Paper

Design of Decentralized Iterative Learning Controllers for Linear Large Scale Dynamical Systems

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The problem of decentralized iterative learning control is considered for a class of linear time–invariant large scale interconnected dynamical systems. In the paper, it is shown that the method of iterative learning control can be applied to such large scale interconnected dynamical systems, and a class of decentralized local iterative learning control schemes is proposed. It is aslo shown that under given conditions, the proposed decentralized local iterative learning controllers can guarantee the asymptotic convergence of the local output error between the given desired local output and the actual local output of each subsystem through the iterative learning process. Finally, a numerical example is given to demonstrate the validity of the results.

Keywords: Iterative learning control, large scale interconnected systems, decentralized control.

1. Introduction

It seems that the main advantage of the iterative learning control strategy is to require less a priori knowledge about the system dynamics and less computational effort than many other types of control strategies. Therefore, since the concept of iterative learning control is introduced in Ref. (1) for repetitive dynamical systems, the problem of iterative learning control has received considerable attention, and many results have been obtained (see, e.g., Refs. (2)∼(7), and the references therein). In particular, there are some works in which an iterative learning control scheme has been applied to the analysis and design of time–delay dynamical systems. In Ref. (8), for example, the design of an iterative learning controller is considered for a class of linear dynamical systems with time delay, and an iterative learning control algorithm is proposed such that the output of the considered time–delay dynamical systems can track a given desired trajectory. In Ref. (9), a class of proportional–integration–derivative (PID)–type iterative learning control schemes is proposed for uncertain nonlinear dynamical systems with state delays, and the convergence conditions for the proposed high–order iterative learning control are derived. However, few efforts are made to consider the problem of iterative learning control for large scale dynamical systems. It seems that for large scale dynamical systems, the similar results have not been reported yet in the control literature.

It is well known that a large scale system can be characterized by a large number of variables representing system, a strong interaction between the system variables, and a complex structure. In particular, a large scale system is often considered as a set of interconnected subsystems, and referred to as large scale interconnected systems. The advantage of this aspect in controller design is to reduce complexity and this therefore allows the control implementation to be feasible. Therefore, the problem of decentralized control of large scale interconnected dynamical systems has also received considerable attention, and many approaches have been developed to synthesize some types of decentralized local state (or output) feedback controllers (see, e.g., Refs. (10)∼(14), and the references therein). Thus, it is obviously meaningful to apply the iterative learning control strategy to large scale interconnected dynamical systems, and to develop some types of decentralized local iterative learning control schemes.

In this paper, we consider the problem of decentralized iterative learning control for a class of linear time–invariant large scale interconnected dynamical systems. We want to show that the method of iterative learning control can be applied to such large scale interconnected dynamical systems. In particular, for such large scale interconnected systems, we propose a class of decentralized local iterative learning control schemes. We also show that under given conditions, the proposed decentralized local iterative learning controllers can guarantee the asymptotic convergence of the local output error between the given desired local output and the actual local output of each subsystem through the iterative learning process.

The paper is organized as follows. In Section 2, the decentralized iterative learning control problem to be tackled is stated and some standard assumptions are introduced. In Section 3, we propose a class of decentralized local iterative learning control schemes for large scale interconnected systems. In Section 4, a numerical example is given to illustrate the use of our results. The paper is concluded in Section 5 with a brief discussion of the results.

2. Problem Formulation and Assumptions

Consider a class of large scale systems S composed

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of N interconnected subsystems S_i, $i = 1, 2, \ldots, N$, described by

$$\frac{dx_i(t)}{dt} = A_i x_i(t) + B_i u_i(t) + \sum_{j=1}^{N} A_{ij} x_j(t) \quad \cdots\cdots (1a)$$

$$y_i(t) = C_i x_i(t) \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (1b)$$

where $t \in R^+$ is the time, $x_i(t) \in R^{n_i}$ is the state vector, $u_i(t) \in R^{m_i}$ is the control (or input) vector, $y_i(t) \in R^{l_i}$ is the output vector, A_i, B_i, C_i are constant matrices of appropriate dimensions, and the matrix A_{ij} accounts for the interconnection between the subsystems S_i and S_j, which is assumed to be unknown constant matrices of appropriate dimensions. In addition, $x(t) \in R^n$ denotes $\begin{bmatrix} x_1^\top(t) & x_2^\top(t) & \cdots & x_N^\top(t) \end{bmatrix}^\top$, where $n = n_1 + n_2 + \cdots + n_N$, and the initial state $x_i(t_0)$ for each subsystem is assumed to be unknown.

For each subsystem S_i, $i \in \{1, 2, \ldots, N\}$, it is supposed that a desired local output trajectory $y_i^m(t) \in R^{l_i}$ is given for a finite time interval $t \in [t_0,\ T]$. Then, the error between the desired local output and the actual local output trajectories of each subsystem can be represented by

$$e_i(t) \;=\; y_i^m(t) - y_i(t), \quad i \in \{1, 2, \ldots, N\} \quad \cdots\cdots (2)$$

where $t \in [t_0,\ T]$ and $e_i(t) \in R^{l_i}$.

Throughout this paper, we use the superscript k to denote the iteration number of learning. Therefore, $x_i^{[k]}(t)$, $u_i^{[k]}(t)$, $y_i^{[k]}(t)$ represent the corresponding vectors at the kth iteration.

Now, the main objective of this paper is to find the decentralized local iterative learning control laws for each subsystems with unknown local desired initial state $x_i^m(t_0)$ such that the local output error $e_i(t)$ between the given desired local output $y_i^m(t)$ and the actual local output $y_i(t)$ is identical for all $t \in [t_0,\ T]$, through the iterative learning process. That is, starting from an arbitrary continuous local initial control input $u_i^{[0]}(t)$ and an arbitrary local initial state $x_i^{[0]}(t_0)$, we want to obtain the next local control input $u_i^{[1]}(t)$ and local initial state $x_i^{[1]}(t_0)$, and the subsequent series $\left\{u_i^{[k]}(t),\ x_i^{[k]}(t_0);\ k = 2, 3, \ldots\right\}$ for ecah subsystem such that for all $t \in [t_0,\ T]$,

$$\lim_{k\to\infty} \left\| e_i^{[k]}(t) \right\| \;=\; 0 \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (3a)$$

and

$$\lim_{k\to\infty} \left\| x_i^m(t_0) - x_i^{[k]}(t_0) \right\| \;=\; 0 \quad \cdots\cdots\cdots\cdots\cdots (3b)$$

where $i \in \{1, 2, \ldots, N\}$.

Before giving our decentralized iterative learning control laws, we introduce for large scale system (1) the following standard assumptions.

Assumption 2.1 For each subsystem S_i, $i \in \{1, 2, \ldots, N\}$, the desired local output trajectory $y_i^m(t)$ is continuous differentiable vector function on $[t_0,\ T]$.

Assumption 2.2 For each $i \in \{1, 2, \ldots, N\}$, the matrix $C_i B_i$ is full rank.

Remark 2.1 It is obvious that Assumption 2.1 is standard, and by this assumption we means that one wants for each subsystem to track a continuous output trajectory. Moreover, it is possible from Assumption 2.1 that a class of derivative–type decentralized local iterative learning control laws is designed for each subsystem. Assumption 2.2 guarantees the existence of decentralized iterative learning control laws, which will be known from the conditions derived in the next sections.

Throughout this paper, $\| \cdot \|$ denotes any vector (or matrix) norm for any vector (or matrix).

3. Decentralized Learning Control

In this section, for the problem stated in Section 2, we propose a local input updating law for decentralized iterative learning control as follows.

$$u_i^{[k+1]}(t) \;=\; u_i^{[k]}(t) + \Gamma_i^{[k]}\, \dot{e}_i^{[k]}(t) \cdots\cdots\cdots\cdots\cdots (4)$$

where for each $i \in \{1, 2, \ldots, N\}$, $\Gamma_i \in R^{m_i \times l_i}$ is an iterative learning control gain matrix which will be determined later, together with an initial state learning algorithm described by

$$x_i^{[k+1]}(t_0) \;=\; x_i^{[k]}(t_0) + B_i \Gamma_i^{[k]}\, e_i^{[k]}(t_0) \cdots\cdots\cdots (5)$$

where $t \in [t_0,\ T]$, $u_i^{[0]}(t)$ is an arbitrary continuous initial control input, and $x_i^{[0]}(t_0)$ is an arbitrary initial state, which may be different from the unknown desired initial state $x_i(t_0)$ for each dynamical subsystem.

Then the following theorem can be obtained which shows that the decentralized local iterative learning control laws given in (4) with (5) can guarantee the asymptotic convergence of the local output error of each dynamical subsystem S_i.

Theorem 3.1 Consider the large scale interconnected systems described by (1a) and (1b) which satisfies Assumption 2.2. Given the desired local output trajectory $y_i^m(t)$, which satisfies Assumption 2.1, over the finite time interval $[t_0,\ T]$, by employing the decentralized local iterative learning control law described by (4) and the initial state learning algorithm described by (5), the local output error $e_i(t)$ of each subsystem can be guaranteed to asymptotically converge to zero, i.e. for each $i \in \{1, 2, \ldots, N\}$ and any $t \in [t_0,\ T]$,

$$\lim_{k\to\infty} e_i^{[k]}(t) \;=\; \lim_{k\to\infty} \left(y_i^m(t) - y_i^{[k]}(t) \right) = 0 \quad \cdots\cdots (6)$$

if there exists an iterative learning control gain matrix Γ_i such that

$$\left\| I_i - C_i B_i \Gamma_i \right\| \;<\; 1, \quad i \in \{1, 2, \ldots, N\} \quad \cdots\cdots\cdots (7)$$

where $I_i \in R^{l_i \times l_i}$ is an identity matrix.

Proof: For any given local control input $u_i(t)$, $t \in [t_0,\ T]$, the general solution $x_i(t)$ to each subsystem S_i, described by (1), can be written in the following form:

$$x_i(t) = \exp\{A_i(t-t_0)\}\, x_i(t_0) + \int_{t_0}^{t} \exp\{A_i(t-\tau)\}\, B_i u_i(\tau) d\tau + \sum_{j=1}^{N} \int_{t_0}^{t} \exp\{A_i(t-\tau)\}\, A_{ij} x_j(\tau) d\tau \cdots (8)$$

where $\exp\{A_i(t-\tau)\}$ is the state transition matrix of the ith unforced isolated subsystem.

Thus, for the kth iteration, it can be known from (8) that for any $t \in [t_0,\, T]$,

$$x_i^{[k]}(t) = \exp\{A_i(t-t_0)\}\, x_i^{[k]}(t_0) + \int_{t_0}^{t} \exp\{A_i(t-\tau)\}\, B_i u_i^{[k]}(\tau) d\tau + \sum_{j=1}^{N} \int_{t_0}^{t} \exp\{A_i(t-\tau)\}\, A_{ij} x_j^{[k]}(\tau) d\tau \cdots\cdots (9)$$

Furthermore, by making use of (9) together with (4) and (5), the state error can be expressed as

$$x_i^{[k+1]}(t) - x_i^{[k]}(t) = \exp\{A_i(t-t_0)\}\, B_i \Gamma_i\, e_i^{[k]}(t_0) + \int_{t_0}^{t} \exp\{A_i(t-\tau)\}\, B_i \Gamma_i\, \dot{e}_i^{[k]}(\tau) d\tau + \sum_{j=1}^{N} \int_{t_0}^{t} \exp\{A_i(t-\tau)\}\, A_{ij} \times \left(x_j^{[k+1]}(\tau) - x_j^{[k]}(\tau)\right) d\tau \cdots\cdots (10)$$

By integrating the term $\dot{e}_i^{[k]}(\tau)$ in (10) by parts, we can obtain that for any $t \in [t_0,\, T]$,

$$x_i^{[k+1]}(t) - x_i^{[k]}(t) = B_i \Gamma_i\, e_i^{[k]}(t) + \int_{t_0}^{t} A_i \exp\{A_i(t-\tau)\}\, B_i \Gamma_i\, e_i^{[k]}(\tau) d\tau + \sum_{j=1}^{N} \int_{t_0}^{t} \exp\{A_i(t-\tau)\}\, A_{ij} \times \left(x_j^{[k+1]}(\tau) - x_j^{[k]}(\tau)\right) d\tau \cdots\cdots (11)$$

Taking the norm of both sides of (11) and making use of the general properties of norms, we can obtain that for any $t \in [t_0,\, T]$,

$$\left\|x_i^{[k+1]}(t) - x_i^{[k]}(t)\right\| \le \|B_i \Gamma_i\| \left\|e_i^{[k]}(t)\right\| + \int_{t_0}^{t} \|\exp\{A_i(t-\tau)\}\|\, \|A_i\|\, \|B_i \Gamma_i\| \left\|e_i^{[k]}(\tau)\right\| d\tau + \sum_{j=1}^{N} \int_{t_0}^{t} \|\exp\{A_i(t-\tau)\}\|\, \|A_{ij}\| \times \left\|x_j^{[k+1]}(\tau) - x_j^{[k]}(\tau)\right\| d\tau \cdots\cdots (12)$$

Moreover, letting

$$\mu_i := \sup_{t,\tau \in [t_0,\, T]} \|\exp\{A_i(t-\tau)\}\|, \quad i = 1, 2, \ldots, N$$

it follows from (12) that for any $t \in [t_0,\, T]$,

$$\left\|x_i^{[k+1]}(t) - x_i^{[k]}(t)\right\| \le \|B_i \Gamma_i\| \left\|e_i^{[k]}(t)\right\| + \mu_i \int_{t_0}^{t} \|A_i\|\, \|B_i \Gamma_i\| \left\|e_i^{[k]}(\tau)\right\| d\tau + \sum_{j=1}^{N} \mu_i \int_{t_0}^{t} \|A_{ij}\| \left\|x_j^{[k+1]}(\tau) - x_j^{[k]}(\tau)\right\| d\tau \cdots\cdots (13)$$

By multiplying both sides of (13) by $\exp\{-\gamma(t-t_0)\}$ where γ is any positive constant, and by making use of some trivial manipulations, we can easily obtain that for any $t \in [t_0,\, T]$,

$$\left\|x_i^{[k+1]}(t) - x_i^{[k]}(t)\right\| \exp\{-\gamma(t-t_0)\} \le \|B_i \Gamma_i\| \left\|e_i^{[k]}(t)\right\| \exp\{-\gamma(t-t_0)\} + \mu_i \int_{t_0}^{t} \|A_i\|\, \|B_i \Gamma_i\| \exp\{-\gamma(t-\tau)\} \times \left\|e_i^{[k]}(\tau)\right\| \exp\{-\gamma(\tau-t_0)\} d\tau + \sum_{j=1}^{N} \mu_i \int_{t_0}^{t} \|A_{ij}\| \exp\{-\gamma(t-\tau)\} \times \left\|x_j^{[k+1]}(\tau) - x_j^{[k]}(\tau)\right\| \times \exp\{-\gamma(\tau-t_0)\} d\tau \cdots\cdots (14)$$

Letting

$$\tilde{e}_i^{[k]}(t) := \left(\sup_{\rho \in [t_0,\, t]} \hat{e}_i^{[k]}(\rho)\right)$$

$$\tilde{z}_i^{[k]}(t) := \left(\sup_{\rho \in [t_0,\, t]} \hat{z}_i^{[k]}(\rho)\right)$$

where

$$\hat{e}_i^{[k]}(\rho) := \left\|e_i^{[k]}(\rho)\right\| \exp\{-\gamma(\rho-t_0)\}$$

$$\hat{z}_i^{[k]}(\rho) := \left\|x_i^{[k+1]}(\rho) - x_i^{[k]}(\rho)\right\| \exp\{-\gamma(\rho-t_0)\}$$

it follows from (14) that for any $i \in \{1, 2, \ldots, N\}$ and any $t \in [t_0,\, T]$,

$$\left\|x_i^{[k+1]}(t) - x_i^{[k]}(t)\right\| \exp\{-\gamma(t-t_0)\} \le \|B_i \Gamma_i\|\, \tilde{e}_i^{[k]}(t) + \mu_i \|A_i\|\, \|B_i \Gamma_i\|\, \tilde{e}_i^{[k]}(t) \int_{t_0}^{t} \exp\{-\gamma(t-\tau)\} d\tau + \sum_{j=1}^{N} \mu_i \|A_{ij}\|\, \tilde{z}_j^{[k]}(t) \int_{t_0}^{t} \exp\{-\gamma(t-\tau)\} d\tau$$
$$\le \|B_i \Gamma_i\| \left(1 + \mu_i \|A_i\| / \gamma\right) \tilde{e}_i^{[k]}(t) + (1/\gamma) \sum_{j=1}^{N} \mu_i \|A_{ij}\|\, \tilde{z}_j^{[k]}(t) \cdots\cdots (15)$$

First of all, notice such a fact that for any real function $f(t)$ and for any nondecreasing real function $g(t)$,

$$f(t) \le g(t), \quad t \in [t_0, T]$$

implies

$$\tilde{f}(t) := \left(\sup_{\rho \in [t_0, t]} f(\rho) \right) \le g(t), \quad t \in [t_0, T]$$

Now, it is obvious from the definitions that for any $i \in \{1, 2, \ldots, N\}$, the functions $\tilde{e}_i^{[k]}(t)$ and $\tilde{z}_i^{[k]}(t)$ are some nondecreasing functions on t. It follows that for any $i \in \{1, 2, \ldots, N\}$, the right–hand side of inequality (15) is also nondecreasing. Therefore, in the light of the definitions of the functions $\tilde{e}_i^{[k]}(t)$ and $\tilde{z}_i^{[k]}(t)$ and the fact stated above, we find from (15) that for any $i \in \{1, 2, \ldots, N\}$ and any $t \in [t_0, T]$,

$$\tilde{z}_i^{[k]}(t) \le \|B_i\Gamma_i\| \left(1 + \mu_i \|A_i\| / \gamma\right) \tilde{e}_i^{[k]}(t) + (1/\gamma) \sum_{j=1}^{N} \mu_i \|A_{ij}\| \tilde{z}_j^{[k]}(t) \cdots\cdots (16)$$

Moreover, if we define that for any $t \in [t_0, T]$,

$$\tilde{z}^{[k]}(t) := \max_i \left\{ \tilde{z}_i^{[k]}(t); \ i = 1, 2, \ldots, N \right\}$$

then, from (16) we can further have that for any $i \in \{1, 2, \ldots, N\}$ and any $t \in [t_0, T]$,

$$\tilde{z}_i^{[k]}(t) \le \|B_i\Gamma_i\| \left(1 + \mu_i \|A_i\| / \gamma\right) \tilde{e}_i^{[k]}(t) + \sum_{j=1}^{N} \left(\mu_i \|A_{ij}\| / \gamma\right) \tilde{z}^{[k]}(t) \cdots\cdots\cdots (17)$$

Similarly, since the right–hand side of inequality (17) is nondecreasing we can obtain that for any $t \in [t_0, T]$,

$$\tilde{z}^{[k]}(t) \le \|B_i\Gamma_i\| \left(1 + \mu_i \|A_i\| / \gamma\right) \tilde{e}_i^{[k]}(t) + \sum_{j=1}^{N} \left(\mu_i \|A_{ij}\| / \gamma\right) \tilde{z}^{[k]}(t) \cdots\cdots\cdots (18)$$

where γ is any positive constant. Letting γ be chosen such that for any $i \in \{1, 2, \ldots, N\}$,

$$\gamma - \mu_i \sum_{j=1}^{N} \|A_{ij}\| > 0$$

Then, we can find from (18) that for any $i \in \{1, 2, \ldots, N\}$ and any $t \in [t_0, T]$,

$$\tilde{z}^{[k]}(t) \le \frac{\gamma + \mu_i \|A_i\|}{\gamma - \mu_i \delta_i} \|B_i\Gamma_i\| \ \tilde{e}_i^{[k]}(t) \cdots\cdots\cdots (19)$$

where

$$\delta_i := \sum_{j=1}^{N} \|A_{ij}\|, \quad i = 1, 2, \ldots, N$$

On the other hand, in the light of the definition of the output error $e_i(t)$ and (11), we can have that for any $i \in \{1, 2, \ldots, N\}$ and any $t \in [t_0, T]$,

$$\begin{aligned} & e_i^{[k+1]}(t) \\ & = e_i^{[k]}(t) - C_i \left(x_i^{[k+1]}(t) - x_i^{[k]}(t) \right) \\ & = [I_i - C_i B_i \Gamma_i] \, e_i^{[k]}(t) \\ & \quad - \int_{t_0}^{t} C_i A_i \exp\{A_i(t-\tau)\} B_i \Gamma_i e_i^{[k]}(\tau) d\tau \\ & \quad - \sum_{j=1}^{N} \int_{t_0}^{t} C_i \exp\{A_i(t-\tau)\} A_{ij} \\ & \quad \times \left(x_j^{[k+1]}(\tau) - x_j^{[k]}(\tau) \right) d\tau \cdots\cdots\cdots\cdots\cdots (20) \end{aligned}$$

Taking the norm of both sides of (20) and making use of the general properties of norms, we can obtain that for any $t \in [t_0, T]$,

$$\begin{aligned} & \left\| e_i^{[k+1]}(t) \right\| \le \left\| I_i - C_i B_i \Gamma_i \right\| \ \left\| e_i^{[k]}(t) \right\| \\ & \quad + \int_{t_0}^{t} \|\exp\{A_i(t-\tau)\}\| \, \|C_i A_i\| \\ & \quad \times \|B_i \Gamma_i\| \left\| e_i^{[k]}(\tau) \right\| d\tau \\ & \quad + \sum_{j=1}^{N} \int_{t_0}^{t} \|\exp\{A_i(t-\tau)\}\| \, \|C_i\| \, \|A_{ij}\| \\ & \quad \times \left\| x_j^{[k+1]}(\tau) - x_j^{[k]}(\tau) \right\| d\tau \\ & \le \left\| I_i - C_i B_i \Gamma_i \right\| \ \left\| e_i^{[k]}(t) \right\| \\ & \quad + \mu_i \int_{t_0}^{t} \|C_i A_i\| \, \|B_i \Gamma_i\| \left\| e_i^{[k]}(\tau) \right\| d\tau \\ & \quad + \sum_{j=1}^{N} \mu_i \int_{t_0}^{t} \|C_i\| \, \|A_{ij}\| \\ & \quad \times \left\| x_j^{[k+1]}(\tau) - x_j^{[k]}(\tau) \right\| d\tau \cdots\cdots\cdots\cdots\cdots (21) \end{aligned}$$

Similar to the method employed above, by multiplying both sides of inequality (21) by $\exp\{-\gamma(t-t_0)\}$ and by making use of some trivial manipulations, we can also obtain from (21) that for any $i \in \{1, 2, \ldots, N\}$ and any $t \in [t_0, T]$,

$$\begin{aligned} & \left\| e_i^{[k+1]}(t) \right\| \exp\{-\gamma(t-t_0)\} \\ & \le \left\| I_i - C_i B_i \Gamma_i \right\| \left\| e_i^{[k]}(t) \right\| \exp\{-\gamma(t-t_0)\} \\ & \quad + \mu_i \int_{t_0}^{t} \|C_i A_i\| \, \|B_i \Gamma_i\| \exp\{-\gamma(\tau-t_0)\} \\ & \quad \times \left\| e_i^{[k]}(\tau) \right\| \exp\{-\gamma(t-\tau)\} d\tau \\ & \quad + \sum_{j=1}^{N} \mu_i \int_{t_0}^{t} \|C_i\| \, \|A_{ij}\| \exp\{-\gamma(\tau-t_0)\} \\ & \quad \times \left\| x_j^{[k+1]}(\tau) - x_j^{[k]}(\tau) \right\| \exp\{-\gamma(t-\tau)\} d\tau \\ & \le \left\| I_i - C_i B_i \Gamma_i \right\| \tilde{e}_i^{[k]}(t) \end{aligned}$$

$$+\frac{\mu_i}{\gamma}\|C_iA_i\|\|B_i\Gamma_i\|\tilde{e}_i^{[k]}(t)$$
$$+\frac{\mu_i}{\gamma}\sum_{j=1}^{N}\|C_i\|\|A_{ij}\|\tilde{z}_j^{[k]}(t)$$
$$\leq\left(\left\|I_i-C_iB_i\Gamma_i\right\|+\frac{\mu_i}{\gamma}\|C_iA_i\|\|B_i\Gamma_i\|\right)\tilde{e}_i^{[k]}(t)$$
$$+\frac{\mu_i}{\gamma}\sum_{j=1}^{N}\|C_i\|\|A_{ij}\|\tilde{z}^{[k]}(t)\cdots\cdots\cdots\cdots\cdots(22)$$

Furthermore, substituting (19) into (22) yields

$$\left\|e_i^{[k+1]}(t)\right\|\exp\{-\gamma(t-t_0)\}$$
$$\leq\left[\left\|I_i-C_iB_i\Gamma_i\right\|\right.$$
$$+\frac{\mu_i}{\gamma}\|B_i\Gamma_i\|\left(\|C_iA_i\|+\frac{\mu_i\delta_i\|C_i\|\|A_i\|}{\gamma-\delta_i}\right)$$
$$\left.+\frac{\mu_i\delta_i\|C_i\|\|B_i\Gamma_i\|}{\gamma-\delta_i}\right]\tilde{e}_i^{[k]}(t)\cdots\cdots\cdots\cdots(23)$$

It is obvious from the definition of $\tilde{e}_i^{[k]}(t)$ that the right–hand side of inequality (23) is nondecreasing on time t. Therefore, we find from (23) that for any $i\in\{1,2,\ldots,N\}$ and any $t\in[t_0,T]$,

$$\tilde{e}_i^{[k+1]}(t)\leq\left[\left\|I_i-C_iB_i\Gamma_i\right\|\right.$$
$$+\frac{\mu_i}{\gamma}\|B_i\Gamma_i\|\left(\|C_iA_i\|+\frac{\mu_i\delta_i\|C_i\|\|A_i\|}{\gamma-\delta_i}\right)$$
$$\left.+\frac{\mu_i\delta_i\|C_i\|\|B_i\Gamma_i\|}{\gamma-\delta_i}\right]\tilde{e}_i^{[k]}(t)$$

That is,

$$\tilde{e}_i^{[k+1]}(t)\leq\eta_i\,\tilde{e}_i^{[k]}(t),\quad i\in\{1,2,\ldots,N\}\cdots\cdots(24)$$

where

$$\eta_i:=\left\|I_i-C_iB_i\Gamma_i\right\|+\rho_i(\gamma)\cdots\cdots\cdots\cdots\cdots\cdots(25)$$

and where

$$\rho_i(\gamma):=\frac{1}{\gamma}\mu_i\|B_i\Gamma_i\|\|C_iA_i\|$$
$$+\frac{1}{\gamma}\mu_i\|B_i\Gamma_i\|\frac{\mu_i\delta_i\|C_i\|\|A_i\|}{\gamma-\delta_i}$$
$$+\frac{\mu_i\delta_i\|C_i\|\|B_i\Gamma_i\|}{\gamma-\delta_i}\cdots\cdots\cdots\cdots\cdots(26)$$

If the condition described by (7) is satisfied, it is obvious from (26) that there exists a positive constant γ^* such that for any $\gamma\geq\gamma^*$, $\eta_i<1$. Therefore, we can obtain from (24) that for any $t\in[t_0,T]$,

$$\lim_{k\to\infty}e_i^{[k]}(t)=0,\quad i\in\{1,2,\ldots,N\}$$

Thus, we can complete the proof of this theorem. ■

Remark 3.1 It is obvious from Assumption 2.2 that one can always choose a decentralized iterative learning control gain matrix Γ_i for each subsystem such that the condition given in (7) is satisfied. Therefore, under Assumption 2.2, the existence of decentralized local iterative learning control laws is well guaranteed for each dynamical subsystems.

Remark 3.2 In this paper, we have employed a class of D–type iterative learning control laws described by (4) with (5), which may regarded as the first–order updating laws. It is not difficult, from the method used in this paper, that the obtained result is extended to the problem of decentralized iterative learning control with high–order updating laws.

4. An Illustrative Example

In this section, we consider the two identical pendulums which are coupled by a spring and subject to two distinct inputs [15][16] as shown in Fig.1.

We choose the state vectors as

$$x_1(t)=\begin{bmatrix}\theta_1(t) & \dot{\theta}_1(t)\end{bmatrix}^{\mathrm{T}}$$
$$x_2(t)=\begin{bmatrix}\theta_2(t) & \dot{\theta}_2(t)\end{bmatrix}^{\mathrm{T}}$$

Then, the systems can be described by

$$\frac{dx_1(t)}{dt}=\begin{bmatrix}0 & 1\\ g/l & 0\end{bmatrix}x_1(t)+\begin{bmatrix}0\\ 1/(ml^2)\end{bmatrix}u_1(t)$$
$$+\begin{bmatrix}0 & 0\\ -ka^2/(ml^2) & 0\end{bmatrix}x_1(t)$$
$$+\begin{bmatrix}0 & 0\\ ka^2/(ml^2) & 0\end{bmatrix}x_2(t)$$
$$y_1(t)=\begin{bmatrix}1 & 1\end{bmatrix}x_1(t)\cdots\cdots\cdots\cdots\cdots\cdots(27a)$$

$$\frac{dx_2(t)}{dt}=\begin{bmatrix}0 & 1\\ g/l & 0\end{bmatrix}x_2(t)+\begin{bmatrix}0\\ 1/(ml^2)\end{bmatrix}u_2(t)$$
$$+\begin{bmatrix}0 & 0\\ ka^2/(ml^2) & 0\end{bmatrix}x_1(t)$$
$$+\begin{bmatrix}0 & 0\\ -ka^2/(ml^2) & 0\end{bmatrix}x_2(t)$$
$$y_2(t)=\begin{bmatrix}1 & 1\end{bmatrix}x_2(t)\cdots\cdots\cdots\cdots\cdots\cdots(27b)$$

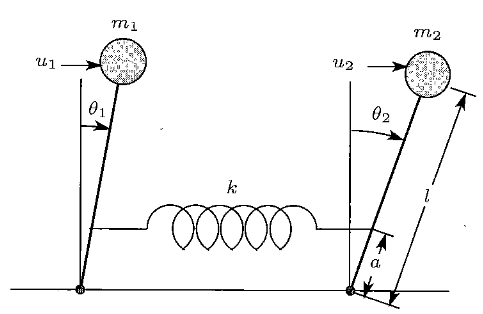

Fig. 1. The coupled inverted pendulums.

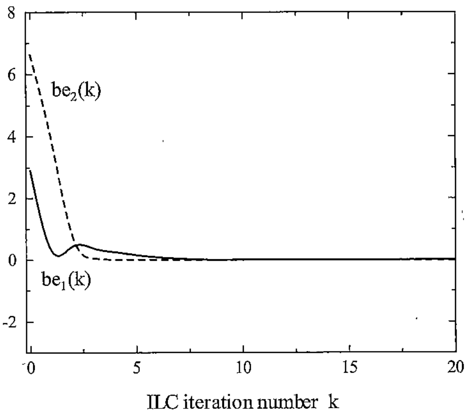

Fig. 2. The tracking error bounds.

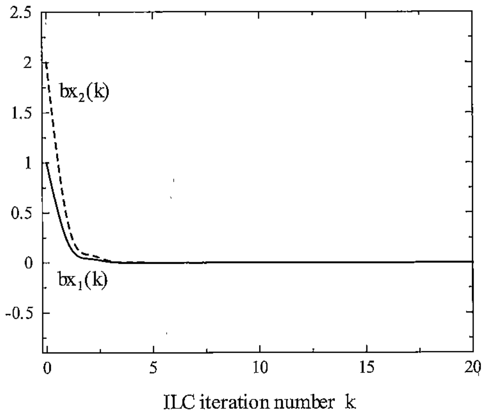

Fig. 3. The learning processes of initial states.

where k and g are spring and gravity constants. For simulation, we give the following parameters:

$$g/l = 1.0, \quad 1/(ml^2) = 1.0$$
$$k/m = 2.0, \quad a\,/\,l = 0.5$$

For the decentralized local iterative learning control laws given in (4) and (5), in the light of the condition described by (7), we can select the decentralized iterative learning cantrol gains as follows.

$$\Gamma_1 = 0.8, \quad \Gamma_2 = 0.9$$

In additions, for iterative schemes (4) and (5), we give in this simulation the following initial conditions.

$$u_1^{[0]}(t) = 1.0, \quad x_{11}^{[0]}(0) = 0.0, \quad x_{12}^{[0]}(0) = 1.0$$
$$u_2^{[0]}(t) = 1.0, \quad x_{21}^{[0]}(0) = 0.0, \quad x_{22}^{[0]}(0) = 2.0$$

For system (27), the desired local output trajectories $y_i^m(t)$, $i = 1, 2$, are given as follows.

$$y_1^m(t) = \sin(0.1t) \quad \cdots\cdots (28a)$$
$$y_2^m(t) = \sin(0.2t) \quad \cdots\cdots (28b)$$

where $t \in [0,\ T]$ and $T = 1.0$.

Here, let the final local tracking error for each dynamical subsystem be defined as

$$be_1(k) := \sup_{t\in[0,\ T]} \left|e_1^{[k]}(t)\right|$$
$$be_2(k) := \sup_{t\in[0,\ T]} \left|e_2^{[k]}(t)\right|$$

and for the learning processes of initial states, we define

$$bx_1(k) := \left\|x_1^{[k]}(t_0)\right\|, \quad bx_2(k) := \left\|x_2^{[k]}(t_0)\right\|$$

Then, the results of a simulation are shown in Fig.2 and Fig.3 for this coupled identical pendulum system with the chosen parameter settings.

It can be observed from Fig.2 that by using the proposed decentralized local iterative learning control laws, we can guarantee the asymptotic convergence of the local output error between the given desired local output and the actual local output for all $t \in [t_0,\ T]$. In particular, from Fig.3 we can also know that the decentralized local initial state learning schemes are effective. That is, the initial state for each subsystem tracks finally the desired one.

5. Concluding Remarks

The problem of decentralized iterative learning control for a class of large scale interconnected systems has been discussed. Here, the considered large scale systems have been assumed to be linear time–invariant, and the interconnections between each subsystem to be unknown. For such large scale interconnected systems, a class of decentralized local iterative learning control schemes is proposed. It has also been shown that the proposed decentralized local iterative learning controllers can guarantee the asymptotic convergence of the local output error between the given desired local output and the actual local output of each subsystem through the iterative learning process.

Finally, a numerical example is given to demonstrate the synthesis procedure for the proposed decentralized local iterative learning control schemes. It is shown from the example and the results of its simulation that the results obtained in the paper are effective and feasible. Therefore, our results may be expected to have some applications to practical decentralized iterative learning control problems of large scale interconnected systems.

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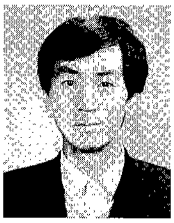

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