quasi-static condition. At the time interval for each time step, the medium and source conditions are assumed to be constant and these conditions are renewed by the resultant field values to be used at the next time step. In this analysis in the discharge process, the conductivity is changed by the Rompe-Weizel model⁽¹⁾⁽²⁾ given as

$$\frac{\partial \sigma(t)}{\partial t} = \frac{\alpha}{p} \sigma(t) E_u^2 \quad (9)$$

here, α and p are the fire constant and the air pressure, respectively. The following difference form is used in the iterative computation on the time domain.

$$\sigma^{k+1} = \frac{2 + (\alpha/p) E_u^{k2}}{2 - (\alpha/p) E_u^{k2}} \sigma^k \quad (10)$$

here, "u" gives the direction of discharge and "k" is the time step number as "t/Δt". This conductivity is connected as a conductance at the center node assumed as the discharge region.

3. Analyzed Results

In the following every analyses⁽¹²⁾⁻⁽¹⁴⁾, the analyzed region of the spatial networks for both vector potential and scalar potential is the cube having each sides of $200\Delta d$ (Δd is the spatial discretization) as shown in Fig. 4. The all boundary planes of the

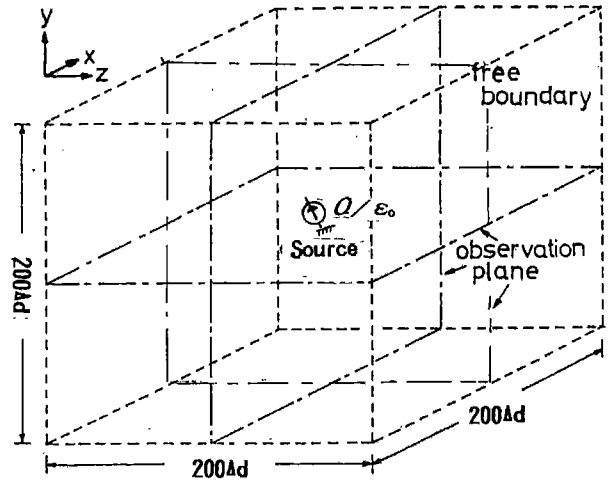
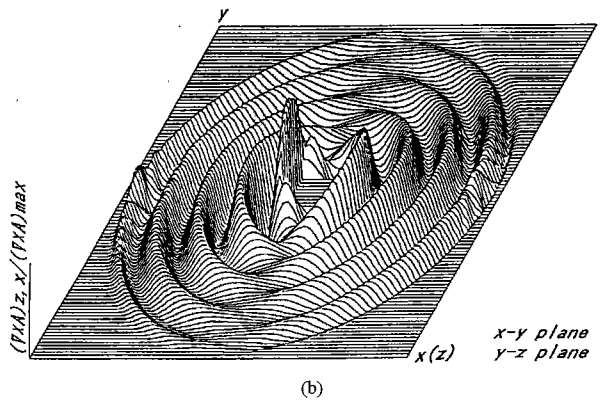
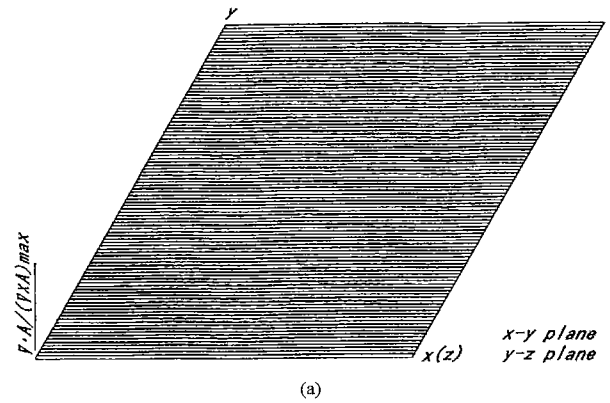


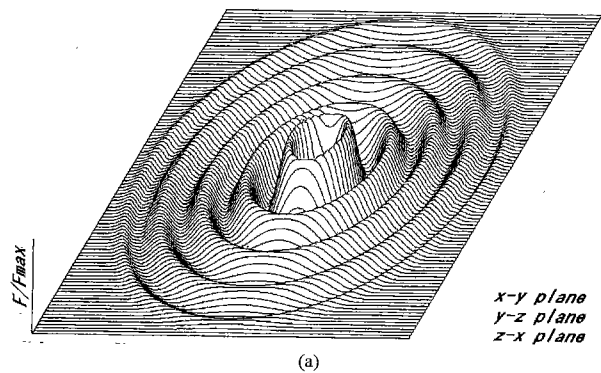
Fig. 4. Analyzed region and an example of source



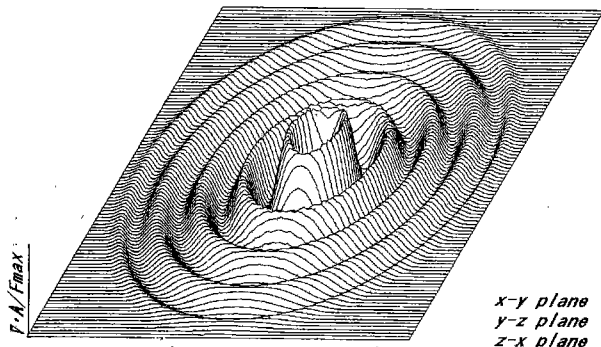
(a) "∇·A" component, (b) "∇×A" component

Fig. 5. Instantaneous vector potential distribution for conventional monopole current source with continuous solenoidal wave form; $\mathbf{J}_0(t) = \sin(2\pi n \Delta t / 40 \Delta t)$

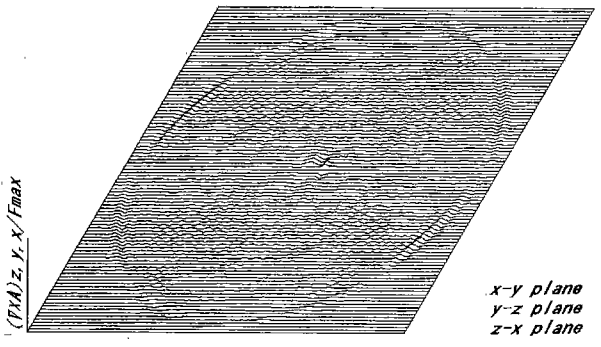
region are terminated by the free space impedance to approximate the free boundary condition. The parameters such as ϵ_0 and μ_0 are normalized as to be unity for the efficient computation. To clear the difference of the spatial distribution between the both non-solenoidal and solenoidal fields, the observation planes are supposed as the perpendicular y - $x(z)$ plane for the components such as "F", "∇·A" and "∇×A" except the case for a point change in Fig.6. Also, these every planes contain the center point in which the source such as a point charge, a current source, or a charge-pair is arranged, respectively. At first, the fundamental difference between the fields caused by the



(a)



(b)

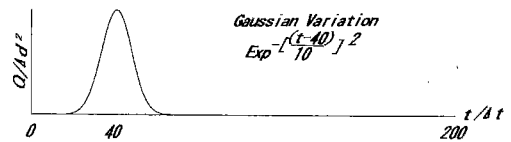


(c)

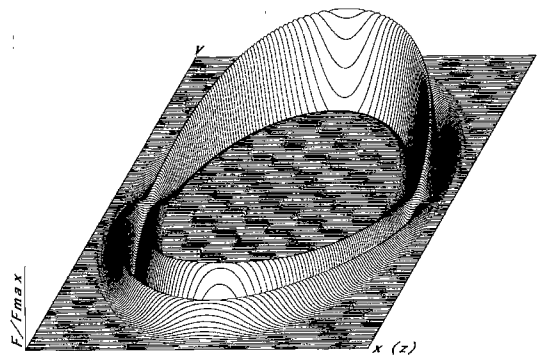
- (a) "F" field in scalar potential spatial network
 (b) " $\nabla \cdot \mathbf{A}$ " component, (c) " $\nabla \times \mathbf{A}$ " component

Fig. 6. Instantaneous scalar and vector potential distribution for a changing point charge with continuous solenoidal wave form; $Q(t) = \sin(2\pi n \Delta t / 40 \Delta t)$

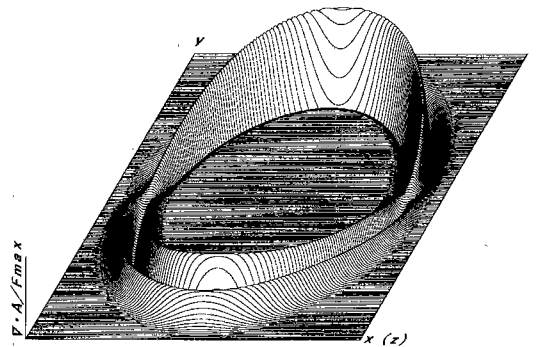
conventional current source and a point charge is considered. Fig. 5 presents the instantaneous field distributions near the sinusoidally changing current source of y-direction located at the center node in the spatial network for the vector potential. In the figure, (a) shows the non-existence of the " $\nabla \cdot \mathbf{A}$ " component and (b) shows the solenoidal field " $\nabla \times \mathbf{A}$ ". These field conditions are the conventionally used for the usual current excitation in the analyses of electromagnetic wave guides and devices. Fig. 6 presents the "F" in (a), " $\nabla \cdot \mathbf{A}$ " in (b), and " $\nabla \times \mathbf{A}$ " in (c) due to the changing point charge located at the center node in the spatial network for the scalar potential field. The field magnitude in each figure is normalized by the maximum value in "F". The figures (a) and (b) give the same distribution because of the formulation based on the Lorenz gauge condition and validate the realization of the non-solenoidal field corresponding to the retarded scalar potential field in the spatial network for the vector potential. The figure (c) shows the



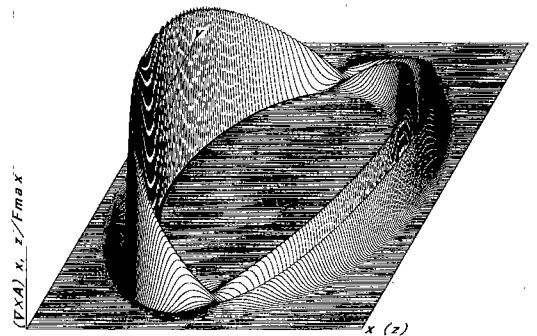
(a)



(b)



(c)

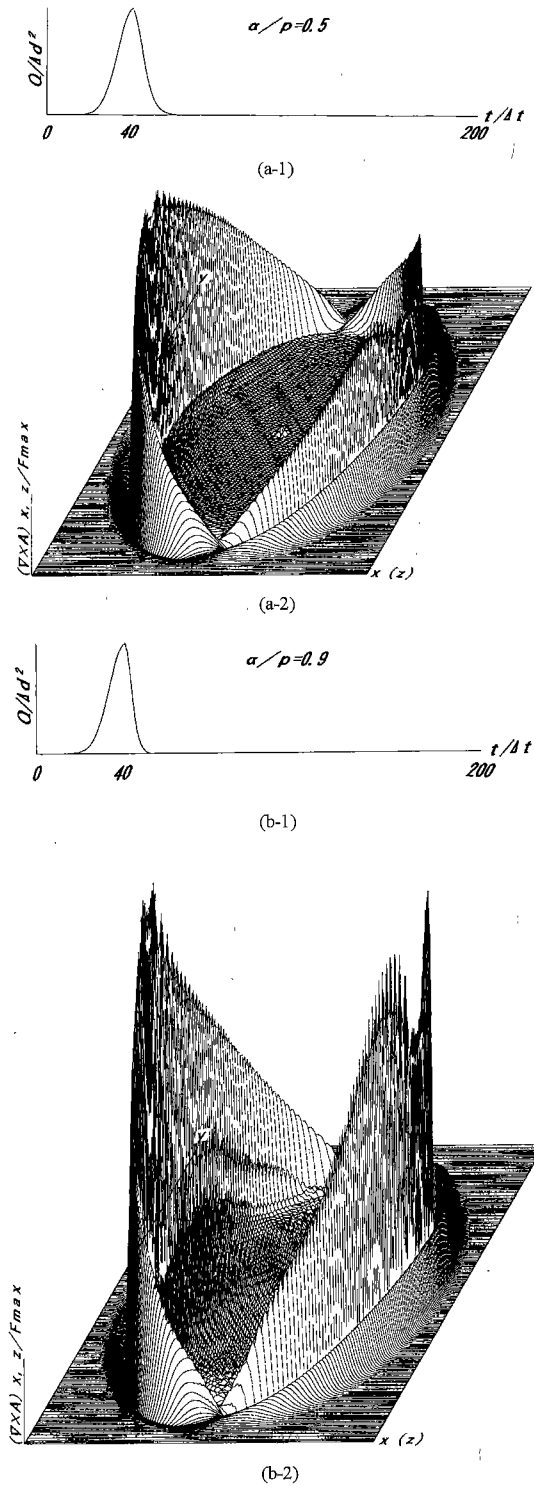


(d)

- (a) Gaussian form of dipole charge [C/m²]
 (b) "F" field in scalar potential spatial network
 (c) " $\nabla \cdot \mathbf{A}$ " component, (d) " $\nabla \times \mathbf{A}$ " component

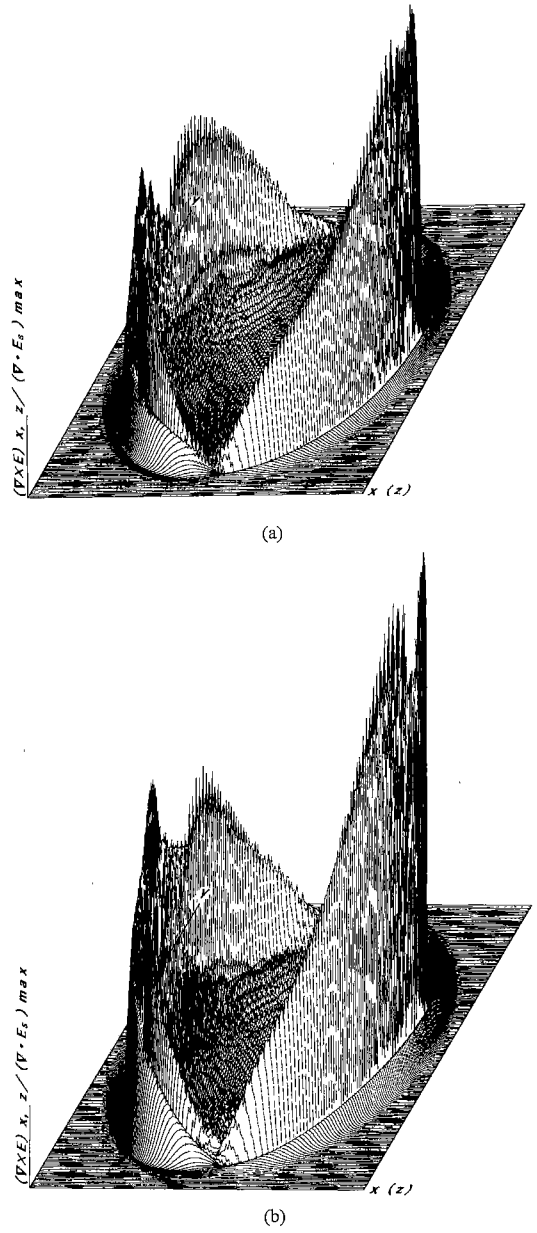
Fig. 7. Instantaneous scalar and vector potential distributions for changing dipole with gaussian wave form; $Q(t) = \pm \text{Exp}(-((t-40\Delta t)/10\Delta t)^2)$

non-existence of the solenoidal component for this source condition. Next, the near field around the dipole is analyzed,⁽¹²⁾⁻⁽¹⁴⁾ which is given as a pair of positive and negative changing charges of y-direction with $2\Delta d$ spatial interval putting in the center node, as the simplified model of the electrostatic induction phenomena. The time-dependent changing of the charge quantities is assumed as the Gaussian form. In this case, not only the non-solenoidal field to the space charge but also the solenoidal field due to the



(a-1) $\alpha/p=0.5$ ($t \geq 40\Delta t$), (a-2) “ $\nabla \times \mathbf{A}$ ” component
 (b-1) $\alpha/p=0.9$ ($t \geq 40\Delta t$), (b-2) “ $\nabla \times \mathbf{A}$ ” component
 Fig. 8. Total variation of “ $\nabla \times \mathbf{A}$ ” of vector potential from charging process with gaussian waveform; $Q(t) = \pm \text{Exp}(-((t-40\Delta t)/10\Delta t)^2) : t \leq 40\Delta t$ to Discharge Process (Equ.10: $t \geq 40\Delta t$)

dipole current due to changing charge pair. Fig. 7 shows the instantaneous field distribution for this case. In the figure, (a) gives the changing waveform of the charges expressed by the surface density and the absolute value. The figures (b) and (c) present the same instantaneous non-solenoidal field distribution



(a) Only considering the discharge current
 (b) Considering both discharge current and displacement current
 Fig. 9. Variation of solenoidal electric field for $\alpha/p=0.5$

corresponding to the retarded potential wave propagation, and figure (d) shows the solenoidal component due to the displacement current. Then, it is confirmed that the spatial network for the vector potential can support both non-solenoidal and solenoidal fields. This field condition is essential in the stage of actual charging process by friction, collision, injection, etc. in the ESD phenomena. Fig. 8 presents the total solenoidal field variation including both charging process and discharge processes on the time domain. In this analysis, it is assumed that the discharge process starts at the $t=40\Delta t$ at which the storage charges of Gaussian form has maximum value. Figures (a-1) and (b-1) show the charge variations for the discharge parameters in Equ. (9); $\alpha/p=0.5$ and $\alpha/p=0.9$, respectively. In the figures (a-2) and (b-2), the outer changing part of each field corresponds to that excited by the displacement current before discharge process occurs. On the other hand, the inner part of each variation

corresponds to the field excited by both the discharge and displacement currents. So, the former part does not be related to the discharge parameter and is almost same as that in Fig.7(c). The each latter part appears to be severely affected by the discharge parameters, and the magnitude in the figure (b-2) is larger naturally than that in figure (a-2). Lastly, to show the advantage of supporting the both non-solenoidal and solenoidal fields, the difference in the usually observed electric field is presented in Fig. 9. The electric field can be easily calculated as the time derivative of the obtained magnetic vector potential. In the figure, (a) is the field simulated by only the discharge current, and (b) is the field simulated by this method. These results clear that the not only the discharge current but also the displacement current have the large influences on the near field around the discharge region. In these analyses, the time discretization condition is almost sufficient as each waveform of the source is approximated by over the 10 divisions.

4. Conclusion

It is performed that the spatial network for the vector potential can support the non-solenoidal field by using the equivalent current source based on the gauge condition, and this property has many advantages in the analyses of complicated EMC problems including both current and charge sources. In the future research, the analysis by using the actual parameter and environment conditions including such metal electrodes will be studied.

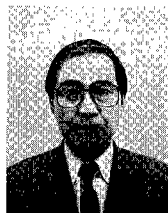
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